A Window into the World of Nucleon-nucleon Interactions*

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An article in *Nature* [1] reports "evidence of mirror-symmetry violation in bound nuclear ground states with the mirror partners strontium-73 and bromine-73", wherein it is found that "a $J^\pi = 5/2^-$ spin assignment is needed to explain the proton-emission pattern observed from the $T = 3/2$ isobaric analogue state in rubidium-73, which is identical to the ground state of strontium-73. Therefore, the ground state of strontium-73 must differ from its $J^\pi = 1/2^-$ mirror bromine-73." The authors add that "this observation offers insights into charge-symmetry-breaking forces acting in atomic nuclei." This article expands a little on the various terms present in the quote above.

Speaking of states: J is the total angular momentum quantum number. π is the parity. So the notation $5/2^-$ refers to $J = 5/2$ and odd parity as explained below. In classical mechanics, we specify the state of motion of, say a planet, by its total energy, momentum, and rotational and orbital angular momentum. In the quantum world, attributes are quantized. So we speak in terms of the number of quanta. Also, there are additional attributes like spin and parity. Changes of state will be specified by selection rules ($\nabla J = \pm 1$ etc.)—which is the way conservation laws appear when we are talking of quanta. Parity was introduced in the early days of quantum mechanics, and it refers to evenness/oddness of the wave function under a change of sign of an odd number of the Cartesian coordinates. In three dimensions, we can write parity $\pi = \pm 1$ as per $\psi(-\mathbf{x}) = \pm\psi(\mathbf{x})$ [2]. In the case reported, the ground states of $^{73}_{38}$Sr and $^{73}_{35}$Br have the same parity, but their angular momentum is different.

Isobaric nuclei: Their atomic mass numbers are the same; they possess equal numbers of nucleons. The nuclei of bromine-73 ($^{73}_{35}$Br), rubidium-73 ($^{73}_{37}$Rb) and strontium-73 ($^{73}_{38}$Sr) are isobaric to each other. In the experiment, strontium-73 first emits a positron to become rubidium-73 which then emits a proton to become krypton-72 in what is called a β-delayed proton-emission process. Rubidium-73 appears in what is called an isobaric analogue state (IAS). IASs are the states in isobars that have the same space and spin wave functions due to charge independence of the strong part of the nucleon-nucleon interaction. These states appear at identical excitation energies, but for the Coulomb interaction. It may be noted that β-decay of $^{73}_{38}$Sr to $^{73}_{37}$Rb would preferentially populate the corresponding IAS in $^{73}_{37}$Rb; it does not guarantee the state in $^{73}_{37}$Rb to be the IAS of $^{73}_{38}$Sr.

Mirror nuclei: Isobaric nuclei which have their proton and neutron numbers switched are called mirror nuclei. Why are they called mirror nuclei?

Mirror-symmetry: In what sense are the mir-

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ror nuclei mirroring each other? To understand that take a top and set it to twirl—in front of a mirror (not *on* it!). The coordinate perpendicular to the surface of the mirror is sign-reversed in the image. Also, using the right-hand rule, it may be seen that the direction of the rotational angular momentum of the reflection is flipped with respect to that of the actual top, i.e., up becomes down/vice versa. It was noticed via experimental results and their analysis that, at the atomic and particle level, there exists a quantized degree of freedom called spin (key names for googling—Stern and Gerlach, Uhlenbeck and Goudsmidt). The wave function for the electron, for example, is not a function of just (x, y, z, t) but of (x, y, z, t, ζ), where the variable ζ takes on only one of two values–up/down. In the case of nucleons too, there are just two possibilities—proton/neutron. Hence the same formalism as for electrons may be used and [1] "Analogously to electrons in atoms exhibiting spin symmetries, it is possible to consider neutrons and protons in the atomic nucleus as projections of a single fermion with an isobaric spin (isospin) of $t = 1/2$". In the internal isospin space, a nucleon with isospin projection $t_3 = \pm 1/2$ is the proton/neutron, like in spin space, an electron with $s_z = \pm\frac{1}{2}$ is the up/down electron. Spin behaves like a vector, and can thus be expressed in terms of three components. The operators for the electron spin and its components (S, S_x, S_y, S_z) have the same algebra as that of angular momentum and its components (J, J_x, J_y, J_z) (the product is anticommutative and the algebra is what is called a Lie algebra as opposed to say real algebra where the product is commutative) (key names for googling—Wolfgang Pauli, Paul Dirac). Similar considerations work for the operators for the total isospin T and its components (T, T_1, T_2, T_3) too (key names for googling—Werner Heisenberg, Eugene Wigner). Isospin is like spin in that their operators obey the same algebra. The difference between them is that, unlike spin, isospin is a dimensionless quantity. Recalling the twirling top and its image, an electron with an up spin can be *looked at* as the mirror image of an electron with a down spin (s_z is sign reversed). (Looked at rather than *considered* because we are talking about a quantum mechanical attribute that is internal to the particle; and it has no connection with spatial coordinates or linear momentum the way angular momentum has). In like fashion, neutrons and protons may be looked at as mirror images of each other in isospin space (t_3 is sign reversed). And that explains the terminology 'mirror nuclei'. Since neutron and proton scattering experiments suggest that the interaction forces between nucleons seem to be identical except for the difference in electric charge, mass and magnetic moment between the n and the p, nucleonic forces are said to have charge-symmetry aka mirror-symmetry.

Symmetries and conservation laws: One of the cornerstones of physics is Noether's theorem (key name for googling—Emmy Noether). The theorem states that symmetries of a physical system manifest themselves in experiments as conservation laws. Thus [1], "Conservation laws are deeply related to any symmetry present in a physical system. Every nuclear state is thus characterized by a total isobaric spin T and a projection T_z—two quantities that are largely conserved in nuclear reactions and decays. A mirror symmetry emerges from this isobaric-spin formalism:

nuclei with exchanged numbers of neutrons and protons, known as mirror nuclei, should have an identical set of states, including their ground state, labelled by their total angular momentum J and parity π." Since isospin is observed to be a conserved quantity, the isobaric spin need not enter into the specification of the ground state (which has total energy zero) for mirror nuclei.

The experiment: The scientists were working with ^{73}Sr. They were looking to study ^{73}Rb, which is extremely short-lived. Their concerns were originally from astronomy—they were looking to see whether ^{73}Rb might be involved in the large energy emission happening in what are called Type I X-ray outbursts. ^{73}Rb is close to the proton drip line. Neutron/proton rich nuclei close to nuclear drip lines, beyond which n/p (whichever highly outnumbers the other compared to their relative numbers in the stable isotopes) starts practically dripping out of the nucleus, are formed/drive evolution in various astrophysical scenarios. An implantation detector with silicon detection plates was used. A beam of nuclei that includes ^{73}Sr is shot at the detector. Nuclei, as they move into the detector, are identified. The ions get implanted in the detector. The locations of the ^{73}Sr nuclei that get implanted may be ascertained. ^{73}Sr will decay into ^{73}Rb*(IAS) by the emission of a positron. The positron will annihilate with an electron, emitting two gamma rays which have energies not too far from the energy equivalent of the electron mass. They come out in practically opposite directions to conserve linear momentum. The gamma rays are detected by gamma-ray counters which surround the implantation detector. ^{73}Rb is very proton-rich and very unstable (stable isotopes of bromine, rubidium and strontium have mass numbers that lie between 79 and 88). It decays soon by emitting a proton.

The half-life of strontium-73 is determined by noting and analysing the times from implantation till the gamma ray detection. The emitted proton (identified by time coincidence with the gamma ray detection), and its energy is determined.

Positron and proton energies were determined for all the 427 identified ^{73}Sr implantations. The plots, for the number of proton detections versus energy and the number of gamma-ray detections versus energy, showed two peaks. Thus two channels were inferred for the β-delayed proton-emission events starting with ^{73}Sr. Theoretical modelling was done, treating the ^{73}Rb*(IAS) as a deformed core (krypton-72) plus a valence proton. It was found that [1] "a $J^{\pi} = 5/2^{-}$ spin assignment is needed to explain the proton-emission pattern observed from the $T = 3/2$ isobaric analogue state in rubidium-73, which is identical to the ground state of strontium-73."

Symmetries, the Hamiltonian and the energy states: A square has certain symmetries; if it is held down at the centre and rotated in its plane by 90/180/270/360 degrees it looks the same; likewise on reflection about the diagonals / the perpendicular bisectors of the sides. Physical systems can also have symmetries. For example, the behaviour of a simple harmonic oscillator (SHO) does not change if, with other factors remaining the same, the position of the oscillator is shifted. The Hamiltonian for the SHO reflects this symmetry. $H(x, y, z, t) = \frac{1}{2}\mathbf{p}^2/m + \frac{1}{2}k\mathbf{d}^2$ is unchanged if the position vector **r** is changed to **r**+**a**. **d**, the displacement vector, is the difference of two position vectors and is hence independent of the origin chosen for the coordinate system. $\boldsymbol{p} = md\boldsymbol{r}/dt$ is unchanged since $d(\boldsymbol{r}+\boldsymbol{a})/dt = d\boldsymbol{r}/dt$. And likewise, the Hamiltonian operator for the SHO $\hat{H} = \frac{-1}{2m}\hbar^2\,\nabla^2 + \frac{1}{2}k\mathbf{d}^2$ also reflects the symmetry, being unchanged under spatial translations:

$$\frac{\partial}{\partial(x+a)} = \frac{\partial}{\partial x}\frac{\partial x}{\partial(x+a)} = \frac{\partial}{\partial x},$$

etc. Thus if it is all the same for the nuclear forces whether the nucleus is made of $(A-Z)$ neutrons and Z protons or Z neutrons and $(A-Z)$ protons, the Hamiltonian for the nucleus should not involve the operator for the total isospin / its component; i.e. T/T_z. Hence, isobars should have identical states since T and T_z don't appear in the Hamiltonian. In particular, since now ground states (which have total energy zero) are distinguished only by the total angular momentum (J) and the parity (π), mirror nuclei should have the same J^π. However, it may be noted that, perturbations to the Hamiltonian due to Coulomb contributions and the np mass difference can affect this conclusion.

Here, what is unusual and was unexpected is that the J^π of the bound, ground state of strontium-73, as ascertained by modelling the experimental results is $5/2^-$, while that of the bound ground state of its mirror bromine-73 is known to be $1/2^-$. In the only other known case of isobars differing in the J^π of their ground states ${}^{16}_{9}\mathrm{F}/{}^{16}_{7}\mathrm{N}$, one of the mirror partners was in a proton unbound state. Since, as per the experiment, two bound mirror nuclei seem to differ in their J^π, it is said in view of Noether's theorem that [1], "This observation offers insights into charge-symmetry-breaking forces acting in atomic nuclei." The concerned researchers are now looking to make direct measurements of the spin of ${}^{73}\mathrm{Sr}$ (by β-NMR or similar methods). Maybe, the nuclei differ in shape. Maybe, it is a case of an unbound valence nucleon. The present discovery will certainly help in deciphering the subtleties of the nucleon-nucleon interaction.

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Suggested Reading

[1] D E M Hoff, A M Rogers, S M Wang, *et al.*, Mirror-symmetry violation in bound nuclear ground states, *Nature*, 580, 52, 2020.

[2] P C Deshmukh and J Libby, Symmetry Principles and Conservation Laws in Atomic and Subatomic Physics – 2, *Resonance: journal of science education*, Vol.15, No.10, pp.926–940, 2010.

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