The Mystery of Dark Energy and Some Revelations*

H K Jassal

Observational data has lead us to believe that the Universe is undergoing an accelerated expansion. This acceleration can only be driven by a component which has a negative pressure, the cosmological constant or other exotic models such as the 'dark energy'. Understanding the nature of dark energy is the focus of many current studies, and its origin being unknown, these studies are largely focused on model building and constraining its parameters. In this article, we discuss how the accelerated expansion of the Universe is driven by dark energy and how dark energy parameters are constrained by observations.

Harvinder Kaur Jassal is working at IISER Mohali with research interests in theoretical and observational aspects of dark energy. In particular, she is interested in fitting cosmological parameters with a variety of observations and works on theoretical modeling and large scale structure formation.

1. Introduction

Observations in the last two decades have led us to believe that the present-day expansion of the Universe is accelerating. This understanding changed the prevalent paradigm that the Universe is composed almost entirely of ordinary matter and a subdominant relativistic component. The concordant belief among astronomers, at present, is that the accelerated expansion is caused not by matter as we know it, but by a mysterious component referred to as the 'dark energy'. One of the most pertinent questions in modern cosmology is: What is dark energy and what is its nature?

The name dark energy is apt as very little, if at all, is known about this component. The reason is that no known form of matter, radiation or field can lead to an acceleration in the expansion of the Universe. The cosmological equations are derived from the general theory of relativity (GTR) which is a well-established and

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observationally verified [1, 2]. These equations allow for an accelerated expansion only if the component dominating the energy density of the Universe has a negative pressure.

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The cosmological constant was again abandoned when the estimates of the rate of expansion were refined, and the problem was resolved without it. In the late nineties, strong evidence from observations showed that the expansion is accelerated; this brought the cosmological constant into prominence once again. Although there had been indications in observations and there were suggestions by cosmologists that there is a need for the cosmological constant, the supernova observations confirmed that this indeed is the case.

At present, the cosmological constant is the concordant model of cosmology and is consistent with all the observations. This model describes the accelerated expansion in the simplest possible manner. However, the cosmological constant has a severe fine tuning problem as the value required to fit the observations is 121 orders of magnitude lower than what any theoretical model can predict (for a review see [3]).

This is an unresolved issue in theoretical physics. Attempts to circumvent the fine-tuning problem in the cosmological constant model led to a large number of alternative suggestions. This resulted in the proposal of a large number of dark energy models based on fluids and on scalar fields (many of the models have been reviewed in [4]). Much effort is being put in the study of dark energy origin and its properties. Since there is no fundamental theory which naturally predicts dark energy and provides a viable model, the approach undertaken is largely phenomenological. Although different models can be tuned to fit the data well, the cosmological constant is emerging to be the favoured model due to its simplicity.

This does not rule out that dark energy may be described by some other forms of matter or by modifying the theory of gravity. In fact, observations do allow for some variation around the cosmological constant. Therefore, it is important to employ larger and diverse datasets to constrain cosmological parameters, including those which describe dark energy. Due to the large amount and variety of data that is available and is likely to be available in coming years, it is now possible to determine different parameters to a precision which had not been achieved in the past. Many surveys are being planned which are aimed at studying the properties of dark energy.

Dark energy may also be described by some other forms of matter or by modifying the theory of gravity. In fact, observations do allow for some variation around the cosmological constant. Therefore, it is important to employ larger and diverse datasets to constrain cosmological parameters, including those which describe dark energy.

In this brief review, we discuss how the accelerated expansion of the Universe is facilitated by dark energy. We also delve into the details of some of the pending issues with different types of models, and how the models compare with the data. First, we review the cosmological model, and in the following section, we discuss the observations that have been used extensively in cosmological parameter fitting and, in particular, in determining dark energy parameters.

2. Cosmological Equations

The background evolution of the Universe is described by the Friedmann equations (cosmological equations). These equations are given by:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \quad (1)$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda}{3}.$$

These two equations contain all the information we need to study the evolution of the Universe. Here, the quantity $a(t)$ is the scale factor. One can imagine a spherical shell that is expanding; the scale factor encodes the information about the amount by which the shell has expanded. We can safely assume the present-day value of the scale factor to be unity, and all cosmological quantities can be scaled with respect to their present-day values. The quantity ρ is the energy density of all the different components of the Universe, the universal gravitational constant is represented by G, and c is the speed of light.

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The total energy density is given by:

$$\rho = \rho_{\mathrm{NR}} + \rho_{\mathrm{R}} + \rho_{\mathrm{DE}}, \tag{2}$$

where the subscripts 'NR', 'R' and 'DE' denote the non-relativistic, relativistic, and the dark energy components respectively. The non-relativistic matter includes ordinary (baryonic) matter and dark matter. Most of the non-relativistic matter is composed of dark matter, whose presence was first inferred by observations of flat galaxy rotation curves[1]. This component of matter manifests only through the gravitational interaction.

[1] Spiral galaxies have a differential rotation, i.e. the orbital velocity of star and gas depends on their distance from the centre of the galaxy. Purely from luminous matter, it is expected that the velocity will rise and then begin to decrease as a function of distance. However, it was found (by Vera Rubin) that the galaxy rotation curve flattens at larger distances; the orbital velocities, therefore, are larger than expected. This can only be explained if there is an invisible form of matter contributing to the gravitational potential of the galaxy.

At present, the relativistic matter is composed of radiation, in earlier stages, neutrinos are included in this component. The constant 'k' determines the spatial geometry of the Universe, where 'k'=0 implies a spatially flat universe and 'k'=1 and 'k'=-1 correspond to a closed or an open geometry. Observations have shown us that the Universe is spatially flat and, therefore, for the rest of our discussion, we will assume a spatially flat universe. The continuity equation, due to the conservation of energy is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{c}^2\rho a^3\right) = -P\frac{\mathrm{d}a^3}{\mathrm{d}t}, \tag{3}$$

where P is the pressure due to the relevant components. If there is no transfer of energy from one component to another, then each component satisfies this equation. It is important to mention here that Newtonian mechanics is valid only for non-relativistic matter ($P \approx 0$), whereas, general relativity treats all constituents consistently. The cosmological equations mentioned above are equations of motion arising out of general relativity.

It can be seen from these equations that the expansion of the Universe or the rate of the expansion of the Universe depends on its contents. Different constituents of the Universe are characterised by different equations of state. The equation of state is the relation between the pressure and the energy density of a particular component, i.e. $P = w\rho c^2$, where P is the pressure. Here, w is the equation of state parameter for the component.

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In the case of non-relativistic matter, the speed of the particles is negligible compared to the speed of light and can, therefore, be neglected. The pressure on the walls of a box containing a gas of particles is due to the thermal motion of the particles. The non-relativistic matter is, therefore, pressureless ($w \simeq 0$). For the relativistic matter or radiation, the relation between the pressure and energy density is given by:

$$P = \frac{1}{3}\rho c^2.$$

This simple relation can be derived from the kinetic theory of gases where instead of gas molecules, the box is filled with photons. Thus for radiation, $w = 1/3$. For the cosmological constant, the energy density remains a constant, and from the continuity equation, we can see that for a constant ρ, the pressure density relation is:

$$P = -\rho c^2.$$

This relation implies that the pressure for the cosmological constant, i.e. the vacuum energy, is negative, and $w = -1$.

Substituting the above three different energy densities in the continuity equation, we can see that for the non-relativistic matter, the energy density goes as $\rho_{NR} = \rho_{0_{NR}} a^{-3}$, where $\rho_{0_{NR}}$ is the present-day matter density (the subscript 0 represents the value at the present epoch). This relation for relativistic component is $\rho_R = \rho_{0_R} a^{-4}$. Of course, in the case of a cosmological constant, the energy density ρ_Λ remains a constant. The three components lead to different dynamics in different epochs: the expansion of the Universe being governed by the dominant component at that epoch.

To understand the dynamics in detail, we rewrite the first equation in equation array (1) in units of the critical density of the Universe. The critical density ρ_c is defined as:

$$\rho_c = \frac{3\mathrm{H}_0^2}{8\pi \mathrm{G}}, \tag{4}$$

where H_0 is the Hubble constant [2]. The value of the critical density is approximately 10^{-29} g/cc. It is also customary to define the density parameter in the following manner:

[2]Hubble's law states that the galaxies recede from us, and the recession velocity is proportional to the distance from us. This is a linear relation with Hubble constant as the slope and is given by $v = \mathrm{H}_0 D$, where D is the distance. The currently accepted value of the Hubble constant is approximately 70 km/s/Mpc. The distance unit 1 pc = 3.1×10^{16} m. The reader is also referred to the *Resonance* Special Issue on Hubble's law and the expanding universe [5].

$$\Omega_i = \frac{\rho_i}{\rho_C}, \tag{5}$$

where i is for one of the components. The first equation then can be rewritten as:

$$\mathrm{H}^2 = \frac{\dot{a}^2}{a^2} = \mathrm{H}_0^2 \left[\frac{\Omega_\mathrm{R}}{a^4} + \frac{\Omega_\mathrm{NR}}{a^3} + \Omega_\Lambda \right]. \tag{6}$$

The Universe is dominated by relativistic matter (or photons) at the earliest stages of its evolution, i.e. for small values of a. The energy density drops faster than that of other components as the Universe expands. At the radiation-matter equality epoch, the radiation and relativistic matter density becomes equal to that of non-relativistic matter, and at later times, matter takes over as the dominant component of the Universe. Through this evolution, the energy density due to the cosmological constant remains at its fixed value. As the Universe expands further, the matter energy density drops below that of the vacuum energy density, and the Universe is then dominated by dark energy and the acceleration of the expansion begins.

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If the Universe is dominated by non-relativistic matter, the Friedmann equation can be solved to show that the scale factor $a(t) = a_0 t^{2/3}$. The corresponding relation for radiation is $a(t) = a_0 t^{1/2}$. For a cosmological constant, the Universe undergoes an exponential expansion, $a(t) = a_0 e^{\mathrm{H}_o t}$. It is easy to see that for ordinary matter and for radiation, the Universe expands with a decreasing expansion rate. In the cosmological constant dominated epoch, the Universe undergoes an accelerated expansion.

Dark energy, if not a cosmological constant, can be effectively described as a fluid. The equation of state parameter of dark energy has to be $w < -1/3$ for accelerated expansion. For a dark energy dominated universe, the energy density as a function of the scale factor is given by $\rho_{\mathrm{DE}} = \rho_{0_{\mathrm{DE}}} a^{-3(1+w)}$.

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The reader can check that the relation is, in general, true for matter and radiation by substituting the relevant value of w in this relation. The value of the equation of state parameter for the cosmological constant is -1. In general, the equation of state parameter can have a value different from -1 and can also be a function of time. In this article, we will consider w to be a constant for simplicity.

Determination of this equation of state parameter for dark energy from observations is used to constrain the properties of dark energy. This parameter is constrained with different observations in conjunction with other parameters of the cosmological model, such as Ω_{NR}, Ω_{DE}, H_0, etc. Typically, the observations are based on the distance measurements to distant objects. It is appropriate here to introduce the concept of redshift, which is defined by:

$$z = \frac{\lambda_{\mathrm{observed}} - \lambda_{\mathrm{emitted}}}{\lambda_{\mathrm{emitted}}},$$

where λ denotes the wavelength of light. The sources away from us show a shift towards the redder part of the spectrum and hence the name redshift. In the cosmological context, $1+z$ is the reciprocal of the scale factor. The cosmological observable quantities are expressed in terms of the redshift. In the next section, we briefly review observations used for cosmological parameter estimation.

3. Observational Evidence

The measurement of distance requires knowledge of luminosity of an object or of the size of an object, and the notion of distances has a different meaning in cosmology as we can only measure distance along the path of a light ray.

The cosmological parameter determination is largely done using measurements of distances. The measurement of distance requires knowledge of luminosity of an object or of the size of an object. The notion of distances has a different meaning in cosmology as we can only measure distance along the path of a light ray. Further, the effective distance measured through angle subtended by an object and from flux measurements of a standard candle have a different variation with the scale factor.

For cosmologists, the brightest standard(ized) candles are the supernovae of type Ia. These supernovae are not of the type we have read about usually, i.e. these are not the explosions which happen at the death of a star. This type of supernova occurs if a white dwarf[3] in a binary system with another star accretes matter from its companion. Once it has accreted enough mass so that the Chandrasekhar mass limit of 1.4 $M_{\odot}$ is exceeded, the system is no longer stable as the gravitational force exceeds the opposing force of what is known as the 'electron degeneracy pressure'. The degeneracy pressure is due to resistance to compressing matter in very small volumes as Pauli exclusion principle states that two electrons with identical quantum numbers cannot occupy the same quantum states.

[3] A white dwarf is a stellar remnant; the remnant after the death of a low mass star. For example, the Sun will end up as a white dwarf star. The Chandrashekhar mass limit is the maximum mass the electron degeneracy pressure can hold up against.

The imbalance leads to an infall of matter and nuclear reactions that leads to an explosion of the outer shells. Since the initial conditions are the same for all such explosions, it is reasonable to assume that these type of supernovae are good candidates for a standard candle. Small variations may be caused by rotation, chemical composition or magnetic field.

Observations of supernovae of type Ia were the first set of observations to give conclusive evidence of acceleration of the expansion of the Universe [6]. These supernovae are 'standard' candles, as their intrinsic luminosity is the same. Therefore, the apparent luminosity depends only on the distance between the supernova and the observer: in this case, the distance being the luminosity distance $D_{\rm L}$. The relevant quantity is the distance modulus which is the difference between the apparent magnitude that is observed and the absolute magnitude (related to the luminosity) of the supernova and is given by:

$$\mu = m - M = 5\log\left(\frac{D_{\rm L}}{1pc}\right) - 5 \ ,$$

where $D_{\rm L} = a(t_o)r(z)(1+z)$, the quantity $r(z)$ being the coordinate distance from the observer to the source.

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Another way to find the distance to an object is to measure the angular diameter distance $D_{\rm A}$ to it, which is a comparison of the angular sizes with physical dimensions. The angular diameter distance is defined as:

$$D_{\rm A} = \frac{\theta}{s}, \qquad (7)$$

where s is the proper size of the object (perpendicular to the line of sight) and equals $a(t)r\theta$. The two distances are related as:

$$D_{\rm L} = (1 + z)^2 D_{\rm A}.$$

Both of these can be used to estimate cosmological parameters.

The cosmic microwave background radiation (CMBR) is the relic radiation from the early Universe when we expect the temperature and density to be high. The early Universe was composed of a very hot plasma where matter coupled strongly with photons due to scattering; photons scatter off electrons. As the Universe expanded, the plasma cooled, and the coupling between the matter and photons weakened until the photons broke free. These photons have been streaming as the background radiation and have cooled to 3 Kelvin today. The photons seem to be coming to us from all directions and apparently from a sphere around us, namely the 'last scattering surface'. The physics of the CMB is well understood.

Perturbations in this baryon-photon fluid behave like sound waves, and these acoustic oscillations can be seen in fluctuations in the CMBR. Anisotropies in the CMBR [7] arising due to acoustic oscillations in the baryon-photon fluid in the early Universe have a fixed scale and are used as a standard ruler.

The acoustic oscillation signature is also carried by the baryons, and this signal shows up in the two-point correlation function of large luminous galaxies [8] and overall matter distribution [9]. The baryon acoustic oscillations (BAO) data holds the promise of constraining the cosmological constants to a very small allowed range.

The cosmological parameters are also determined using the direct measurements of the Hubble parameter at different redshifts. These measurements are coupled with the measurements of either the BAO signal or with the cosmic chronometer measurements such as the age redshift relation of slowly evolving galaxies.

Another set of observations which have been used in determining the cosmological parameters are the direct measurements of the Hubble parameter at different redshifts (data collated in [10]). These measurements are coupled with the measurements of either the BAO signal or with the cosmic chronometer measurements such as the age redshift relation of slowly evolving galaxies.

In *Figure* 1, we show the variation of the luminosity distance and the angular diameter distance as a function of redshift. In this figure, we have restricted ourselves to the cosmological constant model. These quantities have been scaled in terms of the Hubble radius, a characteristic scale which refers to the size of the Universe. The different coloured lines correspond to different values of the matter density parameter. While the luminosity distance is a monotonic function of redshift, the angular diameter distance reaches a maximum and then begins to decrease. This implies that the distant objects appear larger than they would be if it had only been a flat spacetime.

The theoretical models are fitted with the observational data, and the method which is most extensively used is the standard χ^2 minimisation method.

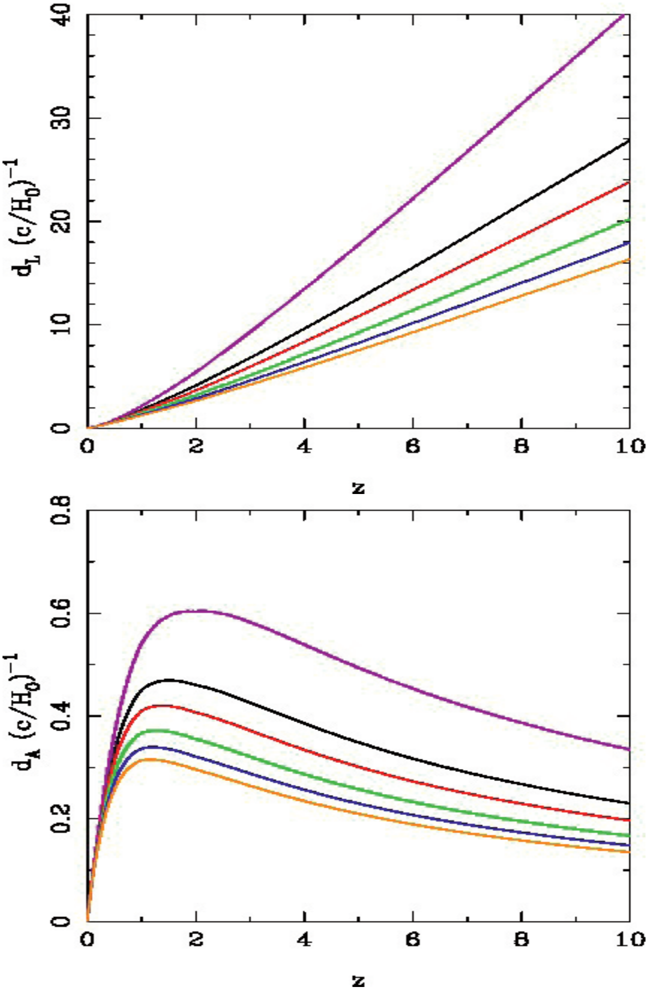

Figure 1. The figure shows the luminosity distance and the angular diameter distance as a function of redshift, for the ΛCDM (cosmological constant) model. The different colours correspond to different values of the matter density parameter Ω_m. The different colours of the curves correspond to Ω_{NR} = 0.1 (pink), 0.27 (black), 0.4 (red), 0.6 (green), 0.8 (blue) and 1.0 (orange) respectively.

The quantity χ^2 is given by:

$$\chi^2 = \sum_{\text{All observations}} \frac{(x_{\text{theoretical}} - x_{\text{observational}})^2}{\sigma^2}, \tag{8}$$

where $x_{\text{theoretical}}$ and $x_{\text{observational}}$ are the theoretical and observational quantities to be compared and σ^2 is the error associated with the measurement. Contribution of each individual observation is added. Minimum χ^2 implies maximum likelihood. The method is an extension of the more familiar least squares method. The connection between χ^2 and maximum likelihood assumes a Gaussian distribution of errors σ. The 1σ, 2σ and 3σ confidence levels correspond to a likelihood (probability) of 68%, 95% and

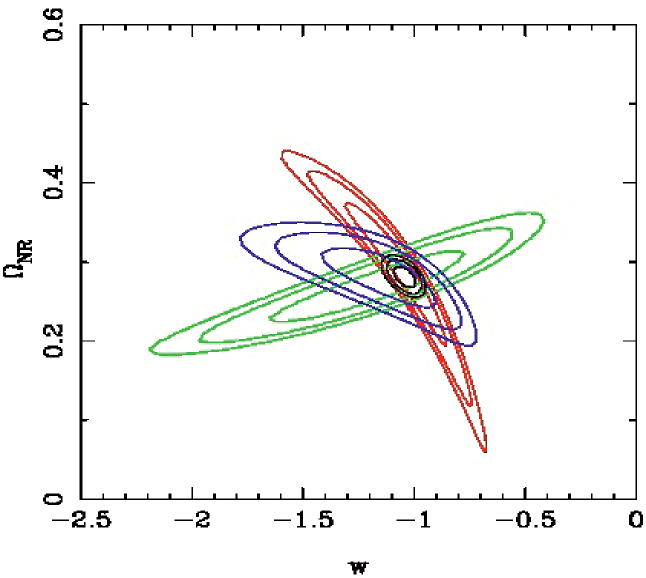

Figure 2. This figure shows the allowed range of the matter density parameter Ω_{NR} and the equation of state parameter w by supernova type Ia (red contours), baryon acoustic oscillations (green), and direct measurements of Hubble parameter observations (in blue). The three contours are for 1σ, 2σ and 3σ confidence level. The black contours are for combined constraints.

99.9% respectively. Searching for a minimum χ^2 can be either done on a grid of parameters or by more Monte Carlo searches based algorithms. The relevant approach is to map out regions of parameter space that are allowed by observations and not restrict attention to the best fit model.

Data from observations is consistent with a narrow range of cosmological parameters. For a spatially flat universe, we neglect the radiation component, and we are then left with only one independent density parameter. This can be either Ω_{DE} or Ω_{NR} and $\Omega_{NR} + \Omega_{DE} = 1$. The second parameter we consider is the equation of state parameter w. We can consider several other parameters, but for simplicity we focus our attention on these two and illustrate the evidence for dark energy. In *Figure* 2, we show the allowed ranges of the matter density parameter Ω_M and the equation of state parameter for dark energy w at 68%, 95% and 99% confidence level for the supernova type Ia [11], BAO (data points collected in [12] and direct measurements of Hubble parameter data [10]. The black contours are constraints obtained by combining these three observations. For a spatially flat universe, the value of the matter and hence dark energy density parameter is consistent with values approximately 30% and 70% respectively

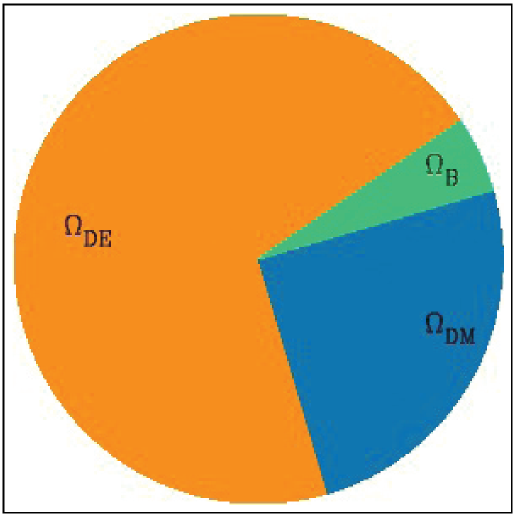

Figure 3. This figure shows the share of the dark energy, dark matter and baryonic matter in the energy budget of the Universe. The non-relativistic matter is composed of both the baryonic and dark matter components and are represented in the green and blue regions respectively. The orange region is the energy contribution due to dark energy.

of the energy budget of the Universe, the allowed range by these three observations put together is between 0.25 and 0.31. The parameter w lies between -1.13 and -0.95. It can be clearly seen from this figure that supernova data puts stronger constraints on the value of the dark energy equation of state parameter while the BAO data constrains the matter density parameter very strongly.

The direct measurement of Hubble parameter data constrains an intermediate range in both. The observations which prefer a lower value of the matter density parameter, favour a higher equation of state parameter of dark energy. For a fluid with a variable equation of state parameter, the best fit value of the matter density is slightly modified and the allowed range increases. The results quoted here are consistent with CMB constrains obtained by the Planck mission [13]. While the individual observations allow for a large range in cosmological parameters, the complementarity of these observations reduce the allowed range significantly.

The energy budget of the Universe can be illustrated well using the pie chart shown in *Figure 3*. The orange region is the fraction of dark energy in the Universe, while the blue and green regions collectively show the contribution of the non-relativistic matter. Dark energy cosmology has also been discussed in [14], wherein, structure formation in such models has been studied.

4. Summary and Conclusion

The goal of understanding the accelerated expansion and to understand the origin of dark energy continues to be challenging for the last two decades. We still lack a fundamental theory which predicts dark energy and leads to a viable cosmology.

In this article, we attempt to understand how dark energy drives the present-day accelerated expansion of the Universe. The goal of understanding the accelerated expansion and to understand the origin of dark energy continues to be challenging for the last two decades. We still lack a fundamental theory which predicts dark energy and leads to a viable cosmology.

These difficulties apart, we can determine the cosmological parameters which are consistent with various observations. The observations are sensitive to different cosmological quantities, and hence give different ranges of cosmological parameters. A combination of them is a better indicator of the range of cosmological parameters as these parameters are consistent with varied observations. A large number of surveys are currently collecting data, and various surveys have been designed especially to study dark energy properties (for example the *Dark Energy Survey* [15]). The coming decade is, therefore, an exciting time to work on aspects of dark energy, both theoretical and observational.

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Address for Correspondence
H K Jassal
Indian Institute of Science Education and Research Mohali,
Sector 81, SAS Nagar, PO Manauli,
Punjab 140 306, India.
Email: hkjassal@iisermohali.ac.in