Apparent Weight of a Photon Box*

Revisiting Mass-Energy Equivalence

Sabarish V M

A photon has zero rest mass. However, it will be shown that when confined in a box, a photon can contribute to the mass of the system. Through this thought experiment, we will revisit the idea of mass-energy equivalence.

Sabarish V M finished his MSc at the Department of Theoretical Physics, University of Madras and is starting his graduate studies at RWTH Aachen University, Germany.

Introduction

Einstein's General Theory of Relativity predicts that photons must be affected by gravitational fields [1, 2], even though they have zero rest mass. It is well established that the frequency of a light wave as it moves away from a gravitational source is red-shifted with respect to the source. Using this result, it will be argued that a box with a trapped photon is heavier than an identical box without the photon.

For the thought experiment, a special case of a single photon bouncing vertically is considered for ease of calculations. An alternate approach has been dealt with by G.W.'t Hooft, et al [3]. Their calculations consider an accelerated box with $a = g$ and Lorentz transformations are used to obtain identical results.

The idea of mass-energy equivalence will be revisited with the help of the above-mentioned thought experiment. There is extensive literature on the origins and the interpretation of the equation $E = mc^2$ [4, 5].

Keywords
Photon box, mass-energy equivalence.

1. Mathematical Section

Consider a box with mirrored walls whose mass is m and let the box be placed on a weighing scale in the lab. Let a photon enter

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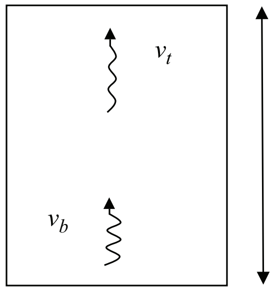

Figure 1. Box with a bouncing photon

For a photon inside a box, due to the gravitational redshift, the momentum of the photon at different distances from the bottom of the box will be different.

the box through the bottom face with a frequency ν_b. The photon bounces back and forth vertically as shown in *Figure* 1. We learn, for instance from [6] that the frequency of a photon in a gravitational field is red-shifted and in our case the potential at the top of the box is gL (taking the potential to be zero at the bottom of the box), therefore, the frequency shift is given by,

$$\nu_b = \nu_t \left[1 + \frac{gL}{c^2}\right]. \tag{1}$$

Here, ν_b is the frequency at the bottom of the box while ν_t is the measured frequency at the top end of the box. We can see from (1) that $\nu_b > \nu_t$, i.e. the photon is red-shifted. Inverting the above equation and retaining terms up to first order in $1/c^2$ in the binomial expansion, we get

$$\nu_t \approx \nu_b \left[1 - \frac{gL}{c^2}\right]. \tag{2}$$

The difference in momentum leads to a difference in the force exerted by the photon upon reflection on the top and the bottom mirror.

The momentum of a photon is given by $p = h\nu/c$. Therefore, the momentum transferred when the photon bounces off the top and bottom mirrors is given by,

$$\Delta p_b = 2h\frac{\nu_b}{c} \quad ; \quad \Delta p_t = 2h\frac{\nu_t}{c} \tag{3}$$

In a time interval Δt, the number of collisions of the photon on each face is given by,

$$n = \frac{c\Delta t}{2L}. \tag{4}$$

The forces acting on the box are the usual gravitational force of the Earth on the empty box plus the rate of change of momentum due to collisions of the photon with the walls. Therefore,

$$F = mg + \frac{\Delta p_b n}{\Delta t} - \frac{\Delta p_t n}{\Delta t}. \quad (5)$$

Substituting (2), (3) and (4) in (5),

$$F = \left(m + \frac{h\nu_b}{c^2}\right) g. \quad (6)$$

2. Results

This implies that the weight of the box registered on the weighing scale will be equal to value given in (6). Therefore the box has an increased apparent weight due to the photon that is contained. Though the photon has zero rest mass, the system as a whole does include an additional term $m' = h\nu_b/c^2$.

The weight of the box with the photon is different from that of an empty one.

Since an accelerating frame of reference is equivalent to a frame in a gravitational field [7], we will get the same results when we try to accelerate the box, i.e. the box will have more inertia.

3. Mass-Energy Equivalence

When the mass of a system is measured, even though the system is at rest, its constituents can be in relative motion. Therefore in our case, the total energy of the photon box is $E_{\text{total}} = mc^2 + h\nu_b$, i.e. mass energy of the box plus the energy of the photon. We observe that the total energy (E_{total}) divided by c^2 is the augmented mass found in the previous section, see (6).

The following examples also illustrate the fact that when the mass of a system is measured, the energy due to the internal dynamics also contributes to the mass.

A person pushing the box (in the absence of gravitational field) from the bottom or the top will find the box to have more inertia than that of an empty one.

1. Mass of a hydrogen atom is less than the mass of its constituents, i.e. a proton and an electron and this is due to the negative Coulombic potential energy.

2. Mass of a proton is approximately 938 MeV/c^2, but its constituents (three quarks) contribute very little to the proton's mass

and it is the quantum chromodynamic binding energy that is responsible for the remainder [8].

3. When a box of gas particles is weighed, their kinetic energy and the potential energy due to their interactions also contribute to the mass of the system. Even though the contributions are negligible, it is still non zero [9].

4. An amusing example would be, "A hot potato is heavier than a cold potato, and a compressed spring is heavier than a relaxed spring" [10].

4. Conclusion

1. We see that even though a photon is massless, it still contributes to the mass of the system that contains it.

2. The other examples given in the previous section also illustrate the fact that the energy due to the internal dynamics of the system also contributes to its mass.

Suggested Reading

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Address for Correspondence
Sabarish V M
Department of Theoretical Physics
University of Madras
Guindy Campus, Chennai 600025
Email:
sabarish.vm@gmail.com