Some Thought Experiments in Physics

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The important role played by some thought experiments in the development of physics has been brought out by expatiating Maxwell's demon, time dilation, length contraction, twin paradox, Heisenberg's microscope and Einstein–Bohr box.

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1. Introduction

Suppose one has a new idea and wants to learn about the feasibility and important effects of its implementation. The simplest approach to achieve the aim would be to devise means for performing some suitable experiment, record the observations and analyse these in a systematic way. However, sometimes it may be difficult or even impossible to carry out a relevant actual experiment, and one might have to deliberate upon various issues involved purely at an intellectual level. This mental process of learning about reality by thoroughly analysing the underlying basic principles, intricacies and possible consequences of the conceived idea is known as a 'thought experiment' or 'gedankenversuch' (in German). As such, a thought experiment is a substitute for real experimentation, and this approach has been commonly followed in philosophy, economics and physical sciences particularly theoretical physics because of its abstract nature. The main feature of such an intellectual endeavour is that the person involved has the facility of an ideal workshop in which any kind of equipment can be fabricated with only requirement that its design and function do not contradict the basic laws of physics.

Keywords
Thought experiments, Maxwell's demon, time dilation, length contraction, twin paradox, Heisenberg's microscope, Einstein–Bohr box.

Though quite a good number of examples of ingenious visualization in physics can be found from the times of Galileo and Newton (and even before that), we plan to describe in this article, few selected cases (from the 19th and 20th centuries only), which have made significant contribution to clarifying the relevant concepts and, therefore, proved to be landmarks in our understanding as well as development of the subject. Interestingly, some of these or their related aspects continue to be topics of lively discussion even in the current research pursuits. It may be added that though the advent of fast computers has made simulations a possible alternative to thought experiments, the two differ significantly at an epistemological level just as two surgeons may be functionally replaceable but may not have the same talent and trustworthiness. Besides, advances in technology have made it possible to experimentally verify some of the earlier thought experiments.

2. Maxwell's Demon

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In his 1860 paper entitled 'Illustrations of the Dynamical Theory of Gases', Maxwell used general principles involving statistical ideas to derive an expression for the velocity distribution of gas molecules with mass m and in thermal equilibrium at temperature T (Kelvin). According to this formula, now known as the Maxwell–Boltzmann distribution because of rigorous studies carried out by the latter, the fractions of the gas molecules with low as well as high speeds are quite small and the plot of the distribution as a function of speed (v) shows a maximum for

$$v_{\text{max}} = (2\text{k}T/m)^{1/2}\,, \tag{1}$$

while the mean speed of the molecules is given by

$$v_{\text{mean}} = (8\text{k}T/m\pi)^{1/2}\,. \tag{2}$$

Here, $\text{k} = 1.38\times10^{-23}\ \text{JK}^{-1}$ is the Boltzmann constant. Following this work and his other publications in this direction, in a letter in 1867, Maxwell put forward a thought experiment to bring out a possible violation of the second law of thermodynamics. He described this idea in a proper perspective in his book *Theory of Heat* in 1871 and used it to bring out the statistical nature of the second law of thermodynamics.

In order to understand this imaginary experiment, we consider a vessel having thermally insulating walls filled with an ideal gas at equilibrium temperature T so that the molecules have a velocity distribution characteristic of this temperature and do not have any intermolecular interactions. We divide the vessel into two equal compartments labelled 1 and 2, with a wall having a hole of such dimensions that only one gas molecule can pass through this at a time. The hole is provided with a tiny shutter that can be opened or closed as per requirement of the situation; *Figure* 1(a). Obviously, the gases in the two chambers will be at the same temperature T and the molecules will have speeds above as well as below the mean value $v_{\text{mean}} = (8kT/m\pi)^{1/2}$. The shutter of the hole in the wall is effortlessly operated by an extremely small but intelligent creature who is looking at the hole and meticulously observes each molecule approaching it from either side; this super entity was called 'finite being' by Maxwell. If the molecule coming towards the hole in chamber 1 has a speed higher than v_{mean}, then the shutter is opened to permit this molecule to enter chamber 2 on the right; *Figure* 1(b). However, a molecule with speed less than v_{mean} is stopped and continues to stay in chamber 1 itself; *Figure* 1(c). In contrast, the molecules with speed lower than v_{mean} in chamber 2 are allowed entry to chamber 1, *Figure* 1(b), while those with higher speeds are not permitted to do so, *Figure* 1(c). As the process of crossing over of gas molecules with lower speeds to chamber 1 and those with higher speeds to chamber 2 progresses, the average speed and hence the average kinetic energy of molecules in the first chamber becomes less and less while these averages in the second chamber become more and more. In other words, the temperature of the gas in chamber 1 is decreasing while that in chamber 2 is increasing as compared to the initial same temperature T in both the compartments. Thus, the discerning action of the little creature is permitting flow of heat from colder chamber 1 to hotter chamber 2. After a sufficiently long time, chamber 1 will have almost all the molecules with speed lower than v_{mean} whereas all the molecules in the other chamber will have a speed higher than v_{mean}; *Figure* 1(d). Thus, the average speed of the molecules in chamber 1 will be less than

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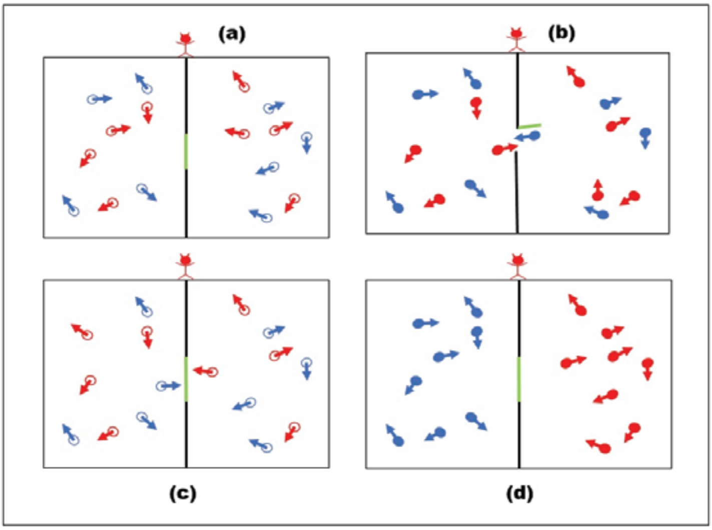

Figure 1. Maxwell's demon sitting on top of the partition of the gas container. Red circles represent molecules having speed $v > v_{\text{mean}}$, while blue circles correspond to $v < v_{\text{mean}}$.

v_{mean} and it will be more than v_{mean} in chamber 2. Consequently, the equilibrium temperatures T_1 and T_2 corresponding to the distribution of velocities in chambers 1 and 2 will be such that $T_1 < T_2$ implying that the gas in the second chamber will be hotter than that in the first. It is pertinent to note that this miraculous segregation of the gas at temperature T into colder and hotter chambers has been brought about by the 'hypothetical little creature' without spending any energy. Now, if we connect a heat engine between the two chambers, the temperature difference $T_2 - T_1$ can be used to produce mechanical work without spending any energy at any stage.

An alternative mode of operation of the little being can be that it opens the shutter when a molecule approaches it in chamber 1 and thus allows the molecules to go from chamber 1 to chamber 2 but not in the opposite direction. Therefore, the number of molecules and hence the pressure of the gas in chamber 2 are continuously increasing at the cost of the corresponding quantities in chamber 1. When sufficient pressure difference is created we can attach a cylinder-piston system between the two chambers and thereby use $p_2 - p_1$ to produce motion of the piston and hence to do work without spending any energy in the process, thanks to the marvellous little creature.

Clearly, all the situations arrived at by the superb act of Maxwell's finite creature are in violation of the second law of thermodynamics. This fantastic being was named 'Maxwell's demon' by William Thomson (later Lord Kelvin) in 1874. Though Maxwell did express his resentment on this name, yet it continues to be used. Not only that, any possible arrangement contravening the second law is generally referred to as Maxwell's demon.

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It may be noted that the passage of the fast or more energetic molecules from chamber 1 to chamber 2 amounts to transferring heat from the colder compartment to the hotter one. If at any instant of time the equilibrium temperatures of these chambers are T_c and T_h, respectively, and heat exchanged in an infinitesimally small time Δt is ΔQ, then $\Delta Q/T_c > \Delta Q/T_h$ because $T_c < T_h$. But the ratio considered here gives the change in entropy ΔS of the corresponding chamber. Therefore, the preceding inequality implies that the decrease in entropy of chamber 1 in time Δt is more than the increase in its value in chamber 2. Accordingly, over any finite time interval, the reduction in entropy of the first chamber is higher than the increase in its value in the other chamber to which hot molecules have got transferred. Thus, the selective action of the demon has produced a net decrease in the entropy of the whole system. In a similar manner, if the demon allows only single way passage from chamber 1 to chamber 2, then the situation effectively being created is that the gas molecules originally occupying volume V are being confined to volume $V/2$. For a perfect gas, the associated change in entropy is given by $N\mathrm{k} \ln (V_{\mathrm{final}}/V_{\mathrm{initial}}) = N\mathrm{k} \ln (1/2) = -0.69N\mathrm{k}$. This too means that the entropy of the system is getting reduced. But the decrease in entropy of the whole system is in contradiction with the second law of thermodynamics, which demands that the entropy of the universe should always increase.

A critique of this contrivance is that in order to have information about the movement of the gas molecules, the demon needs some radiation (light) to be scattered from the molecules and, thus, necessarily uses some photon energy in discriminating and

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separating the molecules into two compartments to develop temperature or pressure difference used for doing some mechanical work. Besides, opening and closing of the shutter does involve the expenditure of some energy. As such the complete process definitely consumes energy and it is found that the total entropy of the system increases. It has been also contended that the demon must erase the information about the measurement of the speed of a molecule from its memory before gathering this for the next molecule. This periodic resetting of memory too involves enhancement in the entropy of the system. In fact, many physicists have put forward different arguments based on information theory, quantum measurement theory, analysis of the limits of computing, etc., to make the things in accord with the second law of thermodynamics. The prominent contributors to these developments were Szilard (1929), Brillouin (1951), Gabor (1951), Landauer (1960), Penrose (1970), and Bennett (1982). In spite of this long history of debates, the issue is not yet settled, and theoretical discussions and experimental efforts to realize the prototypes of the demon in the laboratory continue to thrive even today.

The present technology has made it possible to trap and manipulate individual electrons, atoms and molecules. This, in turn, has enabled physicists to create different versions of Maxwell's demon in the quantum regime. Among the two very recent efforts in this direction, one constitutes the demon monitoring the tunneling of an electron in two capacitively coupled single electron transistors fabricated on the same chip, while in the second, imbalance between two light beams has been used for this purpose. However, we do not plan to go into the details of these setups and refer the interested readers to the relevant references [1, 2]. Nonetheless, it may be mentioned that in such experiments various quantum effects become important, and when the related subtle aspects pertaining to measurement and information erasure are taken into account, the second law of thermodynamics remains intact in a broad sense. These considerations have, in turn, led to better understanding of quantum information processing and quantum heat engines and the development of quantum thermodynamics.

It is worth noting that the persistent extensive debates about Maxwell's demon have not only helped in gaining insight into thermodynamics and statistical mechanics but also in quantum mechanics, classical as well as quantum information theory, cybernetics, and theory of computation. The ups and downs associated with the pursuits concerning the Maxwell's demon and its importance have been appropriately brought out by the starting and the closing sentences of the introductory chapter in the excellent book edited by Leff and Rex [3], which gives a detailed account of different aspects of this fascinating topic investigated by numerous researchers up to 2002, highlights the basic thoughts of the main contributors, and includes reprints of the key publications in various fields from the beginning till the year 2000. These statements read, "After more than 130 years of uncertain life and at least two pronouncements of death, this fanciful character seems more vibrant than ever.... Considering its rich history and present research trends ..., we expect Maxwell's demon to remain a potent teacher for many years to come!"

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3. Time Dilation, Length Contraction and Twin Paradox

One of the postulates of Einstein's radically new concepts based special theory of relativity, presented in 1905, is that the speed of light in vacuum or free space is always the same ($c = 3 \times 10^8$ ms^{-1}) irrespective of the uniform relative motion between the inertial frames of reference, the source and the observer. This constancy of the speed implied that measurement of time intervals and lengths must be affected by the relative motion of the observers. The logistic implications of these kinematic consequences of the theory were two of its many thought-provoking ideas and continued to be out of reach of the actual experimentation for many decades. We introduce these phenomena in the sequel below.

Suppose S and S′ are two inertial frames of reference with parallel axes and the latter is moving along the positive x-direction with uniform high-speed v. Obviously, for an observer in S′ the frame S will be moving along negative x-direction with the same speed. In order to measure a time interval we use clocks (which may be mechanical, electronic, atomic, or light flashes based devices, or even biological processes) kept at fixed points in the two frames and initially synchronised to the highest possible degree of precision with respect to each other when the origins O and O′ of the two frames coincided. We assume that S′ has a clock C′ placed at some point and as this frame moves its clock passes by the synchronised clocks kept in the frame S. Let the time interval between two events, say two light flashes, occurring at the same place at times t'_1 and t'_2 as observed by observer at O′ in S′ be $\tau' = t'_2 - t'_1$. This is 'local', 'rest', 'proper' or 'ordinary' time interval for this observer because the frame of reference in which an observer or a device is stationary is referred to by these adjectives. The corresponding time interval measured in S with the help of the clocks coinciding with the position of C′ when it read t'_1 and t'_2 is given by

$$\tau = \gamma \tau' , \tag{3}$$

where γ, known as 'Lorentz factor', is

$$\gamma = (1 - \beta^2)^{-1/2} \tag{4}$$

with $\beta = v/\text{c}$; c being the speed of light in vacuum. Here, τ is called the 'relative time interval' as this is the observation made in the frame with respect to which the clock C′ is moving.

From (3), we note that as the observer at O′ in S′ records a proper time interval of 1 second in its clock C′, the observer at O in S will measure this interval in its clocks as γ seconds. Since v is always less than c, γ is larger than unity and the difference becomes significant for speeds v close to c, called the relativistic speeds. For example, values of γ for v = 0.1c, 0.8c and 0.95c are 1.005, 1.67 and 3.20, respectively; in fact, γ departs from 1 by less than 5% when $\beta < 0.3$. Therefore, the relative time interval τ

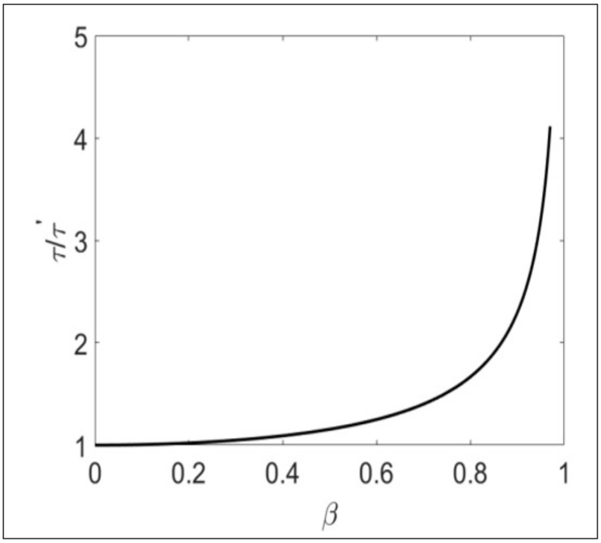

Figure 2. Variation of τ/τ' with β.

is always larger than the proper time interval τ' by a factor γ. This increase in the value of the time interval is called 'time dilation' or 'dilatation'. The trend for variation of τ/τ' with β is shown in *Figure* 2.

The fact that the preceding formula involves β^2 or v^2 implies that a change in sign of v has no effect on the dilation observed. Thus, if τ' (or τ) is the proper time interval in S′ (or S), its relative measure τ (or τ') in S (or S′) will be given by the above expression by interchanging the primed and unprimed quantities. Therefore, the time dilatation of moving clocks is a perfectly symmetrical or reciprocal affair with respect to the two moving inertial frames. It may be mentioned that the proper time interval is the shortest and the relative time intervals are longer meaning thereby that the moving clocks (irrespective of their nature) appear to be running slower or retarded by a factor γ; *Figure* 3 corresponds to $\gamma = 3$ or $\beta = 0.943$. Furthermore, the observations are not affected whether the clock is approaching or receding from the observer.

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The theoretical predictions about time dilation were experimentally confirmed in 1941 when it was found that the apparent half-life τ of the fast-moving μ mesons (now called muons) in the cosmic rays as determined in the laboratory was significantly larger

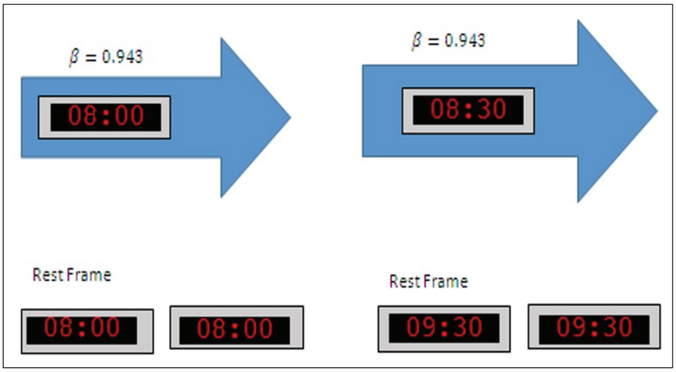

Figure 3. A pictorial depiction of time dilation.

than the rest or proper half-life τ' and the results were in reasonable agreement with the value obtained from (3) by treating these elementary particles as tiny clocks. Since then the phenomenon of time dilation has been verified by numerous different types of experiments including Mossbauer spectroscopy (1960), a modified version of the study of cosmic ray muons (1963) and Laser spectroscopy (2003). Recently, Chou and coworkers [4] have further validated this effect by comparing two optical atomic clocks connected by a 75 m long optical fibre and moving with a non-relativistic speed of about 10 ms^{-1}.

The process of measurement of the length of an object, say a rod, requires placing it alongside a reference scale (meter rod) and noting the reading for the two ends of the rod. This process can be carried out at leisure if the rod and the measuring scale are at rest with respect to each other. However, the situation changes if the two are located in different frames of reference in relative uniform motion. We have to look for an arrangement by which the scale readings for the two ends of the rod are recorded simultaneously. Otherwise, the two will get displaced relative to each other making the measurement meaningless. Once again we assume that the rod is at rest along the x-axis in S′ frame and has its endpoints at coordinates x'_1 and x'_2. The length found by observer at O′ in this frame will be $L' = x'_2 - x'_1$. This gives local, rest, proper or ordinary length of the rod. The observer at O in S notes the simultaneous (at time t) readings x'_1 and x'_2 on the scale in his frame corresponding to the two ends of the rod moving in S′ with respect to him. The length $L = x_2 - x_1$, known as the relative length, found out by him turns out to be

$$L = L'/\gamma = L'(1 - \beta^2)^{1/2}. \qquad (5)$$

A moving meter scale will always look lesser in length along the direction of its motion whether it is coming towards or going away from an observer.

From the remarks made after (4), it is clear that $L < L'$ so that the proper length is always the greatest and the length contraction will be negligible if v is much smaller than c. *Figure* 4 represents the dependence of L/L' on β. Since the expression for the reduced length is the same as proposed by Fitzgerald and Lorentz, this phenomenon is usually called the 'Lorentz–Fitzgerald contraction' or simply Lorentz contraction though they had assumed the decrease to be a real physical happening, whereas, in special relativity it is a consequence of the process of measurement. Once again, like time dilation, the length contraction too is a mutually reciprocal or symmetric phenomenon with respect to the two frames in relative motion with uniform speed. Thus, a moving meter scale will always look lesser in length along the direction of its motion whether it is coming towards or going away from an observer. It may be mentioned that no such contraction is observed if the rod is placed perpendicular to the direction of motion of S′ frame with respect to S frame; i.e., along the y and z directions. In other words, moving rods appear to be shorter by a factor γ in the direction of their motion and there is no effect in the perpendicular dimensions. Thus, a metre scale moving through space at $\beta = 0.866$ would appear to an observer on the earth as a 50 cm scale having the same width and thickness; *Figure* 5. As an extension of this, imagine the appearance of a 1.7 m tall man swimming through space at the same or some higher speed. It may be mentioned that in *Figure* 3, we have not indicated the effect of length contraction on the clock, whose screen along x-direction would be shortened by a factor γ.

However, in contrast with the case of time dilation, direct experimental proof for length contraction has not been possible even after more than 11 decades. This is because even with the present technology, large objects that can yield observable decrease in

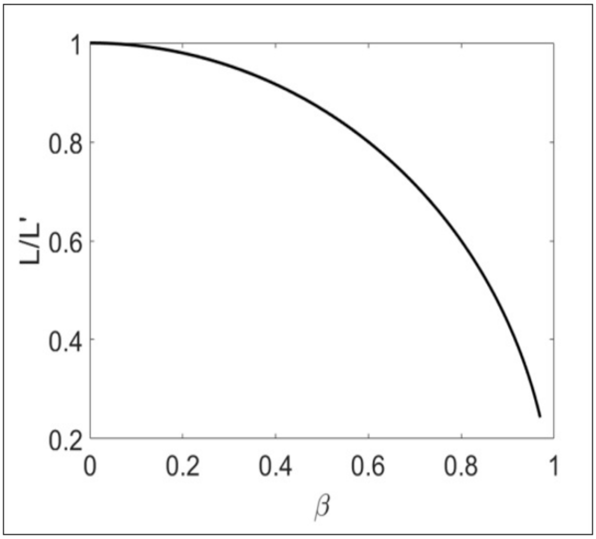

Figure 4. Dependence of L/L' on β.

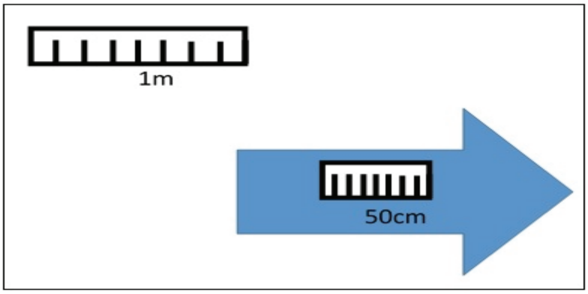

Figure 5. This is how a metre rod moving in space at a speed corresponding to $\beta = 0.866$ would appear on the earth.

length cannot be accelerated to speeds comparable with that of light, and the atomic particles that can travel with relativistic speeds have so small diameters that the reduction in these cannot be measured directly. Nonetheless, numerous indirect pieces of evidence do support this phenomenon. For example, the observed increase in the lifetime of muons can also be explained as follows. To an observer moving in the frame of the muons, the distance measured in this frame as compared to that in the laboratory frame would appear contracted by a factor of γ (i.e., $L_{\text{meson}} = L_{\text{lab}}/\gamma$); and it is covered with speed v in the proper half-life τ', which, in turn, can be interpreted as dilation in the half-life in the frame of the person performing the experiment. Furthermore, the interpretation of the results obtained from the experiments on collision of heavy ions moving with relativistic speeds, performed at Brookhaven National Laboratory (New York) and CERN (Geneva) during the last decade requires incorporating the increase in nucleon density arising from the change in shape to flat discs from the original spherical due to length contraction.

As a follow up of the above discussion, corresponding to the hint given by Einstein in his papers, we consider the situation that observer at O′ in the inertial frame S′ with its clock C′ moves away from the frame S with its clock C on the earth at time $t = 0$, and comes back to it after a long voyage at a relativistic speed. According to the observer at O in S, the clock C′ in S′ was moving at a high-speed and, therefore, it was running slower than his own clock C so that the time interval of their separation as shown by C′ will be less than that recorded by C. In contrast with this, the observer O′ in S′ can equally claim that his inertial frame was at rest and the frame S with its clock C was in motion so that the latter should show lesser measure of time interval. Obviously, both the observers cannot be correct and this contradiction leads to the so-called 'clock paradox'. However, following Langevin (1911), if we replace the clocks C and C′ by some biological processes like heartbeats, pulse, rate of respiration, etc., we get the 'twin paradox' or the 'Langevin effect'. Stated in words, this means that when a twin, named Mohit, travelling in a very fast moving spaceship reunites with his twin brother Rohit on the earth after a long round trip, each of them will think that the other sibling had gone away from him so that his biological clock would lag behind and he would look younger by a factor γ.

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A wide range of different arguments have been put forward to unravel this thought-based paradox. The most popular common solution is based on the idea that the twin travelling in the spaceship, Mohit in our example, experiences acceleration and deceleration during his forward and back journeys so that his frame of reference S′ is non-inertial at least for some time, while the other twin, namely Rohit, continues to be in the inertial frame S. Thus, the distinguishing features of the former lead to loss of inherent

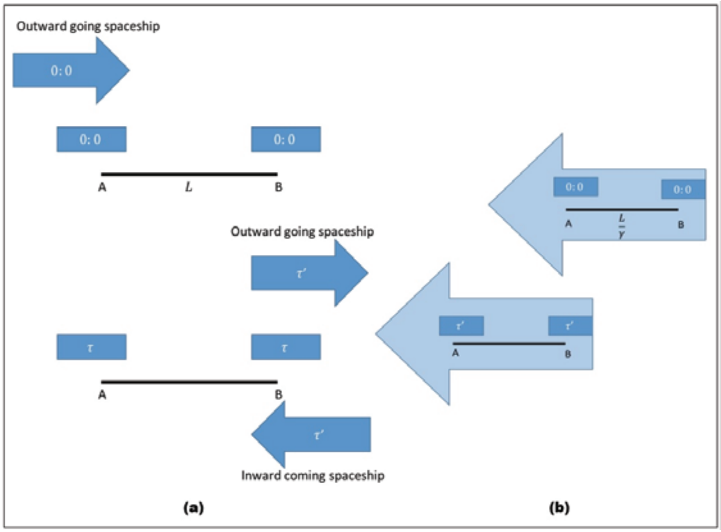

Figure 6. Solution of the clock paradox. **(a)** As measured by the observer at A on the earth (S frame). **(b)** Measurements recorded by the observer in the outward going spaceship (S′ frame).

symmetry of the two systems. While analysing the problem, the effect of field of force associated with the acceleration and deceleration of S′ frame must be taken into account to determine the time elapsed as recorded by Mohit. Proceeding in this manner, it is found that the time interval noted by Rohit is more than that measured by Mohit meaning thereby that the former will look older than the latter in terms of all the corporeal functions.

However, guided by [5] we give a relatively simple treatment in terms of clocks. Suppose the to and fro journey is carried out between two very distant points A on the earth and B in the space. We choose the frame of reference S such that A as well as B are at rest in it and the x-axis of the frame is along the line joining these. We imagine that there are two separate but perfectly identical outward-going and inward-coming spaceships moving with the same uniform relativistic speed v, and the reference frame S′ is assumed to be attached with these for the corresponding situations; *Figure* 6. The outward-going spaceship crosses point A at time $t = 0$ and all the clocks being used in S as well as S′ are synchronised with respect to each other.

For the observer at A in S, the distance AB is proper length L and the spaceship is moving towards B at uniform speed v. So the time interval recorded in his clock for the time taken by the observer in the spaceship to reach point B will be

$$\tau = L/v\,. \tag{6}$$

Next, if we look at things from the point of view of the observer in the moving frame S′ (spaceship), the separation line AB in S moves in the negative direction of motion of the spaceship with the same speed v. Therefore, length L appears to be Lorentz contracted and his measure of (relative) distance between A and B is given by

$$L' = L/\gamma\,. \tag{7}$$

As he covers this distance with speed v, the time interval determined by him for traversing the distance from A to B will be

$$\tau' = \mathrm{L}'/\mathrm{v}, \tag{8}$$

which on being combined with (6) and (7) becomes

$$\tau' = \tau/\gamma\,. \tag{9}$$

This is the same as found from the dilation of time interval τ, namely, (3).

We assume that the moment the outgoing-spaceship arrives at B the inward-coming spaceship crosses this point and their clocks are precisely synchronised. Exactly same time intervals will be obtained for the return journey from point B to point A by the observer located at A (see (6)) and the one travelling in the inward-moving spaceship (see (8)). The observer at A, who is not aware of the arrangement in respect of the two spaceships, will think that he was separated from the observer in the spaceship for a time interval 2τ while the clock in the spaceship will show this time interval as $2\tau'$ such that

$$2\tau' = 2\tau/\gamma \,. \tag{10}$$

Since the time dilation is the same for all types of clocks, the same results will hold good for the biological processes (heart-beats, pulse, rate of respiration, etc.), and bringing in the twins Rohit and Mohit in place of clock at point A and that in the outward-going and inward-coming spaceships, respectively, the parameters for Mohit would indicate that he has grown by time $2\tau'$ whereas those for Rohit will correspond to his growth by time interval 2τ. Here, we are assuming that the jerks felt by Mohit in changing from one spaceship to the other at point B have no biological consequences. In view of (10), we find that the ratio of increase in the ages of Mohit and Rohit is given by

$$2\tau'/2\tau = 1/\gamma = (1-\beta^2)^{1/2} < 1 \,. \tag{11}$$

Mohit has grown less older as compared to Rohit and both of them agree about this. The reasonably good agreement between the predicted and the experimental values has vindicated the assertion about the different ageing of the twins.

Thus, Mohit has grown less older as compared to Rohit or the former will be younger when they reunite after the former's long journey with relativistic speed $v = \beta c$ and in view of the fact that time intervals τ and τ' are, respectively, measured by Mohit and Rohit, (6) and (8), we assert that both of them agree about this.

With a view to check the asymmetry associated with the time intervals recorded by the clock stationary on the earth and that undertaking a forward and back voyage, Hafele and Keating (1972) [6] flew four very accurate cesium beam clocks on jet airplanes around the world – once eastward and once westward – and compared their time intervals for the trips with the similar reference clocks on the earth. Considering the relativistic aspects and taking into account the effect of the west-to-east rotation of the earth about its axis, theoretical calculations showed that the moving clocks would lose $(40 \pm 23) \times 10^{-9}$ s during the eastward flight and would gain $(275 \pm 21) \times 10^{-9}$ s in the westward trip. The corresponding measured results were $(59 \pm 10) \times 10^{-9}$ s and $(273 \pm 7) \times 10^{-9}$ s, respectively. The reasonably good agreement between the predicted and the experimental values has vindicated the assertion about the different ageing of the twins.

4. Heisenberg's Microscope and Einstein–Bohr Box

One of the striking features of quantum mechanics that distinguishes it from the classical description of the physical world is the immensely acclaimed uncertainty principle or principle of indeterminacy. This is essentially a set of mathematical inequalities which impose a limit on the accuracy with which certain pairs of physical observables called the canonically conjugate variables of a particle having size of a molecule, an atom or even smaller, can be measured simultaneously or at the same place under such ideal conditions that there are no instrumental or experimental errors. According to the most popular one-dimensional position-momentum uncertainty rule if position x and the corresponding component of linear momentum p_{x} are determined together, then the uncertainties Δx and Δp_{x} in these quantities are such that their product is greater than or equal to h/4π. Thus,

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$$\Delta x \, \Delta p_{\mathrm{x}} \geq \mathrm{h}/4\pi = \hbar/2; \tag{12}$$

h = 6.626 $\times 10^{-34}$ Js is the Planck's constant and $\hbar$ = h/2π = 1.055 $\times$ 10^{-34} Js is called the reduced Planck's constant. Strictly speaking, Δx and Δp_{x} are the relevant root mean square or standard deviations from the corresponding average values for a large collection of identically prepared quantum systems. A rigorous derivation of the inequality in (12) involving the concepts of vector spaces was put forth in the same year, namely 1927, as Heisenberg presented his heuristic arguments based on a thought experiment using an imaginary microscope and obtained the higher value of the lower bound. He used the German word 'unbestimmtheit' for the principle which has been translated as uncertainty, indeterminacy or indefiniteness.

Equation (12) implies that if the position of a quantum system is measured with high precision (Δx = 0) then the error in the simultaneous determination of the momentum is very large making this extremely inaccurate and vice versa. Thus, we can have only imprecise knowledge or fuzziness about the value of these two quantities when determined together. This inference is in complete contrast with the concepts in classical mechanics, where both the position and momentum of an object can be found exactly except the experimental errors. The uncertainties in the simultaneous measurement of the y and the z components of the position and momentum are given by expressions identical to (12). It may be mentioned that the minimum uncertainty product, which corresponds to equality in (12), is obtained when the wave function describing the quantum system is Gaussian.

Guided by the fact that angular position ϕ and angular momentum L_z are the dynamical variables describing the circular motion of a particle about the z-axis (similar to x and p_x for the linear motion), the inequality describing the indeterminacies in these quantities determined at the same time, was initially given by $\Delta\phi\Delta L_z \geq \hbar/2$. However, realizing the fact that unlike the linear position x the angular position ϕ is a periodic variable (having values, say, in the range θ to $\theta + 2\pi$ radian) and the angular momentum has quantized or discrete values $L_z = m\hbar$ rather than the continuous spectrum of values as for p_x, it has been proved and experimentally established [7, 8] that the correct form of the angular position-angular momentum uncertainty relation is

$$\Delta\phi\,\Delta L_z \geq (\hbar/2)\,|1 - 2\pi P(\theta)|\,. \tag{13}$$

Here, $\Delta L_z = \Delta m\hbar$ and $P(\theta)$ is the angular probability density at the boundary of the selected angular range. Obviously, if the angular momentum for a quantum mechanical state is accurately known (i.e., $\Delta L_z = 0$) then $P(\theta) = 1/2\pi$ implying that the probability for the angular position to have any value between θ and $\theta + 2\pi$ radian is the same. Furthermore, the equality in (13) holds good when $\theta = \pi$.

The uncertainty principle for time t and energy E needs special mention. To begin with, Heisenberg assumed this to read $\Delta t\Delta E \sim \hbar$, but later it was taken to have the form

$$\Delta t\,\Delta E \geq \hbar/2. \tag{14}$$

Despite its similarity of appearance to (12), this expression is interpreted differently and that also in more than one ways depending on the nature of the quantum system under consideration. This situation is a consequence of the fact that, unlike the special theory of relativity, in non-relativistic (quantum) mechanics, time is treated at different footing than position coordinates. It is an independent variable or parameter on which other quantities like linear and angular positions, linear and angular momenta and energy, called the dynamical variables or observables, depend. The dynamical variables can be experimentally measured for the system at any time, and we can determine the relevant standard deviations like Δx, $\Delta\phi$, Δp_{x}, ΔL_{z} and ΔE from the corresponding observed mean values. But nothing such can be done for time; we cannot talk about root mean square deviation for a collection of time measurements. As such, the time-energy uncertainty relation is different in content from the others which involve a pair of dynamical variables.

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While considering the effect of a constant weak perturbing field on the atoms, molecules or nuclei, we say that the probability for a transition with energy change ΔE between the initial (E_{i}) and the final (E_{f}) energy states forming almost a continuous energy spectrum near E_{i} is distinctly discernible only if $\Delta E \Delta t \sim \hbar$, where Δt is the finite time for which the system has been subjected to the radiation field. Similarly, when the excited atoms, molecules and nuclei get deexcited by jumping to the ground state the energy of the photons emitted has a spread $\Delta E \sim \hbar/\Delta t$, where Δt is the lifetime of the excited state. This causes a broadening of the peak in the energy spectrum and is usually referred to as the natural linewidth. The physical explanation of the nuclear forces between the nucleons (protons and neutrons) in the atomic nucleus involves a continuous exchange of pions of mass m within a time interval of $\Delta t \sim \hbar/\Delta E = \hbar/mc^2$. Thus, Δt is the time taken by the system to undergo an observable change rather than being an uncertainty in the measure of time. In particular, it may be mentioned that for an unstable system with life time $\Delta t \sim 10^{-8}$ s, the value of energy is uncertain by at least 5×10^{-27} J. Also, a quantum state with very precise energy ($\Delta E = 0$) is possible only if the time available for its measurement is infinitely long, i.e., the state is stationary.

Recently, Briggs [9] has very nicely brought out the difficulties associated with the earlier attempts for deriving the time-energy uncertainty relation and has obtained the exact expression for the principle by considering a quantum composite of a system and environment wherein the latter is ultimately taken to be classical. It is shown that Δt is the uncertainty in the measurement of time by a classical clock. Nonetheless, the author has emphasized that the meaning attached to Δt is governed by the kind of measurement being talked about.

It may be pointed out that the smallness of the magnitude of $\hbar$ ($\sim 10^{-34}$ Js) precludes the observation of the effects of various indeterminacies on the measurements pertaining to the macroscopic systems.

In order to describe Heisenberg's approach, we take an electron gun, which consists of a cylindrical closed tube having appropriate arrangement for controlled thermionic emission of electrons from an indirectly heated cathode and is provided with necessary focusing and accelerating anodes at desired positive potentials. The tube has a small hole in a plane surface to act as an outlet for a collimated beam of electrons having well-defined speed, linear momentum or kinetic energy. We fix this gun in a completely evacuated chamber barren of even a single air molecule to rule out any scattering on this account. The electron gun is so arranged that the emitted electrons move along the x-direction and it can be made to produce a single electron at a time. To observe the trajectory of the moving electron, we set up a source of illumination capable of giving monochromatic light of arbitrary wavelength, perpendicular to the x-direction and a suitable microscope as projected in *Figure* 7. The process of seeing the electron, which is essentially assumed to be a classical particle, involves its interaction with the incident photons of light and scattering of the latter into the microscope.

Suppose we have got an electron in the focal plane of the micro-

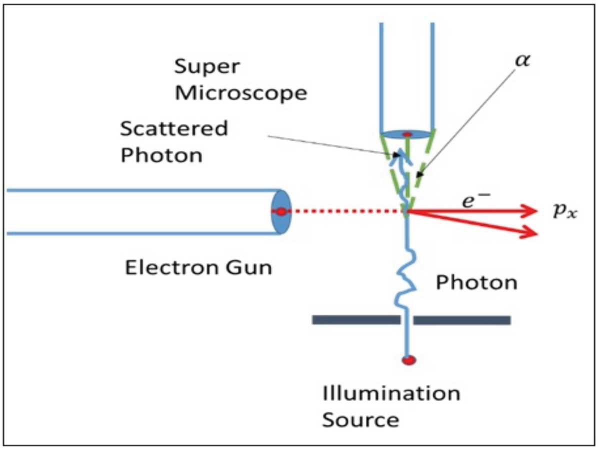

Figure 7. Arrangement for simultaneous determination of position along x-axis and the corresponding component of momentum for an electron.

scope and proceed to find its exact x position by illuminating it with light of wavelength λ. Obviously, we can obtain information about the position of the electron if the angle of scattering θ for the light beam is less than or at most equal to half the angular aperture α radians of the objective lens of the microscope; *Figure* 7. But, we know from the theory of diffraction in optics that the details of the image so formed are resolved in vacuum if the linear dimensions of the object under observation are of the order of $\lambda/\sin\alpha$. Accordingly, the electron can be located anywhere in this spread of length implying that the measure of its position x will be uncertain by

$$\Delta x \sim \lambda/\sin\alpha\,. \tag{15}$$

For small α, which is generally so for a practical optical microscope, this becomes $\Delta x \sim \lambda/\alpha$. Thus, the finite resolving power of this instrument introduces an inevitable uncertainty Δx in the measure of its position x and this can be rendered small using light having shorter wavelength and an objective of higher angular aperture. Since the constraint on the precision of determining position has its origin in the laws of optics it is sometimes referred to as 'optical limitation'.

Since an electron is essentially a point particle with an approximated diameter of about 10^{-16} m or less, the uncertainty in its location should be at most of this order of magnitude. These considerations show that the wavelength of the radiation employed for illumination should be 10^{-16} m or even less. Therefore, the visible light, which has nearly a billion times larger wavelength, cannot be used in the present arrangement. Rather, we must shine the electron with γ-rays of wavelength of the order of 10^{-16} m and assume that this is possible to get these in the laboratory and to record the position as well as the momentum of the electron. Actually, such γ-rays have an energy of more than 10^{10} eV and can be obtained only from astronomical sources. In view of these facts, the microscope being used in this experiment is a γ-ray super-microscope and is usually referred to as the Heisenberg's microscope.

Thus, observing the electron involves shining it with high energy γ-ray photons and their scattering from it into the microscope. Although many of these are required for recording the information, we consider the effect of just one photon. The photon not only has a characteristic wavelength but also linear momentum $p = \mathrm{h}/\lambda$. When it bounces from the electron, it exchanges the linear momentum leading to the recoil of the latter and decrease in its own momentum. In fact, the scattering involved is the 'Compton scattering'[1] discovered for the X-rays in 1923. The exact change in the momentum of the photon cannot be predicted as it will depend upon the angle of scattering θ from the electron and this introduces an uncertainty in the x-component of its momentum. Since the recoil of the electron arises from the scattering of the photon the uncertainty in the momentum of the photon renders the electron momentum along x-direction also inaccurate. But as mentioned above, for being observed the photon must enter the microscope at an angle $\theta \leq \alpha$ so that indeterminacy in the x-component of its momentum will be about $p \sin \alpha$. Therefore, the determination of the x-component of electron momentum will be uncertain by

[1] For more details see the Article-in-a-Box and Classics in this issue.

$$\Delta p_{\mathrm{x}} \sim p \sin \alpha \sim \mathrm{h} \sin \alpha / \lambda \,. \qquad (16)$$

Here, p as well as λ refer to the relevant magnitudes after scattering of the photon from the electron. Obviously, Δp_{x} can be made small using a radiation of higher wavelength and a microscope objective of small angular aperture α; both these conditions are in contradiction with the requirements for making Δx less. It may also be noted that uncertainty Δp_{x} can be used to find indefiniteness Δv_{x} in x-component of velocity. Combining the preceding two relations, we get

$$\Delta x \, \Delta p_{\mathrm{x}} \sim \mathrm{h}\,. \tag{17}$$

These considerations show that the indeterminacy in the simultaneous measurement of x and p_{x} values has its origin in the very process of measurement because the probe (photon) disturbs the system under study. An attempt to make the results for one quantity precise spoils the outcome for the conjugate variable. Using short waves, we can find the position precisely but we interfere greatly with the momentum making this more uncertain. On the other hand, the use of long waves checks the disturbance giving an accurate value of momentum but the information about position becomes less reliable. All we can do is choose a middle ground and compromise for the accuracy of both. This immediately shows that a quantum entity, say an electron, cannot be treated as a point in space and its trajectory is a widened line rather than a mathematical one.

It may be pointed out that uncertainty in the position of electron Δx (see (15)) arises from the constraint of resolving power of the microscope due to diffraction of light which is a wave phenomenon. On the other hand, the indeterminacy Δp_{x} in electron momentum has arisen from the indefiniteness of the photon momentum – a particle aspect. So, there is an intimate relationship between the uncertainty principle and wave-particle duality. In fact, sometimes an expression for the product of the above two uncertainties is also derived by considering the de Broglie wave packet associated with a moving particle.

Though Heisenberg's assertion got recognised, the analysis was

Though Heisenberg's assertion got recognised, the analysis was criticised as it invoked classical as well as quantum concepts.

criticised as it invoked classical as well as quantum concepts. The present-day version of the principle for any pair of noncommuting operators corresponding to relevant observables (said to be incompatible) owes its origin to the sophisticated derivation given in 1929. This has nothing to do with the experimental arrangement for the measurement of the conjugate pairs of variables and thereby in the precision involved. Rather the uncertainties are an inherent aspect of the quantum state and the inequalities in the exact statements give the inherent minimum possible constraint. Thus, the uncertainty relations are manifestations of the law of nature at the atomic scale and are universally applicable.

The uncertainty principle and the counter-intuitive statistical interpretation of quantum mechanics essentially divided the physics community into two groups: votaries and dissenters. Einstein belonged to the latter and because of his convictions about determinism he presented many subtle arguments to highlight the inconsistencies in this theory, which effectively helped in the further development of the subject. One such interesting incidence took place at the Sixth International Solvay Conference held at Brussels in 1930, where he put forward an ingenious thought experiment purported to contradict the time-energy uncertainty.

He imagined an ideal box lined with perfect mirrors, which could hold electromagnetic radiation indefinitely. It had a hole on one side, which could be opened or closed by a shutter itself operated by a precise clockwork placed inside the box; *Figure* 8. The box is weighed very accurately and the shutter is opened at a predecided time with a mechanism controlled by the clock for a very precisely adjusted time interval to release some amount of radiation. This time interval could be made as small as possible so that just one photon is let out. Now, find the exact weight of the box again and the accurately determined decrease in its mass multiplied with c^2 gives the correct measure of the energy carried by the radiation released. Thus, Einstein asserted that one could very precisely measure both the energy and the time during which this was emitted making inaccuracies in these almost zero, defying the uncertainty principle.

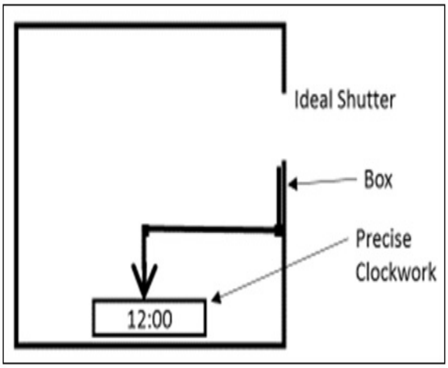

Figure 8. Einstein's ideal box for weighing light radiation.

Bohr, who was one of the main advocates of the then standard interpretation of quantum mechanics, was immensely upset by this seemingly unassailable contention. However, next morning he presented a counter thought experiment with the ideal box having a spring balance for precise weighing and a pointer to read its position on a vertical scale; *Figure* 9. He argued that the release of radiation decreases the weight of the box making the spring contract. Thus, the box would move upwards and there will be indefiniteness in its vertical velocity and hence an uncertainty in its height above the table or the floor. This uncertainty in its elevation above the earth's surface will produce uncertainty in the rate of the clock which depends upon the position of the clock in the gravitational field. He proceeded to prove that the uncertainties of time and the change in weight of the box and, therefore, in mass and hence energy content of the radiation released would indeed be in conformity with the time-energy uncertainty relation. This argument countering Einstein's objection too had contemporary physicists supporting or opposing it.

It is worth mentioning that the uncertainty principle has been very useful in having a rough estimate of some quantities and thereby in solving many fundamental problems in quantum physics. Some of the examples are existence of zero-point energy, the non-existence of electrons but the existence of protons and neutrons in the nuclei, size of elementary cells in phase space, the phenomenon of natural line-width, estimate about the mass of the exchange particles involved in different interactions, elucidation of quantum

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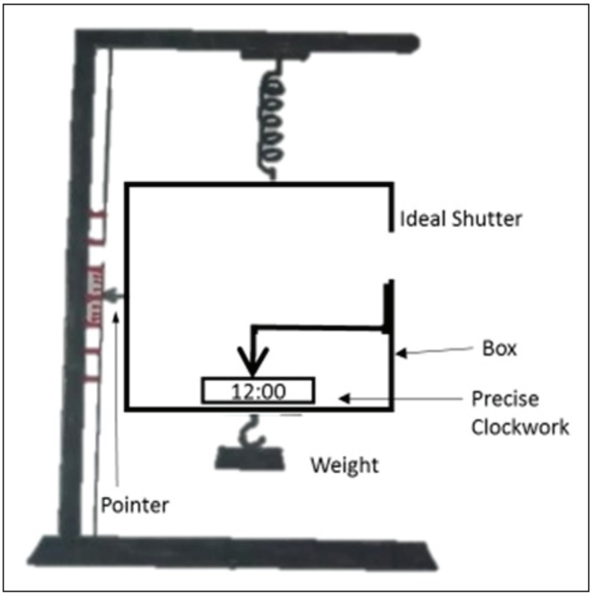
Figure 9. Bohr's ideal box for weighing light.



confinement effect in semiconductor nanocrystals, etc. The successful application of these concepts proves the correctness of this principle. Besides, recently, the uncertainty relations have been found to be relevant in the studies pertaining to quantum information and quantum cryptography. It may be added that references [10] and [11] have dwelt upon some aspects of the uncertainty principle in a very commendable manner.

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The article is dedicated with immense affectionate regards to my dear sisters, Sudesh, Savitri and (late) Sneh and their families.

Suggested Reading

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