ICM Awards 2018*

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The International Congress of Mathematicians (ICM) met in Brazil in August 2018 for its quadrennial conference. During the ICM, the Fields Medals and some other awards are presented. In this report, we briefly recount the personages and the significance of their work.

Some genius and a curiosity that is fanned,
by complete craze for any mystery at hand,
arguably gives us the gist,
of who may be a Fields medalist.
For this dubious summary, I'm sure to be panned!

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1. The Fields Medals

Once in every four years, the International Congress of Mathematicians meets for a conference, and during the glittering open ceremony, the Fields Medals are awarded to two to four mathematicians under the age of forty. This medal is regarded as one of the highest honours a mathematician can receive at a young age. The Fields Medal comes with a prize money of 15 thousand Canadian dollars. The medal was designed by Canadian mathematician J C Fields who established the award.

Lars Ahlfors and Jesse Douglas were the recipients of the first Fields Medals in 1936. The medal aims to recognize breakthrough contributions by young mathematicians and encourage them to make further such discoveries. The Iranian mathematician Maryam Mirzakhani became the first woman Fields Medalist in 2014; tragically, she passed away in 2017 not long after receiving the award. In 2014, Manjul Bhargava became the first Fields Medalist of Indian origin.

In the ICM held in Rio de Janeiro, Brazil in 2018, four mathematicians received the award. The 2018 Fields Laureates are: Caucher Birkar, Alessio Figalli, Akshay Venkatesh and Peter Scholze. Birkar's medal was stolen shortly after the event, and the ICM presented

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Figure 1. (From left) Caucher Birkar, Alessio Figalli, Peter Scholze, and Akshay Venkatesh were awarded Fields Medals at the 2018 International Congress of Mathematicians. Photo Courtesy: ICM 2018.

Birkar with a replacement medal some days later. The technical details of the medalists' works are available elsewhere for those who wish to look at them; the modest aim here is to merely introduce the readers to the areas to which these mathematicians contributed.

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Born as *Fereydoun Derakhshani* in Iran, he migrated to the U.K. and changed his name to *Caucher Birkar* which means 'migrant mathematician' in Kurdish. He won the 2018 Fields Medal for his work in algebraic geometry; more specifically to the minimal model problem and work on Fano varieties. Let us describe this very roughly. The classical conic sections that one learns about in high school are examples of 'algebraic varieties' – i.e., the zero locus of polynomial equations in several variables. One considers 'projective' varieties which involve homogeneous polynomials and are topologically easier to study. Further, smooth varieties (those which do not have singular points; i.e., points where tangent space cannot be properly defined) are easier to study, and according to the famous theorem of Hironaka (for which he won the 1966 Fields Medal), any projective variety is 'birationally isomorphic' to a smooth one (i.e., after removing some proper subvarieties, they are isomorphic). There is a process of 'blowing up' which produces new smooth varieties from a smooth variety which are all birational to it. The important task that arises is to start with a smooth variety and choose a minimal one birational to it – a 'minimal model' – i.e., one which is not a blowup of something. Varieties of one and two dimensions were already well understood a 100 years back. However, in dimensions > 2, there were two problems – minimal model may not exist, and even if it did, it may not be unique. Shigefumi Mori and others including Reid, Shokurov, and Kawamata instituted a program called the 'Minimal Model Program', whereby, he considered not necessarily smooth varieties but those with some mild singularities. Starting with a smooth variety, one tries to systematically blow down to get a minimal model. This process may create singularities but if the singularities are no longer mild, they would hopefully be treated using some operations called 'flips' to improve the singularities without blowing up. Mori won the 1990 Fields Medal for carrying this out in three dimensions. One of the accomplishments of Birkar and some of his collaborators is to carry this out in general dimensions! Although it is still unknown if this process stops in finitely many steps, Birkar showed it does so in many important special cases. Another major work of Birkar is a fundamental result on Fano varieties called the BAB conjecture; this makes positive progress towards constructing a 'moduli space' for (a variety which classifies isomorphism classes of) Fano varieties.

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Peter Scholze won the Fields Medal for his work in the area of arithmetic geometry. His completely novel ideas and work on p-adic Hodge theory has reportedly revolutionized arithmetic geometry. Prize citations prior to the Fields Medals had called Scholze "already one of the most influential mathematicians in the world", and "a rare talent which only emerges every few decades." As a 22-year old graduate student in Bonn, he introduced revolutionary ideas to re-prove in a 37-page paper, a 288-page work of Harris and Taylor in number theory. Scholze's work concerns among other things certain 'cohomology theories'. Cohomology can be roughly described as a way to study the holes in a geometric shape. There are several different types and versions of cohomology. Scholze, together with Bhargav Bhatt of the University of Michigan, has been developing a unified theory called prismatic cohomology; this views the diverse cohomologies akin to 'bands of light in a cohomological rainbow.' Many mathematicians react to Scholze with 'a mixture of awe and fear and exhilaration,' according to Bhargav Bhatt. This is not due to his personality which all who come in contact with him unanimously proclaim as grounded and generous, but due to an almost unnerving ability to perceive with depth the nature of mathematical phenomena. In his doctoral thesis, he had introduced a class of 'fractal-like' spaces, called *perfectoid spaces*. This enabled him to compare the geometry over p-adic fields with geometry in characteristic p. The geometry over p-adic spaces is often counter-intuitive (in the light of our experience with real/complex geometry) due to the fractal nature of these spaces. Scholze was able to construct perfectoid spaces for a wide variety of mathematical structures. These spaces make it possible to shift questions about polynomials from the p-adic world into another mathematical world in which doing arithmetic is much simpler – for instance when adding, we don't have to carry! Since their introduction, perfectoid spaces have already proven extraordinarily useful. A striking application that followed right, in the beginning, was the positive solution of the so-called weight-monodromy conjecture in many general cases. Very Roughly, this conjecture predicts a certain regularity property in the cohomology of algebraic varieties over p-adic fields. The weight-monodromy conjecture was proposed by Pierre Deligne in 1970. Thus, the accomplishment of Scholze to this long-standing conjecture is remarkable, to say the least.

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Akshay Venkatesh won the 2018 Fields Medal for his work which encompasses different areas but is sometimes put down as Number Theory. Most remarkably, Akshay Venkatesh creates tools from Ergodic Theory, Differential Geometry, Algebraic Geometry, Lie Theory, Representation Theory, and Number Theory to prove results in all these areas. His Fields Medal citation reads, "synthesis of analytic number theory, homogeneous dynamics, topology, and representation theory, which has resolved long-standing problems in areas such as the equidistribution of arithmetic objects." Using ergodic theory, Akshay, in collaboration with Ellenberg, made significant contributions to the Hasse principle on integral representations of quadratic forms. Using ergodic theory again, he solved a totally different type of problem in collaboration with Lindenstrauss, Einseidler, and Michel; they proved an old conjecture due to Linnik on the dynamics of torus orbits attached to class numbers of cubic fields. Another outstanding contribution was to the so-called subconvexity problem. To indicate what this is about, consider the classical Riemann zeta function $\zeta(s)$. Weaker than the Riemann hypothesis (RH) is the so-called Lindelöf hypothesis (LH); it asserts that $\zeta\left(\frac{1}{2}+it\right) = O(t^{\epsilon})$ for any $\epsilon > 0$. The *convexity* estimate $\zeta\left(\frac{1}{2} + it\right) = O(t^{1/2})$ can be deduced from knowledge of $\zeta(s)$ on the vertical lines $\Re(s) = 1$, $\Re(s) = 0$, and standard convexity bounds in complex analysis, Therefore, any improvement over the convexity estimate towards LH is called a *subconvexity* result for $\zeta(s)$, and is among the most significant accomplishments in present-day number theory as it may be construed as positive evidence towards the LH, and hence, towards the RH. The RH itself is thought of as 'out of bounds'; hence, subconvexity bounds are sought for as they give new number-theoretic insights. In the Langlands program (which could be thought of as a grand unification of mathematics), one studies more general families of *L*-functions (generalizing $\zeta(s)$). These functions correspond to generating functions of so-called automorphic representations of reductive groups. A major thrust of research in this area is to prove subconvexity results for these *L*-functions. Using the above results of Akshay Venkatesh *et al.*, the subconvexity problem is now resolved completely for GL(1) and GL(2) functions over general number fields.

Using ergodic theory, Akshay, in collaboration with Ellenberg, made significant contributions to the Hasse principle on integral representations of quadratic forms.

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Alessio Figalli won the 2018 Fields Medal for his "contributions to the theory of optimal transport, and its application to partial differential equations, metric geometry, and probability." Roughly

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speaking, optimal transport is the problem of finding the cheapest way to transport a distribution of mass. Figalli is the leading authority on the applications of optimal transport to isoperimetric problems and minimal surfaces. The classical isoperimetric problem is the question of determining among all smooth domains with a given boundary surface area, the shape of the domain that maximizes the volume. This explains the spherical shape of soap bubbles which need to minimize surface tension of the soap film. In the study of crystals where the energy is dictated by the corresponding microstructure, it is again true that they attain a configuration that is energy minimizing. The analogous question of determining the changes in the shape of the crystal due to an application of external energy can be rephrased as a variant of the classical isoperimetric inequality. Figalli and his collaborators used optimal transport to obtain strong quantitative versions of such inequalities and, thereby, established the stability of the configuration of crystals. Figalli also obtained striking regularity results for several nonlinear degenerate problems where classical methods fail. He developed a beautiful regularity theory for elliptic equations whose degeneracy set is allowed to be a large convex set. Further, in collaboration with Alice Guionnet, Figalli introduced and developed new transportation techniques in the area of random matrices and proved universality results in several-matrix models. Finally, in collaboration with Joaquim Serra, Figalli proved the De Giorgi conjecture for boundary reaction terms in dimension at most 5; this led to the improvement of the classical results by Luis Caffarelli on the structure of singular points in the so-called obstacle problem. The problem is to find the equilibrium position of an elastic membrane whose boundary is held fixed, and which is constrained to lie above a given obstacle. It is intimately related to the study of minimal surfaces During the Fields Medal ceremony, Luis Caffarelli summarized his description of Figalli's mathematical contributions as, "Figalli's work is of the highest quality in terms of originality, innovation, and impact both on mathematics per se as well as on its applications. He is clearly a driving force in today's global mathematical community."

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2. Chern Medal, Gauss, Nevanlinna and Leelavati Prizes

The 2018 Chern Medal was awarded to *Masaki Kashiwara* for his outstanding, foundational and sustained contributions over an almost 50 year period to algebraic analysis and representation theory. As the name indicates, 'Kashiwara Crystal Basis' is one of his seminal contributions to representation theory. Kashiwara has several hundreds of notebooks filled with mathematics. The Chern Medal is awarded once in four years during the ICM to an individual "whose accomplishments warrant the highest level of recognition for outstanding achievements in the field of mathematics. All living, natural persons, regardless of age or vocation shall be eligible for the Medal."

'Kashiwara Crystal Basis' is one of the seminal contributions of Masaki Kashiwara to representation theory.

Kashiwara is a skilled ping-pong player and the author can vouch for this from own experience.

The 2018 Gauss Prize was awarded to *David Donoho*. The Gauss Prize is awarded to a scientist whose research in mathematics has had an impact outside the mathematical realm; be it in technology, business, or simply in people's everyday lives. David Donoho is known for his studies in theoretical and computational statistics and developing algorithms that have proved vital to understanding the entropy principle, the structure of robust procedures, and sparse data description. Donoho's plenary lecture at ICM 2018 had several deeply personal anecdotes that brought to the fore the need for faster and better MRI sensing. One of the anecdotes concerned the brain surgery his wife had undergone earlier, and which inspired his son to become a neurosurgeon.

During the plenary address, Donoho mentioned a personal anecdote concerning a brain surgery his wife had undergone earlier, and which inspired his son to become a neurosurgeon.

The 2018 Nevanlinna Prize was awarded to *Constantinos Daskalakis*. The Rolf Nevanlinna Prize is awarded once every 4 years at the ICM for outstanding contributions to mathematical aspects of information sciences including: (i) all mathematical aspects of computer science, including complexity theory, logic of programming languages, analysis of algorithms, cryptography, computer vision, pattern recognition, information processing and modelling of intelligence, (ii) scientific computing and numerical analysis, (iii) computational aspects of optimization and control theory, and (iv) computer algebra. An interesting Indian connection is that Manindra Agarwal was part of the Nevanlinna Prize Committee for 2018. Daskalakis was given the Nevanlinna Prize for "transforming our understanding of the computational complexity of fundamental problems in markets, auctions, equilibria, and other economic structures." His work provides both efficient algorithms and limits on what can be performed efficiently in these domains. Daskalakis works in the interface of computer science and economics and is interested in using mathematics to understand humans. In his PhD thesis, he thanks his supervisor and says, "Christos once told me that I should think of my PhD research as a walk through a field of exotic flowers. You should not focus on the finish line, but enjoy the journey. And, in the end, you'll have pollen from all sorts of different flowers on your clothes."

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The 2018 Leelavati Prize was awarded to *Ali Nesin*. The Leelavati Prize is intended to accord high recognition and great appreciation of contributions for increasing public awareness of mathematics as an intellectual discipline and the crucial role it plays in diverse human endeavors. Started as a one-time award during the ICM 2010 which was held in Hyderabad, India, it was converted into a prize to be awarded once in four years at the ICM. The 2018 Leelavati Prize recognizes Ali's "outstanding contributions towards increasing public awareness of mathematics in Turkey, in particular for his tireless work in creating the 'Mathematical Village' as an exceptional, peaceful place for education, research and the exploration of mathematics for anyone." Ali's journey to developing and creating the Mathematical Village began after his father, Aziz Nesin, a renowned writer and socialist, died in 1995. Ali left the University of California, at Irvine, to return to Turkey to look after the foundation that his father had begun. The Nesin Mathematical Village, nestled in a verdant hillside, in a remote part of Turkey attracts students from high school onwards to enjoy mathematics in beautiful environs, with no pressure of exams. The underlying principle is that it is more important to understand the problem than to just solve it.

The Leelavati Prize is intended to accord high recognition and great appreciation of contributions for increasing public awareness of mathematics as an intellectual discipline and the crucial role it plays in diverse human endeavors.

Acknowledgment

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Suggested Reading

[1] A two-hour video: https://www.youtube.com/channel/UCnMLdlOoLICBNcEzjMLOc7w

[2] https://www.quantamagazine.org/tag/2018-fields-medal-and-nevanlinna-prize-winners/

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