V S Varadarajan at the Indian Statistical Institute, Calcutta*

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This is a short compilation of snapshots of memories about an eminent mathematician who was a great friend, philosopher and guide for the author.

In the summer of 1956, V. S. Varadarajan arrived in Calcutta as a teenager after completing his B.Sc. Honours in Statistics at the Presidency College of Madras University to embark on a research career in Mathematics at the Indian Statistical Institute. The institute was located in a long multi-storeyed structure raised on grey hollow bricks, specially manufactured by its administrative department. Its campus consisted of three adjacent compounds with the address at 203–205, B.T. Road. The campus was lush green with mango, jackfruit, magnolia, royal palm and areca nut trees around half a dozen ponds with the biggest one in front of the main building at 203, B.T. Road. There were a few derelict bungalows and many sheds raised on grey bricks and steel frames with asbestos roofs. The sheds partitioned by sheets made from sliced bamboo constituted the rooms of students' hostels as well as innumerable 'Units' and minor offices for an overflowing crowd of employees called workers who had a powerful Union. All theoretical research in diverse areas of statistics was carried out in the third floor of the main building called the RTS (an acronym for Research and Training School). This floor was again partitioned into class rooms, offices for the faculty and administration and rooms for research students. The main teaching programme was called 'Three year advanced professional statisticians' training course'. Varadarajan, called Raja hereafter started his research and teaching career in such a setting with enthusiastic encouragement from Dr C. Radhakrishna Rao

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who was the Head of RTS. The Director of the Institute Professor P.C. Mahalanobis, was simply addressed as 'Professor'. All the other employees including the faculty were called 'workers'. The Institute was, indeed, a veritable beehive.

In 1956, there was no research activity in mathematics, except for probability theory as in volume I of Feller's book [4] and integration theory as in Cramer's *Mathematical Methods of Statistics* [3]. Sophisticated measure theory appeared in the works of Professor R.R. Bahadur. There was no faculty doing research in stochastic processes, particularly in the art of studying probability distributions in path spaces. Raja entered such a milieu, studied the famous little red book of A.N. Kolmogorov with the title '*Foundations of the theory of probability*' [6] and the influential 1956 paper of Yu. V. Prohorov [8] and began his own investigations of the theory of weak convergence of probability measures in topological spaces, which included sequence spaces and function spaces of infinite dimension. After all, a stochastic process is a probability measure in the space of trajectories traced out in discrete or continuous time. Raja was very fond of 'Kolmogorov's Consistency theorem' and in his lectures he derived Riesz's representation theorem for linear functionals from Kolmogorov's theorem on stochastic processes [12]. Later, I saw how Kolmogorov's theorem appeared in basic number theory as well as the independence of continuum hypothesis in logic. By 1958, Raja had published six papers in the theory of probability and measure and submitted his famous PhD thesis 'Convergence of stochastic processes' to Calcutta University [11]. This Doctoral dissertation took the shape of his great paper 'Measures in topological spaces', Mat. Sbornik [19] (in Russian) in 1961. Its English translation appeared in 1965 in AMS Translation Series.

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I too joined the ISI in 1956 as a student of its training programme and towards the end of my second year I did a project on the moments problem based on the book by Shohat and Tamarkin [10]. Fortunately, for me Raja happened to be my tutor for this project. Walking around the ponds in the evenings Raja explained to me a whole lot of theorems in measure and topology as well as

weak convergence with prodigious energy. Prior to his departure for Princeton, Raja presented a series of seminars in the theory of operators in Hilbert space covering the spectral theorem for a bounded selfadjoint operator as well as the Hahn-Hellinger multiplicity theorem for spectral measures, and he wrote a beautiful set of cyclostyled lecture notes prepared with the assistance of R.T.S's great typist Gour Babu. Thus mathematics took root in RTS as a way of life, thanks to the tremendous freedom in research bestowed by C.R. Rao and P.C. Mahalanobis.

In the U.S., Raja attended the seminars of G.W. Mackey at the University of Washington at Seattle on the mathematical foundations of quantum mechanics and group representations. Already in 1954 as a first year undergraduate student he had the opportunity of listening to a talk by the eminent physicist P.A.M. Dirac at Madras University. In the course of his lecture Dirac made the statement 'the world is noncommutative'. It was just as 'loud as a heartbeat' but its full significance came to light during the Mackey seminars. It was a meeting point of fundamental laws of nature, group representations, Lie groups and Lie algebras and probability theory. Raja had several long discussions with Mackey during which he mentioned that a deeper understanding of group representations must be sought in the 'formidable Lie algebraic machinery of Harish-Chandra'.

On his return to Calcutta in 1962, Raja started a project on Lie algebras in collaboration with S.R.S. Varadhan who had just completed his Ph.D. thesis containing his famous Lévy-Khinchine formula for infinitely divisible probability measures and central limit theorem in an infinite dimensional Hilbert space. The project came to a halt after a year when Varadhan left Calcutta for good and joined the Courant Institute. In the year 1963, R. Ranga Rao, who had collaborated with Raja during 1958–1960, returned to Calcutta from the Illinois University at Urbana. The same year I returned to Calcutta after spending a year at the Steklov Mathematical Institute in Moscow. Then Raja, Ranga Rao and I worked at the Indian Statistical Institute for a period of two years. Raja started a seminar on complex Lie algebras and starting from

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scratch, proved the classification theorem of all semisimple Lie algebras and covered the famous 1951 paper [5] of Harish-Chandra. Once again Raja's seminars concluded with the typical ISI cyclostyled notes on Lie Algebras typed by Gour Babu. These notes followed by a huge volume of computations, conversations during walks around the ISI ponds along with delicious dishes from the kitchen of Veda Varadarajan provided the ground work of the paper [7]. Much later in his life Raja wrote his famous book on *Lie groups, Lie algebras and Their Representations* [15].

During his last two years at ISI he lectured on the mathematical foundations of quantum mechanics following the Mackey seminars at Washington University. He wrote his ISI lecture notes *Geometry of Quantum Theory* which became his famous book [16] with the same title and thus 'Noncommutativity of the World' came to light for me. To this motto Raja added 'supersymmetry' in his quest to understand Nature through the mathematics of quantum theory.

Prof. Varadarajan's sketch by Shyamala Parthasarathy

In 1965 Raja left ISI for good and joined the University of California at Los Angeles as a permanent faculty member. Here, his fertile pen never ceased to flow. His 'Selected Works' running to nearly 2000 printed pages were published in three volumes [14],

[1, 2] by AMS and Hindustan Book Agency, Delhi. He had authored more than a dozen books, edited the collected papers of Harish-Chandra and worked as the chief editor of the Pacific Journal of Mathematics. Raja breathed his last on 25 April 2019.

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Suggested Reading

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