On the Front Cover

The first expression on the front cover is the simple continued fraction expansion of e written more briefly as

$$e = [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, \cdots]$$

It is an open question whether the continued fraction expansions of irrational, algebraic numbers of degree > 2 have unbounded partial quotients (degree 2 corresponds to eventually periodicity of the continued fraction). The continued fraction expansions of transcendental numbers like e are related to the best rational approximations they afford. The irrationality measure (the supremum of all $d > 0$ such that, infinitely many rational numbers p/q are at a distance less than $1/q^d$) of e is 2. For the simple continued fraction of the number π, no such pattern as above for e is known:

$$\pi = [3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, \cdots]$$

is known to less than 10^{10} terms; its irrationality measure is known to be at the most 7.6063 although expected to be equal to 2. On the other hand, the easily proved non-simple continued fraction expansion for $4/\pi$ has a beautiful pattern:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cdots}}}$$

The second expression on the front cover shows that the transcendental number $e^{\pi\sqrt{163}}$ is very close to an integer; this is a consequence of the theory of complex multiplication.

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