In Memoriam – V. S. Varadarajan*

Ramesh Gangolli

Ramesh Gangolli has been serving as the faculty of the Department of Mathematics at the University of Washington, Seattle, since 1962. He is an active mathematician and educationist, and has served as the Sloan Foundation Fellow (1966–68); Member, Institute for Advanced Study, Princeton (1966–67); In 1966 he was awarded the Prix Paul Levy by the Academy of Sciences of France. He has served on advisory committees of the National Science Foundation, the National Academy of Sciences and the National Research Council, as a Trustee of the American Mathematical Society, Chair of the AMS's Committee on Education, and in other similar roles. Ramesh Gangolli also maintains a serious interest in the classical music of North India and teaches in the Ethnomusicology Program in the School of Music.

1. Preface

I first met Varadarajan in January 1958, and maintained a close friendship with him till he passed away in April 2019. He was known to all his friends as Raja, and that is how I shall refer to him in the rest of this article. I learned much from him and collaborated with him on two book-sized works.

I am honoured to be asked to write about him. In my view, he was one of the best mathematicians of Indian origin to have worked in the second half of the 20th century. I have divided this article into three parts. The first gives a brief description of his life. The second part deals with some highlights of his work. The last part is devoted to some personal recollections. There I will describe some vignettes of our long friendship.

2. His Life

Raja was born in Bengaluru, but grew up in a number of different places, because his father had job as a school inspector, which was subject to transfers. His high school education was completed in Tiruchirapalli and Salem. His father was transferred to Chennai (then called Madras) just when Raja finished high school, so his undergraduate college education took place in Chennai, initially at Loyola College and later, for the honours degree, at Presidency College. The degree course was in statistics, but it had a great deal of mathematics included in it. Raja was an exceptional student. Speaking about his undergraduate

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experience, he has mentioned that he was inspired by a lecture on quantum mechanics given by P. A. M. Dirac who visited his college in 1954. He mentions that although he did not understand much of it, it inspired in him a keen desire to learn more about the connection between mathematics and physics. After he finished his Bachelor's degree, he went to the Indian Statistical Institute (ISI) as a graduate student, intending to work for his Ph.D. He has written in a couple of places about how he landed in ISI. During his final year at Presidency College, the eminent statistician C. R. Rao came to give a talk there. Raja was very impressed by the talk, and after graduation he wrote to C. R. Rao asking whether he could work at ISI, and was accepted. (This is just one example of C. R. Rao's taste and judgement in spotting talent). Around the same time, C. R. Rao also recruited two other young mathematicians as graduate students: R. Ranga Rao and K. R. Parthasarathy. These two and Raja became a tightly knit group, about which I will say more below.

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Raja finished his PhD. at ISI, early in 1960, and immediately went to Princeton on a short postdoctoral fellowship. During this period, he met Victor Bargmann, who encouraged him to learn the work of Harish-Chandra. He spent the following academic year (1960–61) at the University of Washington in Seattle. The academic year that followed (1961–62) was spent largely at the Courant Institute in New York University, during which he made a fleeting acquaintance with Harish-Chandra, whose work would later dominate his interests.

Raja returned to ISI in the latter half of 1962 and resumed his collaboration with his old buddies Parthasarathy and Ranga Rao. With his new-found interest in representation theory, he led the group to start studying the work of Gelfand, Naimark and Harish-Chandra, on infinite-dimensional representations of semi-simple Lie groups and Lie algebras. This led to a paper that has become quite famous [1]. The work was essentially finished late in 1964 or early in 1965, but for various reasons was published only in 1967. He also published several other papers, including one on the logic of quantum physics,and was now getting some international renown as a rising young mathematician. He soon got an offer of a position as Associate Professor at UCLA, and he moved there in 1965 with his wife Veda, his college sweetheart, whom he had married the year before.

Raja had a distinguished career at UCLA, contributing in many capacities to the life of the department. He continued to work on multiple fronts and was recognized with many distinctions. He became a universally respected and beloved figure in a department teeming with many excellent mathematicians. He also interacted with many colleagues in the physics department at UCLA. Besides this, he had developed working relationships with physicists and mathematicians at the Universities of Genoa and Trondheim, and collaborated with colleagues there in significant projects.

In the last two years, his health took a turn for the worse. Beset with diabetes and accompanying ailments of the heart and kidneys, he faced several bouts of hospitalization. He anticipated that his end was near and spoke to me about it just days before his passing. Two months before he passed away, he and Veda announced a major gift to UCLA, funding a Visiting Professorship in the Department of Mathematics. The Professorship is being named in memory of Srinivasa Ramanujan, whom Raja admired immensely. Although beset with challenging ailments, he was functional till the end, which came peacefully on April 24, 2019.

3. His Work

Raja's work is wide-ranging and copious. I have decided mainly to highlight a few pieces of work in the area of Representation Theory and Harmonic Analysis, with which I am most familiar. In an article such as this, one cannot dwell on detail. A general appreciation of the taste displayed and the challenges surmounted in a few significant contributions is all one can try to convey. I hope that the few facets that I highlight will give the reader an inkling of both the depth and the good taste that suffuses his work.

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In the decade after India's independence from British rule in 1947, there was a concerted move to build up scientific institutions, and the Indian Statistical Institute (ISI) was one of them, founded under the aegis of P. C. Mahalanobis, the well-known statistician. Mahalanobis had not only built up a strong group of statisticians at ISI, but also had decided to encourage related fields, especially Mathematics. In this, he was helped by the brilliant statistician, C. R. Rao, who had impeccable taste and judgement for identifying talent, and who was the Head of the Research and Training School of ISI. As I mentioned above, C. R. Rao recruited Raja in 1957. This was the start of Raja's research career, fresh out of undergraduate studies.

Raja was very young, just 20 at that time, and soon after his arrival he was joined by two other promising young mathematicians, also recruited by C. R. Rao, namely K. R. Parthasarathy and R. Ranga Rao, both about the same age as Raja. Although ISI had several very good statisticians on the faculty at that time, there was no one there who could direct the research of this young group of mathematicians, whose interest were decidedly in pure mathematics. They worked on their own, and studied areas that were then emerging, using the resources of a reasonable good library at ISI, and managed to choose their own thesis topics.

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Raja chose to work on some problems concerning measures on topological spaces for his thesis. The problems he was studying arose directly from the work of Kolmogorov in the preceding two decades, in which probability theory had been given a clear and precise foundation through its formulation in terms of measure theory. Through the work of Feller, Doob and others, it was becoming clear that stochastic processes were best studied as temporal evolutions of probability measures on suitable function spaces. In his thesis, which Raja completed in early 1960, he obtained many interesting new results and also solved two problems on convergence of measures on topological spaces that were posed by Kolmogorov and Prokhorov. Questions of convergence of measures are natural in the context of studying limit theorems for stochastic processes, as is clear from what was said above. Kolmogorov, who was the external referee for Raja's thesis, remarked how impressed he was by the extraordinary maturity of the work. I have always been astonished by this work, done in almost complete isolation, relying on his own capacity to learn and formulate. Many years later, C. R. Rao told me that he regarded this as a superb example of raw talent, uninformed by any serious training, and knew immediately that Raja was destined to scale great heights.

I have referred above to the next two years in Raja's development. Two encounters that he had during this period were decisive in framing the course of his future work. One was his contact with Victor Bargmann, who urged him to study Harish-Chandra's work. Raja has said somewhere that Bargmann was struck by the fact that Harish-Chandra had established the notion of a distribution character for infinite-dimensional representations of a semi-simple Lie groups, and was convinced that the entire course of harmonic analysis on such groups was going to be dominated by a deeper understanding of them. This was a prophetic insight, and affected Raja deeply. As we will see below, this led to his lifelong interest in Harish-Chandra's work, culminating in a steadfast friendship with Harish-Chandra.

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The second influence was through his encounter with George Mackey. During the summer of 1961, Mackey visited Seattle and gave a course of lectures on the mathematical foundations of quantum physics, and more particularly the connections between representation theory and quantum mechanics. Raja was deeply influenced by these lectures and started studying the subject in earnest. He has mentioned that Mackey also prodded him to study Harish-Chandra's work.

His way of studying a subject seems to have been to pose a nontrivial problem in the subject and start attacking it. This pattern emerges immediately in his work over the two years after his return to ISI from the US in 1962, when he published a paper [2] on the logic of quantum mechanics, based on discussions with Mackey in Seattle, and Warren Hirsch at NYU. He also read and thought about the formulations that were in the work of Hermann Weyl and John von Neumann and, as became his custom later, organized his thoughts in a two-volume work on the *Geometry of Quantum Theory* [3], which was very well-received and continues to be influential. In the direction of representation theory, he started studying the work of Harish-Chandra, both on the representations of semi-simple Lie groups as well as the parallel theory of representations of semi-simple Lie algebras. Soon after he arrived back in ISI, he started collaborating with his erstwhile buddies, Parthasarathy and Ranga Rao. They started working on a problem concerning irreducible infinite-dimensional representations of a complex semi-simple Lie algebra. This work brought the first wide international notice to this young trio.

The PRV Conjecture

The paper I refer to above is [4]. In this paper, the authors get many interesting results concerning infinite-dimensional representations of a complex semi-simple Lie algebra $\mathfrak{g}$. One result, that drew the immediate attention of experts in the field was the irreducibility of any unitary representation of the non-degenerate principal series of G that contains the trivial representation of K.

Although there are several other results in this paper, I do not go into them here. I will only describe one problem they had to grapple with that has led to several other investigations. This concerns the decomposition of the tensor product of two finite dimensional irreducible $\mathfrak{g}$-modules into irreducible $\mathfrak{g}$-modules. They had to study this in the course of their investigation and were led to the following conjecture, which has come to be known as the PRV Conjecture.

The PRV Conjecture: Let $\mathfrak{g}$ be a finite dimensional complex semi-simple Lie algebra, and let W be the Weyl group associated with $\mathfrak{g}$. Let $V(\mu)$ and $V(\nu)$ be two finite dimensional irreducible $\mathfrak{g}$-modules with highest weights μ and ν respectively. Then, for any $w \in W$, the irreducible $\mathfrak{g}$-module $V(\overline{\mu + w\nu})$ with extremal weight $\mu + w\nu$ occurs at least once in $V(\mu) \otimes V(\nu)$.

Here, $\overline{\mu + w\nu}$ is the highest weight in the W-orbit of $\mu + w\nu$.

Later, Kostant made a strengthened form of this conjecture, which he also called by the same name, which says that the irreducible $\mathfrak{g}$-module $V(\overline{\mu + w\nu})$ with extremal weight $\mu + w\nu$ occurs *exactly* once in $V(\mu) \otimes V(\nu)$.

This conjecture has, as they say, "attracted a lot of press". Both the PRV conjecture and its strengthened form proposed by Kostant were proved by Shrawan Kumar in 1988, and investigations related to it are frequent even today. One reason is that the analogous question occurs also in the theory of representations of infinite-dimensional Lie algebras, and these techniques carry over to that context.

Transforms of L_1 *Spherical Functions*

Let G be a semi-simple Lie group with finite centre, and let K be a maximal compact sub-group. a function f on G is spherical if $f(k_1 x k_2) = f(x),\ x \in G, k_1, k_2 \in K$. If $S(G)$ is any space of functions on G, the subspace of spherical elements of $S(G)$ is denoted by $S(K\backslash G/K)$. In particular, we have the space $\mathcal{C}(G)$, the Schwarz space of rapidly decreasing smooth functions in $L_2(G)$, and its spherical subspace $\mathcal{C}(K\backslash G/K)$ defined by Harish-Chandra.

In a series of important papers in the years 1954–56, Harish-Chandra studied the problem of deriving the Plancherel theorem for the space $\mathcal{C}(K\backslash G/K)$. The techniques developed in these papers acted as inspirations for those that he developed later for his assault on the Plancherel theorem for $\mathcal{C}(G)$. At the end of this series of papers, the explicit Plancherel theorem is *almost* proved for a function in the space $\mathcal{C}(K\backslash G/K)$. The gap that remained amounted essentially to showing that not matrix-elements of a representation of the discrete series of G could be spherical. Naturally, this was not resolved until Harish-Chandra had finished his monumental construction of the discrete series of G in 1962–64.

Although this resolved the issue, there was something a bit unsatisfactory about this state of affairs, because, to accomplish this last step, one had to go outside the domain of *spherical* functions and representations into the more general domain of all functions and representations. In the paper [5], Trombi and Varadarajan study the space $C^1(K\backslash G/K)$, the subspace of L_1 functions in $C(K\backslash G/K)$. A complete characterization is obtained in terms of the Harish-Chandra spherical transform. This paper is notable for two reasons. One is that it clarifies and carries out the technique of descent that Harish-Chandra pioneered in his papers on spherical functions mentioned above in a different context. The other is that it completes the last step in Harish-Chandra's Plancherel theorem while staying within the domain of the spherical functions, without having to appeal to the theory of the discrete series.

Infinitesimal Form of a Discrete Series Representation.

The relevant paper here is [6], whose title says it all. The discrete series are pivotal for obtaining the Plancherel theorem in Harish-Chadra's work. In this paper, Enright and Varadarajan obtain a characterization of the representation of the Lie algebra $\mathfrak{g}$ that arises from an irreducible discrete series representation of G. This paper has become very well-known and has spawned some papers on what are now called Enright–Varadarajan modules.

A Vanishing Theorem for Certain Distributions

In [7] Kolk and Varadarajan study the following problem: Let X be a manifold, and E a Frechét space. An E-valued distribution on X is an element of the dual space of $C_c^\infty(X, E)$, endowed with the usual topology. Now let O be a closed submanifold of X, and G a Lie group acting on X by diffeomorphisms. The problem is to study the behaviour of E-valued distributions that are supported by O and invariant under the action of G. Using ideas from the theory of micro-local distributions, and their derivatives, they prove a vanishing theorem for such distributions under certain conditions. This result is then used to provide simple new proofs for two theorems of Bruhat on the irreducibility of certain induced representations of G, and also two important theorems of Harish-Chandra whose full proofs have never appeared, although they were informally described by Harish-chandra to various colleagues. One of these two theorems is about the irreducibility of certain parabolically induced representations of a semi-simple Lie group G, and extends an analogous theorem earlier proved by Bruhat. The second theorem concerns a vanishing theorem about certain distributions that are encountered in Harish-Chandra's work on his Whittaker theory. This vanishing theorem plays an absolutely critical role in that theory.

The paper [7] is important for two reasons: (a) It provides the natural global framework that unifies the major step in each of these two theorems of Bruhat and Harish-Chandra, and (b) the approach taken in this paper makes transparent the motivation behind the complicated calculations that are undertaken by Bruhat and Harish-Chandra, and then avoids those calculations, arriving at the conclusion much more simply.

Because of the importance of the first theorem of Harish-Chandra in representation theory, I will describe it briefly. I use standard notation. Let G be a semi-simple Lie group with finite centre. Let (P_0, A_0) be a minimal parabolic p-pair. Let (P, A) be another p-pair such that $(P, A) > (P_0, A_0)$, and let $P = MAN$ be the Langlands decomposition of P. Given an irreducible double representation σ of M, a unitary character $e^{i\lambda}$ of A, one can regard $\sigma \otimes e^{i\lambda} \otimes id$ as a unitary representation of P, from which one induces the representation $\pi_{\sigma,\lambda} = \mathbf{Ind}_P^G\, \sigma \otimes e^{i\lambda} \otimes id$ of G. The issue is to prove the irreducibility of the representation $\pi_{\sigma,\lambda}$ under suitable conditions, and to understand the relation between two representations, say π_{σ_1,λ_1} and π_{σ_2,λ_2} arising from two different p-pairs (P_1, A_1) and (P_2, A_2). Bruhat had studied this question for the *minimal* parabolic pair (P_0, A_0). The irreducibility theorem for an arbitrary p-pair was essential for Harish-Chandra's work, and was proved by Harish-Chandra. The proof was sketched by Harish-Chandra in a famous letter to van Dijk in 1983, just before Harish-Chandra passed away. A complete proof is given in [7], with due credit to Harish-Chandra of course. As far as I know, the proof in this paper is the only complete published proof of the irreducibility theorem. Moreover, the formulation given in this paper yields a much clearer picture of the essential ingredients of these theorems of Bruhat and Harish-Chandra. The paper also has some related results, but I will not describe them here, because of the space restrictions of this article.

Harish-Chandra His work, and Legacy

As I have said above, Raja was strongly influenced by Harish-Chandra and his work. As many have remarked, the total corpus of Harish-Chandra's work is a magnificent edifice. Langlands has described his work as "....like a Gothic cathedral, heavily buttressed below but, in spite of its great weight, light and soaring in its upper reaches, coming as close to heaven as mathematics can". To get a grip on these "buttresses" is not an easy task. Harish-Chandra himself never launched into explanations that revealed the basic philosophy underlying his work. Raja's close study of Harish-Chandra's work led him to write a number of articles, explaining and commenting on that philosophy. A study of these articles by Raja is, I think, indispensable for anyone who would like to understand Harish-Chandra's work. There are many articles by Raja in this genre, of which I cite just one [8], which I believe lays out an excellent guide to Harish-Chandra's work. Together with [9], one has a negotiable road into this territory.

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Reviews and Historical Articles

A mathematician's research accomplishments have always ranked high in the eyes of peers. But articles such as reviews, memorabilia, historical expositions, etc., reveal a different view of the mathematician's personality and taste. This is beautifully apparent in Raja's case. Many such citations possible, but I confine myself to just a few. [10] is a tribute to George Mackey, but is also a very nice road map to the study of the mathematical formulation of quantum physics. [11] is a memorial article about

André Weil. [12] and [13] are gems cut to perfection, while [12] is a tasteful article on Euler's work on infinite series, providing many insights of this old theme as presented in Euler's work, [13] is a historical account of algebra, which includes, among many other topics, a perspective on the work of Indian mathematicians in the sixth and seventh centuries CE, usually missing from Western accounts on this subject.

I am very fond of these articles because, on the one hand, they reveal a poetic aspect of Raja's view of mathematics and its practitioners, and on the other hand, reveal an implicit attitude that regards mathematicians great and small as contributors to a noble stream.

4. Some Personal Recollections

I first met Raja when I went to the Indian Statistical Institute (ISI) in January 1958. At that time he was the leader of an extraordinary group of young mathematicians who were assembled there by the brilliant statistician C.R. Rao, who was the Director of the Research and Training School of ISI. My presence at ISI was somewhat accidental. I had finished my studies at Cambridge University, finishing the Mathematics Tripos there, which roughly corresponds to a Master's degree in India. I returned to India in June 1957, but had not decided what I would like to do. I had grown up in a family of modest means, and attended Cambridge only because I was lucky enough to get a scholarship. Although education was very much valued by my parents, their horizons did not extend beyond securing a stable job and following the traditional path of a middle-class family. I had done well at Cambridge, and when I returned to India, I was offered positions in the business world. In my state of uninformed indecision, I had accepted an offer from Tata's as a management trainee. However, fate intervened felicitously at that point. I had gone to Delhi in October 1957, and through my friend Amartya Sen, who was at the Delhi School of Economics at that time, I met P. C. Mahalanobis. When Mahalanobis heard of my Cambridge record and my plans to join Tata's, he was very blunt. "Don't be foolish" he said. "You must go into research. You owe it to yourself to find out what you can do in mathematics. You can always join any business you want later, but now is the only time you can pursue the path towards research" – that was his message. He also generously offered me a graduate student internship at ISI with a modest stipend, no strings attached, just so that I could come there and mark time while deciding my future course. I slept on it for a couple of days, and fortunately was convinced by his forthright honesty and passion. When I returned to Bombay, and announced my decision to my parents, they were taken aback, but fortunately let me follow my own star. Thus I joined ISI in January 1958.

My time at ISI was extremely instructive. There were several formally registered graduate students, in mathematics, statistics as well as in some other sciences. I had no formal status, and was an outsider. All graduate students had desks in a large bare room with many blackboards, where we worked on whatever caught our fancy. Access to the library was easy. There was no regular curriculum, but the faculty would announce courses now and then and intensive study followed. As I said above, Raja, Parthasarathy and Ranga Rao were the core mathematics group. They would organize seminars on a subject they wanted to learn, take turns at studying and lecturing on a facet of the subject and engage in fierce discussions while doing so. This method was new to me. I had learned to do well in examinations but had not learned to how to learn on my own. I watched in fascination, and soon found that Raja, clearly the un-anointed leader of this group, was definitely not your ordinary guy. They were far ahead of me. But watching them, I began to learn how to learn. I was not immediately accepted in the group but was not snubbed either. Their initial reluctance was a natural facet of the dynamics of any established group. Slowly it faded, and I developed, though numerous conversations, and uncounted hours of discussion, a warm and easy friendship with Raja, which stood the test of time.

At this point, I had decided that I would commit to a career of teaching and research in mathematics. I had also realized that I was not yet ready to launch into research on my own, as the ISI group had done and had decided to go to the US for a doctoral degree. I did not know much about US universities but had heard of Harvard and MIT as great centres of mathematics. I applied for admission to the PhD. program plus teaching assistantship to both institutions and was offered admission and a teaching assistantship at both. But the stipend that MIT offered was much more. This was important because my wife Shanta was also intending to go with me as a student, and the difference in my stipend would be of great significance to us. Thus i accepted the offer from MIT, and we proceeded together to Boston in 1958.

After finishing my PhD. in 1961 at MIT, I taught there as an Instructor for a year and then came to the University of Washington at Seattle in 1962. I was finding my feet in the world of mathematics, and soon found myself being attracted to the area of Analysis on Symmetric Spaces, which was intimately connected with Lie Groups and representation theory, and which was now also one of Raja's principal interests. Thus, after Raja came to UCLA, we renewed our contact, and began to be in close touch. Both of us being on the West Coast enabled us to see each other more often, and this allowed us to spend many happy hours talking and laughing. Raja had married his college sweetheart Veda after his return to India in 1962, and my wife Shanta and I became close to Raja and Veda over the succeeding years, forging bonds of deep affection that have never weakened.

We collaborated on two book-sized volumes in these years, the second one completed just last year. Raja and I shared many hours laughing and chatting. He had wide interests. For some years he took up the study of the clarinet, and enjoyed trying to play some snatches of Mozart's clarinet pieces, especially his Clarinet concerto K622. Shakespeare, *The Mahabharata*, Mozart, Bach, Wodehouse, Tin Tin, and the fortunes of the Los Angeles Lakers in the basketball championships were his unfailing passions. I shared all of them except the last one, but could not help but enjoy the total enthusiasm with which Raja would cheer his team while watching their games on TV. They were always referred to as the "beloved Lakers" while the epithet "hated" was used for any team that had the gumption to vie against them.

Most mathematicians are content to spend their research career on one of two areas of mathematics. Raja contributed to a host of them, at a high level. His work ranges over probability theory, measures on topological spaces, representation theory of semi-simple Lie algebras, Analysis on semi-simple Lie groups and their quotients, algebraic geometry of differential equations, mathematical foundations of quantum theory, and supersymmetry and related questions of mathematical physics. In addition, he has written beautifully crafted historical articles, lecture notes, and reviews, all done in an impeccably clear and elegant, almost poetic, style.

Raja had a unique gift for communicating with his doctoral students, and younger colleagues who came under his spell. Almost all of them became his friends. On the one hand, he inspired them to excel, by the force of his example and his expectations; on the other hand, he showered them with unreserved friendship.

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Collaborating with Raja, learning from him, and enjoying his friendship and humor has been a great gift for me. Raja was a great admirer of the work of Harish-Chandra. In turn Harish-Chandra also recognized Raja's powers, and they had become good friends. Harish died in 1983, alas too early in his life. He had left behind a large body of manuscripts dealing with many topics. For many years Raja had wanted to see what they contained, and whether the contents could be organized into a coherent account of the topics they addressed. A couple of years after his 70th birthday, he and I decided to collaborate on this project, encouraged by Robert Langlands, who had attended the conference. We worked in fits and starts, but never abandoned the project, and were able to salvage a considerable amount of valuable material. That material was published in July of 2018 as the fifth (posthumous) volume of *Harish-Chandra's Collected Works* [14], by Springer Verlag, (the same publisher that has published the first four volumes) with the two of us cited as Editors. Raja was very gratified to see the completion of this project, and I am happy to have contributed to his joy.

The last three years were very trying for Raja and Veda. Both have had health issues of different kinds. Raja was harried by diabetes and accompanying afflictions that affected his heart and kidneys. But fortunately, he was able to function, and he kept on working on mathematics till the end, which came peacefully, on the 24th of April 2019, mercifully without the pain and indignities which are often our lot. As I know from my collaboration with him on the Harish-Chandra volume, he lost none of his incisiveness and clarity till the end.

His passing is a great personal loss for me. He was a friend and an inspiration. But age brings with it an ability to face the bitterness of the loss of friends by taking comfort in the gentleness of their passing, if they are so favoured.

Suggested Reading

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