Veeravalli S Varadarajan*

18th May 1937 – 27th April 2019

Varadarajan was one of the "famous four" at the Indian Statistical Institute, Kolkata during the golden period 1956–1965 – the others being K R Parthasarathy, R Ranga Rao, and S R S Varadhan. He obtained his doctorate in 1960 at the Indian Statistical Institute under the supervision of C R Rao. After holding postdoctoral positions at the Institute for Advanced Study, Princeton, University of Washington at Seattle and the Courant Institute, NYU, Varadarajan joined the Faculty of the University of Califiornia at Los Angeles in 1965.

Even though Varadarajan had a wide spectrum of mathematical interests, an underlying theme of much of his research had always been to understand the role of symmetry in mathematics and physics. A large part of this was realized through his work in the representation theory of Lie groups and its applications to harmonic analysis and quantum mechanics. Varadajan had also worked extensively in the theory of differential equations with irregular singularities. His more recent work centered around supersymmetry and representations of super Lie groups and super homogeneous spaces.

Varadarajan was also much influenced by the seminal work of Harish-Chandra. There is an amusing folklore about this rather deep influence. Akin to Harish-Chandra whose research papers sometimes started by fixing i a square root of -1, Varadarajan was also seen to do so. On an occasion, M S Narasimhan (another eminent Indian mathematician) asked Varadarajan if the choice of the square root of -1 was the same as Harish-Chandra's! Varadarajan edited the Collected Works of Harish-Chandra and often recalled the interactions between them over the years. For instance, Varadarajan recollected that at the beginning of this long association, when sought for advice in Princeton about working on semisimple groups, Harish-Chandra had mentioned that one must "know the three volumes of Chevalley backwards" (a typically Indian phrase signifying one has to learn it in and out!). The other mathematician whose work had a lasting influence on Varadarajan was George Mackey about whom also he has written extensively.

He had an abiding interest in the history of mathematics and in the Indian contributions, in particular. The books "*Euler Through Time: A New Look at Old Themes*" and "Algebra in Ancient and Modern Times" are masterpieces – the treatment of the Chakravala method in the latter is crystal clear and understandable to an undergraduate student. He has written several

*DOI: https://doi.org/10.1007/s12045-019-0860-y

text books among which his book on '*Lie groups, Lie algebras and their Representations*' is considered the best book by anyone on the subject. His book "*An Introduction to Harmonic Analysis on semisimple Lie groups*" is similarly a classic. These books will continue to be the most popular ones to be used by graduate students all over the world who wish to work in this subject. They are technically powerful and yet, pleasurable to read and learn from. The deep research contributions of Varadarajan in a variety of areas is the tour-de-force that facilitated the communication of rather technical mathematics through text books.

Varadarajan's name is associated with some important discoveries. One of them is the so-called PRV conjecture that emerged from path-breaking work (jointly with Parthasarathy and Ranga Rao). The work by PRV began in the early 1960's by trying to generalize Harish-Chandra's results on infinite-dimensional representations of semisimple Lie groups when the underlying group is complex which was thought to be very difficult or even out of reach at that time. It has proved to be of fundamental importance in Representation Theory - the PRV conjecture from 1967 was solved in 1988 by Shrawan Kumar, a mathematician of Indian origin at the University of North Carolina at Chapel Hill. In collaboration with Thomas Enright, Varadarajan introduced and studied what are now known as Enright-Varadarajan modules in Representation Theory. Also, the PRV triumvirate and Kostant introduced a determinant named after them.

Ramesh Gangolli and K R Parthasarathy have written articles in this issue carrying more mathematical content based on their relationship with and first hand knowledge of the mathematics of Varadarajan. We describe here some of the lighter writings of Varadarajan that are expository or historical in nature. In his 2011 book "*Reflections on Quanta, Symmetries, and Supersymmetries*", Varadarajan mentions at one point:

"The creations of the mathematician reflect an esthetic that is purely internal and yet, miraculously, these very same constructions are precisely the ones we need when we try to understand the physical world. The examples are many and well known: (a) the theory of ordinary differential equations and their uncanny applications to the Newtonian theory of celestial motion and universal gravitation; (b) Riemannian geometry and its use in Einstein's general theory of relativity; (c) fiber bundles, connections, and their emergence as the basic tools in gauge theories like Yang-Mills; (d) the use of spin geometry in Dirac's equation for the electron and in unified theories; and so on. In several conversations I had with Harish-Chandra he often used to refer to this phenomenon as the coincidence of the inner and outer realities."

He goes on to arrive at the interesting point of view that as a guiding principle what is not forbidden must be true and that only beautiful theories have a chance of being also true. Elsewhere, he writes:

"This period, 1963-65, was one of the most wonderful in my career. Calcutta was a very difficult place to live, with recurring power outages, rationing of essentials like rice and sugar, and other disruptions of daily life. The war in Vietnam was heating up and Calcutta, one of the most politicized centers in India, was in an uproar over this. The only constant was our work. We would work nonstop every day. We would read statements by Robert McNamara that it was only a matter of time before the war would end since half of Hanoi's electrical capacity was destroyed. We would laugh at this since we were always working in Calcutta much below capacity and yet the city and the people carried on heroically."

When PRV communicated their results in a letter to Harish-Chandra, they found that he had already left on a tour of India under the auspices of the UGC. Varadarajan recalles:

"To illustrate how little the monumental stature of Harish-Chandra was appreciated in India at that time, his visit made absolutely no waves, while the visit of a certain cosmologist, who was staying as a special guest of Lal Bahadur Shastry, the Prime Minister, was played up in the newspapers. To be fair, things are not very different here in the US even today, as the headlines and coverage in even great newspapers like the New York Times have very little correlation to the scientific importance of the topics discussed."

One interesting conversation Varadarajan recalls with Harish-Chandra concerning learning new thungs goes as follows:

"I remember once a conversation in which he was telling me how difficult it was for him to learn new things. He said that every time he wanted to go a little deeper into algebraic geometry he was told that he should learn about schemes. He said that this is like a patient who has an iron deficiency and is asked to swallow a pound of nails every day."

Varadarajan wrote about "arithmetic physics" that propounds the idea that there are connections between quantum physics and number theory. In particular, one could formulate some of the mathematical questions arising in quantum physics over realms other than the real field like the rational numbers. The basic tenet is that experimental evidence being discrete, the mathematical structures over the rational numbers should serve as good models. The corresponding theories over the real numbers is an idealization and just as the real numbers represent a completion of the rational numbers (allowing for convergence etc.) there are other realms like the p-adic numbers for various prime numbers which could also be considered. These are also completions of the rationals and one ought really to consider the so-called adeles that incorporate all the completions simultaneously. Varadarajan quotes Yu Manin as saying:

"On the fundamental level our world is neither real nor padic; it is adelic. For some reasons, reflecting the physical nature of our kind of living matter (e.g. the fact that we are built of

massive particles), we tend to project the adelic picture onto its real side. We can equally well spiritually project it upon its non-Archimedean side and calculate most important things arithmetically. The relation between 'real' and 'arithmetical' pictures of the world is that of complementarity, like the relation between conjugate observables in quantum mechanics."

Varadarajan's writings express a facility with diverse aspects of mathematics in this manner.

Varadarajan won several honours and accolades. To mention a couple, one is the Onsager medal and the other is being honoured by the Fellows of the American Mathematical Society program along with 23 Indian and Indian American members 'who have made outstanding contributions to the creation, exposition advancement, communication, and utilization of mathematics'. In March 2019, Varadarajan and his wife Veda donated one million dollars to establish a Ramanujan Visiting Professorship at the Department of Mathematics in UCLA during the university's 100th anniversary year.

One might summarize:

Varadarajan is one of the famous four
that comprise I.S.I's math folklore.
Inspired by work of Harish
he truly found his niche.
Since then all he did was soar, soar and soar!

B Sury
Indian Statistical Institute Bangalore
Email: surybang@gmail.com