Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

A Single Measurement Method to Find Refractive Index*

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The article discusses a simple method to find the angle of minimum deviation of a general prism, not necessarily equilateral or isosceles. The method uses a single measurement and requires a compass, ruler, some pins, and no additional material. It is precise and accurate. We present the method first and then the justification.

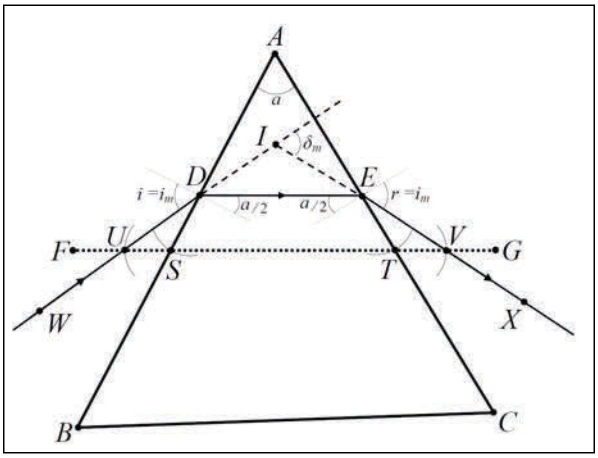
Introduction

Among the various methods to find the refractive index of a transparent material, the minimum deviation method is a simple and preferred one. Also, finding the angle of minimum deviation is a very popular high school experiment. In the existing method, for a prism with a known apex angle, the angle of deviation for various angles of incidence are measured and plotted on a graph to find the minimum. Here we point out how this method can introduce inaccuracy and also suggest a simpler method that requires only one single measurement, increasing precision and accuracy.

Keywords
Prism, refraction, deviation, incidence, angle of minimum deviation.

*Vol.25, No.4, DOI: https://doi.org/10.1007/s12045-020-0972-4

Figure 1. Minimum deviation.



Method

Refer to *Figure* 1. Let ABC be a prism with apex angle a at vertex A and refractive index μ. Trace the outline of the prism '$\triangle ABC$' on a sheet of paper. Remove the prism and take a measure of length less than the smaller of the sides $\overline{AC}$ and $\overline{AB}$ in a compass. With this length and A as the center, draw arcs cutting $\overline{AB}$ and $\overline{AC}$ at S and T respectively. So $\overline{AS} = \overline{AT}$. Draw a line $\overline{FG}$ passing through S and T. Again using the compass with S and T as centers draw arcs cutting $\overline{FG}$ at U and V respectively so that $\overline{SU} = \overline{TV}$ and the points U and V are nearer to the prism. Mount pins vertically at U and V. Place the prism back coinciding with the outline of $\triangle ABC$. By changing the angle of view through the side AC, align the pin-V with the image of pin-U seen through the side AC. For this same angle of view through side, AC, mount a third pin behind (or in front of) pin-U such that all the three pins, i.e., the pin-V the image of pin-U and the image of the third pin, are aligned. Mark the position of the third pin as W. Remove the prism and draw a line passing through U and W intersecting the triangle at D. Draw a perpendicular at D to the line AB, which will also be the normal at D. Measure the angle of incidence, we assure that this will yield the incident angle i_m corresponding to the angle of minimum deviation δ_m. Then $\delta_m = 2i_m - a$ and the

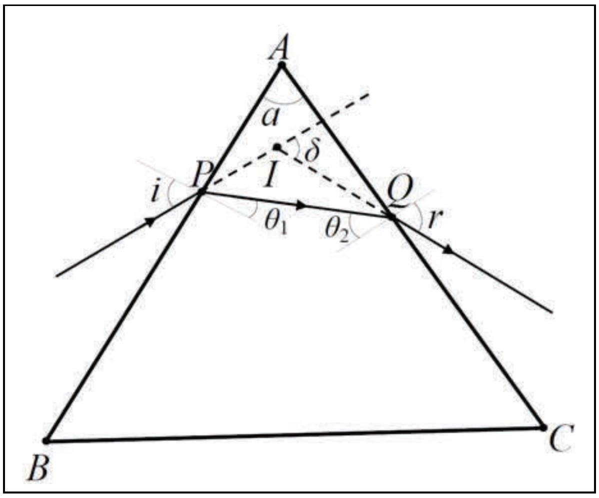


Figure 2. Angular deviation.

refractive index is given by $\mu = \sin([\delta_m + a]/2)/\sin(a/2)$.

To confirm that this method gives δ_m, keep back the prism and viewing through side $\overline{AB}$ place a fourth pin behind (or in front of) pin-V so that the image of the fourth pin is aligned with pin-W, pin-U, and the image of pin-V. Mark the position of this fourth pin as X. Draw a line passing through V and X intersecting $\overline{AC}$ at E. Measure the angle of refraction r at E to find it equal to i_m, the condition for minimum deviation. Now let us understand how the above method gives the angle of minimum deviation.

Relation between the incident angle *i* and the angle of deviation *dl*

Refer to *Figure* 2. A ray of light with incident angle i to the normal at P is refracted with an angle θ_1 inside the prism. After refraction, it makes an incident angle θ_2 to the normal at Q and is again refracted with an angle of emergence r out of the prism. The extension of the incident ray and the emergent ray intersect inside the prism at I. Since in $\triangle IPQ$, $\delta = \angle IPQ + \angle IQP = i - \theta_1 + r - \theta_2$ and in $\triangle APQ$, $a + \angle APQ + \angle AQP = 180°$ with $\angle APQ + \theta_1 = 90° = \angle AQP + \theta_2$, we get

$$a = \theta_1 + \theta_2 \quad \text{and} \quad \delta = i + r - a. \qquad (1)$$

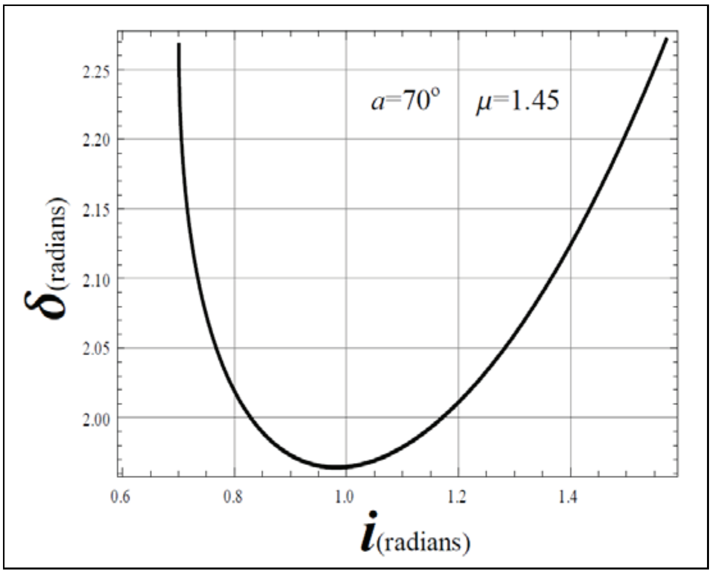


Figure 3. Deviation (δ) vs. Incident (i).

At the points of refraction P and Q, we have

$$\sin i = \mu \sin\theta_1 \quad \text{and} \quad \sin r = \mu \sin\theta_2. \tag{2}$$

Substituting for $\sin\theta_2$ from (1) in the above equation $\sin r = \sin a\, \mu \cos\theta_1 - \cos a\, \mu \sin\theta_1$. Using this relation in (1) gives:

$$\delta = i - a + \sin^{-1}\left[\sin a \sqrt{\mu^2 - \sin^2 i} - \cos a \sin i\right].$$

The usual misconception is that the relation between i and δ is quadratic.

The usual misconception is that the relation between i and δ is quadratic. But from the above equation, we can see that it is not quadratic. For example, plotting δ Vs i for $a = 70°$, $\mu = 1.45$ (refer to *Figure* 3) clearly shows the highly non-linear nature of the relation. So despite δ_m being unique for a given prism, when found graphically by plotting the values of i and δ from an experiment, there is a possibility that at different times one may find a variation. The reason is, had the relation between δ and i been quadratic, a minimum of three points would be sufficient to uniquely define the curve and hence the point of minimum. But due to the highly non-linear nature of the relation, trying to locate the minimum of the curve with few experimental points is the reason for the variation. Further, the probability to experimentally observe the point of minimum deviation is very low since this can happen only if by chance, while varying, i takes the value corresponding to δ_m. So to locate δ_m with more accuracy, several experimental points around and nearer to the point of minimum is required, that too in very small steps. But the best would be to devise a technique to make the minimum an experimentally observable point. To do that we first require to understand what happens at the minimum.

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Condition for the Angle of Minimum Deviation δ_m

Most books on optics discuss the minimum deviation in a prism, but few give all the details like in Eugene Hecht's book [1]. We present the derivation here to make this note complete and also one need not then look for a reference. Using $d\delta/di = 0$, the condition for minimum, in (1) gives:

$dr/di = -1$ and $d\theta_1/di = -d\theta_2/di$.

From (2) we get:

$\cos i = \mu \cos\theta_1 (d\theta_1/di)$ and $\cos r (dr/di) = \mu \cos\theta_2 (d\theta_2/di)$.

Taking the ratio of the above equations and squaring gives:

$$\cos^2 i \cos^2 \theta_2 = \cos^2 r \cos^2 \theta_1. \tag{3}$$

Squaring (2) we get:

$\mu^2 \cos^2\theta_1 = \mu^2 - 1 + \cos^2 i$ and $\mu^2 \cos^2\theta_2 = \mu^2 - 1 + \cos^2 r$.

Substituting these in (3) and simplifying gives:

$$\cos^2 i(\mu^2 - 1) + \cos^2 i \cos^2 r = \cos^2 r(\mu^2 - 1) + \cos^2 r \cos^2 i \Rightarrow \cos^2 i = \cos^2 r.$$

Since i can vary only from 0° to 90° and similarly for r, we get:

$$i = r \Rightarrow \theta_1 = \theta_2$$

as the condition for the minimum. This condition can also be easily achieved by doing ray tracing if we have the knowledge that there is only one minimum. Since it is observed from the graph in *Figure* 3 that there is only one minimum, if $i \neq r$ then there will be two angle of incidences for which δ is minimum, a contradiction so $i = r$. Let i_m be the incident angle for δ_m. Since $\theta_1 = \theta_2$ from eq(1), $\theta_1 = a/2 = \theta_2$ and $i_m = (\delta_m + a)/2$. Substituting in (2) gives

$$\delta_m = 2\sin^{-1}\left[\mu \sin\frac{a}{2}\right] - a,$$

which is a constant for a given prism. So when $i = i_m$ we see the refracted ray inside the prism forms an isosceles triangle with the vertex A and emerges with $r = i_m$. It is this property we have used, to devise this technique to experimentally observe the minimum.

Explanation for the Method to Find δ_m

Refer to *Figure* 1. Only a ray parallel to $\overline{FG}$, say ray $\overline{DE}$ will emerge with the same angle of refraction at D and at E out of the prism so that $\overline{SU} = \overline{TV}$.

Acknowledgment

I thank Prof. Vijay A Singh for encouraging me to publish this note and for his valuable suggestions. Thanks to Prof. Vinayak Katdare for his critical comments.

Suggested Reading

[1] Eugene Hecht, *Optics*, *Pearson Global Edition*, pp.99–201, 2017.