Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Unscientific Determination of Fermi Energy of Copper by Heating and/or Cooling a Copper Wire*

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Fermi energy, which is a theoretical concept, has been explored in an experiment for a copper wire in a *Laboratory Course of Engineering Physics* [1]. In this article, the scientific validity of this experiment is discussed and a method to determine Fermi energy is discussed.

Keywords
Fermi energy, electron density, relaxation time, temperature dependence of resistance, mean free time.

Introduction

The simple model that describes electron conduction in a metal considers no interaction between electrons. Introduction of quantum mechanical ideas into this model gives rise to the concept of Fermi energy of metals. In metals, valence electrons are loosely bound to the nucleus. Due to this, valence electrons can easily get dissociated from the parent atom leaving behind a positive ion. The Coulombic repulsion between large number of free electrons can be considered as balanced by the attraction by the positive ions. The screening of positive charge occurs because of increased negative charge in its surroundings. Hence, free electrons

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can be assumed to be moving in a region of constant potential energy instead of a region of periodic Coulombic potential. Under such conditions, electrons can be treated as particles confined in a three-dimensional infinite potential well. In metals, the highest energy level occupied by the electrons when all the lower energy levels are filled is known as Fermi energy.

The average number of Fermions at temperature T in a state of energy E is given by

$$n_F = \frac{1}{e^{(E-E_F)/KT} + 1} \; . \tag{1}$$

At $T = 0$ K, as pointed out earlier, the electrons fill up the energy levels up the the Fermi energy E_F.

Fermi Energy of the Metal

As discussed in [2], the Fermi energy, E_F, of metal is given as

$$E_F = \frac{h^2}{8\pi^2 m}(3\pi^2 n)^{2/3} \; , \tag{2}$$

where n is the density of electrons, m is the mass of electron and h is the Planck's constant.

Fermi energy of a metal can be calculated by measuring the density of electrons in a Hall effect experiment.

If this model is applied to a metal, Fermi energy of a metal can be calculated by measuring the density of electrons in a Hall effect experiment.

Fermi Energy of Copper

The experimental method to determine Fermi energy is described in [3]. In this method, the Fermi velocity of electron is expressed in terms of mean free path, λ_F, and relaxation time, τ as

$$v_F = \frac{\lambda_F}{\tau} \; . \tag{3}$$

Thus, the Fermi energy is expressed as

$$E_F = \frac{m\lambda_F^2}{2\tau^2} \; . \tag{4}$$

The relation between relaxation time and conductivity

$$\tau = \frac{m\sigma}{ne^2}, \quad (5)$$

is utilized to get Fermi energy as

$$E_F = \left(\frac{ne^2 \lambda_F R\pi r^2}{L\sqrt{2m}}\right)^2, \quad (6)$$

where conductivity, $\sigma = L/R\pi r^2$, with L is the length of the copper wire, r is the radius of wire and R is the resistance of the wire utilized. It appears that above formula has been rewritten as

$$E_F = \left(\frac{ne^2 \lambda_F T\pi r^2}{L\sqrt{2m}}\right)^2 \times \left[\frac{\Delta R}{\Delta T}\right]^2, \quad (7)$$

to include temperature dependence of resistance of metals. With reference temperature, $T = 318$ K, Fermi energy of various metals, including that of copper, have been reported in [3].

Drawbacks of the Method

The above method has the following drawbacks.

1. Mean Free Path λ_F and Relaxation Time

The relaxation time is usually referred as the time taken by a free electron to decrease its velocity, v_d, by a factor of $1/e$, when the electric field is turned off. The drift velocity of electron decreases as the electric field is switch off. In case of (5), the relaxation time is defined when the metal wire is subjected to electrical potential.

The relaxation time is usually referred as the time taken by a free electron to decrease its velocity, v_d, by a factor of $1/e$, when the electric field is turned off.

Apart from this, the relation between Fermi velocity and relaxation time in (3) is inappropriate as it combines two different concepts of drift velocity and Fermi velocity. So, application of (5) to calculate the mean free path, λ_F, is incorrect.

Besides, λ_F is calculated for Fermi velocity, which is obtained from the theoretical value of Fermi energy of copper. This approach is also incorrect as the aim of the experiment is to find Fermi energy experimentally.

2. Fermi Energy and the Density of Electrons

Since expression in (7) is obtained from the inappropriate application of relaxation time, the equation is invalid to get the Fermi energy. As shown in (2), measurement of the density of electrons is enough to calculate the Fermi energy of a metal. Surprisingly, (7) uses the theoretical value of the density of electrons to estimate Fermi energy.

Despite these errors, how are correct theoretical values for Fermi energy being reported in the experiment carried out in [3]? The key is that the mean free path, λ_F, is calculated from the theoretical value of Fermi energy. If (7) is rearranged as

$$E_F = \frac{v_F^2}{\tau^2}\Big(\frac{ne^2T\pi r^2}{L\sqrt{2m}}\Big)^2 \times \Big[\frac{\Delta R}{\Delta T}\Big]^2 , \quad (8)$$

then the equation reduces to $E_F = mv_F^2/2$. Thus, the measurements that are carried out are irrelevant to get the final result because these measurements get cancelled in calculations.

Conclusions

The heating and/or cooling of a copper coil to study the dependence of resistance on temperature is not a very good option to determine the Fermi energy.

The heating and/or cooling of a copper coil to study the dependence of resistance on temperature is not a very good option to determine the Fermi energy. The correct way of finding Fermi energy is to measure the density of electrons in Hall effect experiment. Another way of finding the density of electrons is to utilize mobility, μ, of electrons at room temperature. The current density and mobility of electrons can give the electron density as:

$$n = \frac{IL}{e\mu V\pi r^2} , \quad (9)$$

where V is the electric potential applied to the wire.

Suggested Reading

[1] ***Engineering Physics Laboratory Course*** **of Visveswaraya Technological University, Belagavi, Karnataka (http://vtu.ac.in/pdf/cbcs/12sem/engphylab.pdf)**

[2] C Kittle, *Introduction to Solid State Physics*, John Wiley and sons, pp.137–141, 2005.

[3] *The Fermi Energy Experiment Kit* (FE-501) is made available by Kamaljeeth Instruments. The product manual can be accessed by visiting http://kamaljeeth.net