Extracting Fuel Efficiency Information From the Car Dashboard Display
An Application of Interpolation Technique

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We discuss an application of interpolation of curves to extract fuel efficiency information of cars. Input data is taken from the mileage meter in the dashboard display of cars equipped with such a facility. This is also an application of harmonic mean.

1. Introduction

Many of us are owners and regular users of personal vehicles. Most of these vehicles have an on-board computer chip and an assortment of sensors to measure and report various things about the vehicle. Some of these are age-old objects like speedometer and odometer. Many modern cars are also equipped with a mileage meter. It usually reports a measure of the fuel efficiency of the car. For the on-board computer chip, it is very easy to estimate this figure as the ratio of distance traveled to the fuel injected into the engine to cover that distance. In India, it is often reported in units of kilometers per liter (km/l). In many other markets, it is often reported in miles per gallon. Different markets have their own preferences or customs. For the purposes of this article, we assume that the cars are purchased in India. For other markets, a similar analysis works with appropriate changes.

There is usually a switch to toggle the mileage meter display between two modes – instantaneous and overall. The instantaneous mode shows the mileage on a per instant basis. It is a very rapidly fluctuating number from which it is not easy to extract useful information about the fuel efficiency of the car. The overall mileage is the ratio of distance traveled so far and the volume of fuel used

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so far. As can be expected, this is a very slowly changing number. Again of not much practical use as it doesn't reflect the current mileage of the car. We usually desire to know the mileage of the car for the last few kilometers. We also want to know the mileage for one particular type of trip, say, city driving, long driving, hilly driving, congested city driving, etc. There is also a switch to reset the numbers so that fresh estimates can be obtained but the driver must remember to reset it before the trip starts. Not a very practical way. We need a figure which changes at an intermediate rate, a kind of short-term average but not exactly instantaneous.

To begin with, let us unambiguously understand the terms used in this article. Our use of these terms may differ from textbooks. We will use the term 'fuel efficiency' of a car to measure the car's ability to convert fuel into distance covered. To clarify, we do not mean the thermodynamic energy efficiency (ability to convert heat/chemical energy of the fuel into work done) of the engine. Though it is a good guess that both the notions of efficiency[1] stand as good proxies of each other. We use the term 'mileage' to refer to the distance traveled by the car divided by volume of the fuel used to cover that distance. It goes without saying that mileage is a very good proxy for fuel efficiency as defined by us. Higher the mileage, better the fuel efficiency and vice versa. Our preferred unit for mileage is kilometers per liter (km/l). We use the term 'average' [2] to imply the usual dictionary meaning which generally corresponds to arithmetic mean in mathematics.

[1] It is very interesting to note when these two efficiencies are similar/dissimilar.

[2] In India, the term 'average' is also used for mileage, though we do not do so in this article.

Let us now see if we can extract the current mileage of a car out of the reported overall milegae using some simple high school mathematics. Let us first recall.

2. Harmonic Mean

Definition 1.1. Harmonic Mean (H.M.) of two numbers a and b is defined as the reciprocal of the arithmetic mean (average) of their respective reciprocals.

$$\text{H.M}(a, b) = \frac{2ab}{a + b}\,.$$

Definition 1.2. Weighted Harmonic Mean (W.H.M.) of two numbers a and b with weights w_a and w_b is defined as the reciprocal of the weighted arithmetic mean (average) of their respective reciprocals.

$$\text{W.H.M}(a, b) = \frac{(w_a + w_b)ab}{w_b a + w_a b}\,.$$

Note the respective positions of w_a and w_b. If the weights are equal, then we get the ordinary H.M. Next, suppose we have the following problem.

Suppose a car delivers a mileage of A_1 for traveling 1000 kilometers and A_2 for traveling the next 1000 kilometers. Assuming the mileage to be uniform in these two trips, what is the overall mileage of the car for the entire 2000 kilometer trip?

Problem 1.

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Solution:

Say the desired figure is A. We estimate the total fuel consumed in two different ways and equate them. The first trip consumed $\frac{1000}{A_1}$ liters of fuel and the second trip used $\frac{1000}{A_2}$ liters of fuel. So the total fuel consumed is $\frac{1000}{A_1} + \frac{1000}{A_2}$ liters.

Since A is the overall mileage at the end of both the trips (of total 2000 kilometers), the total fuel consumed is $\frac{2000}{A}$ liters. We can now equate the two estimates.

$$\frac{2000}{A} = \frac{1000}{A_1} + \frac{1000}{A_2}\,.$$

From this A can be easily computed to be,

$$A = \frac{2A_1A_2}{A_1 + A_2}\,. \quad (1)$$

□

(1) shows that A is the Harmonic mean of A_1 and A_2. If the two trips were not of equal distance, then we would have got a weighted harmonic mean with the respective distances used as weights. This can be clearly seen from the following problem.

Problem 2.

Suppose a car delivers a mileage of A_1 for traveling 1000 kilometers and A_2 for traveling the next 2000 kilometers. Assuming the mileage to be uniform in these two trips, what is the overall mileage of the car for the entire 3000 kilometer trip?

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Solution:

Let us again assume the desired figure to be A. We proceed as before and estimate the total fuel consumed in two different ways. One of the way gives us: $\frac{1000}{A_1} + \frac{2000}{A_2}$ liters. The other way gives us $\frac{3000}{A}$ liters. Equating the two gives us:

$$A = \frac{3000A_1A_2}{2000A_1 + 1000A_2},$$

which is clearly the weighted harmonic mean.

□

In real life situation with a commercially available car, A_1 and A are usually the known quantities and A_2 is the unknown one. (1) can of course take care of it. In both the above situations, we had assumed a single trip to deliver a uniform mileage however long the trip may be. In practice, this is hardly the case. The mileage of the car varies considerably throughout any trip. In a traffic light it could be zero whereas in a downhill empty road one could possibly coast with infinite mileage by turning off the engine.[3] All kinds of intermediate cases may be present even for very short trips. I hope the reader appreciates the need for the concept of 'continuous weighted harmonic mean'.

[3] A very dangerous practice for all type of vehicles but especially extreme for a car equipped with power steering/brakes – certainly never to be practiced.

3. The Continuous Case – An Idealized Situation

Let us say we know the overall mileage $a(x)$ expressed as a function of the distance x traveled so far. i.e.,

$$a(x) = \frac{\text{Total distance traveled so far (in kilometers)}}{\text{Total volume of fuel consumed so far (in liters)}} = \frac{x}{V}. \tag{2}$$

Let $i(x)$ be the instantaneous mileage of the car.

$$i(x) = \frac{\mathrm{d}x}{\mathrm{d}v}$$

Our objective is to extract this function from known information.

As we have done in the two problems before, we again estimate the total volume of fuel consumed in two different ways. Firstly from (2), $V = \frac{x}{a(x)}$. Also from the definition of $i(x)$,

$$V = \int_0^x \frac{ds}{i(s)} .$$

Equating the two gives us:

$$\frac{x}{a(x)} = \int_0^x \frac{ds}{i(s)} .$$

The perceptive reader will notice that we are really computing the continous weighted harmonic mean. Differentiating both sides of the above equation w.r.t. x gives us:

$$\frac{1}{i(x)} = \frac{xa'(x) - a(x)}{a^2(x)} .$$

We finally get,

$$i(x) = \frac{a^2(x)}{xa'(x) - a(x)} . \tag{3}$$

A slightly different case presents itself when the mileage meter is reset leading to a changed $a(x)$. Suppose the mileage meter is reset when the odometer shows x_0 then our $a(x)$ shows overall mileage starting from $x = x_0$ instead of $x = 0$. Imitating the above steps leads us to $V = \frac{x-x_0}{a(x)} = \int_{x_0}^x \frac{ds}{i(s)}$ and from this we get the modified formula:

$$i(x) = \frac{a^2(x)}{(x - x_0)a'(x) - a(x)} \tag{4}$$

We should apply the formula (3) or (4) as applicable.

The real world is complicated. All meter readings are only approximate since they have rounding off errors. Moreover the meter doesn't display any continuous function, but just a single number which is changed by the on-board computer chip.

4. Real World Case – Judicious Choice of Interpolation

The real world however is still more complicated. All meter readings are only approximate since they have rounding off errors. Moreover the meter doesn't display any continuous function, but just a single number which is changed by the on-board computer

Table 1. Sampled mileages.

x	$a(x)$
x_1	a_1
x_2	a_2
x_3	a_3
x_4	a_4

chip. Due to this, we are unlikely to be in possession of the continuous function $a(x)$. Instead we have samples of $a(x)$ somewhat similar to *Table* 1.

From this data we need to estimate the function $a(x)$ in order to apply either formula (3) or (4) as applicable. There is no unique way to do this. Some of the options we have are regression line, step function, least square polynomial, curve fit polynomial, etc. Since $a(x)$ models a real physical phenomenon, we must choose a physically plausible function estimate. We shall keep the following caveats in mind while choosing such a function.

1. **Incidence:** In order not to oversimplify (or in some cases, overcomplicate) things, we insist on all the sample points a_i's being incident on the curve $a(x)$. We are really demanding,

$$a(x_i) = a_i \,.$$

   This rules out various regression lines or curves based on least square estimates. From now on, we must restrict our attention to interpolation curves.

2. **Continuity:** For simplicity, the physical situation is best modeled by a continuous function, not a step function.

3. **Differentiability:** Our formulae (3) and (4) demand this as well. We further impose continuity of the derivative in order to get a continous $i(x)$.

4. **Average:** The on-board chip of the car computes $a(x)$ as a ratio of two numbers, both of which are monotonic increasing. Further, it is an overall mileage. Hence it is unlikely to have wild fluctuations between sample points. This rules out a high degree polynomial as an interpolation curve. We want minimal number

Figure 1. Digital display of the car dashboard.

of crests and troughs for another reason too. Recall the discussion in the 2nd paragraph of this article. We wish to have a function which changes faster than the overall mileage but slower than the instantaneous mileage as displayed on the car dashboard. Choosing a sufficiently smooth interpolation function is equivalent to the instantaneous mileage $i(x)$ not being exactly instantaneous but an average mileage over the last few kilometers of a running trip.

The above caveats leave us with a choice of various spline techniques. For our examples that follow, we have chosen the cubic spline. The rounding off of the digital displays present another problem. Any given number displayed in the mileage meter will be stuck for a wide range of x. For example, the mileage might show 19.5 km/l for odometer starting from 4000 to 4200 kilometers. The problem is choosing x_1 (say between 4000 to 4200) for a given a_1 (say 19.5). Someone may choose exactly the midpoint of this range or someone else may choose one of the end points. Whatever we do, we must be consistent for the entire data. This author has chosen a different approach. It is much easier to just capture a snapshot[4] (of the type in *Figure* 1) of the dashboard from a mobile camera at the very moment the mileage meter changes. This gives us x_i's as well as a_i's simultaneously.

[4]But not before bringing the vehicle to a complete halt. Safety must never be compromised.

Table 2. Sample points of a real car.

Outskirts data		Inner city data	
x	$a(x)$	x	$a(x)$
2487	19.0	4213	20.8
2549	19.3	4518	20.6
2701	19.8	4686	20.5
2848	20.1	4829	20.4
2946	20.4	5032	20.2
3088	20.5	5442	20.1
3649	20.9	5665	20.0
3916	21.1	5743	19.9

5. Example – Swift dZire

The author owns a petrol powered 2015 model Maruti Swift dZire car. He collected some data on his own car for a period of few months while driving in the city of Hyderabad, India. Please look at *Table* 2 for the collected data. All the subsequent data analysis was done for only this car for obvious reasons. There are two different phases when this analysis was done. One ranged from 2400 kilometers to 4000 kilometers of mainly outskirts driving, while the other ranged from 4200 kilometers to 6000 kilometers of mainly city driving. The outskirts driving included a fair bit of multi-lane highways. Both the data is displayed together. All the driving was done while getting on with everyday life. Hence in either of the phases, only one type of road was not driven on. It certainly cannot be said to be a controlled experiment but the uncontrolled nature of the analysis makes it more fun. If nothing, it does correspond with the real life.

In this example, (4) was applicable as the author had reset the mileage meter at x_0 = 1900km. A spreadsheet on LibreOffice was used to interpolate the curves and obtain the graphs (*Figures* 2 and 3) for the two phases. The cubic spline curve was implemented as an extension module to LibreOffice written in the OOBasic language. All of it can be done on Microsoft Excel using Visual Basic with very minor changes. In both the graphs, solid bullets

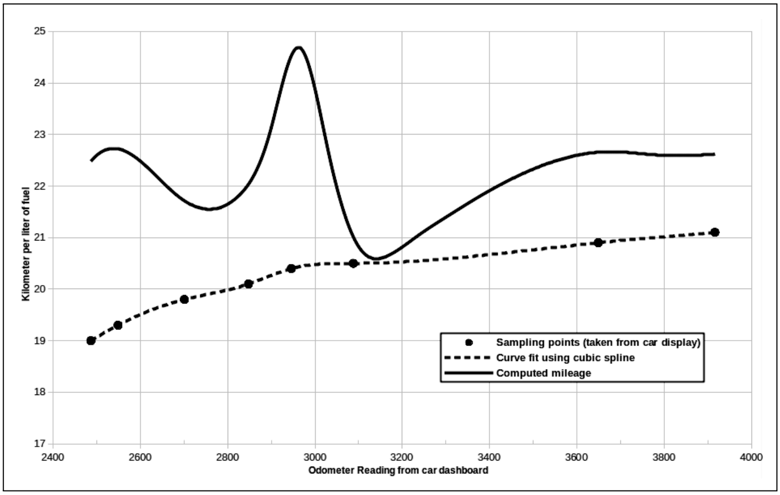

Figure 2. Driving in the outskirts of the city.

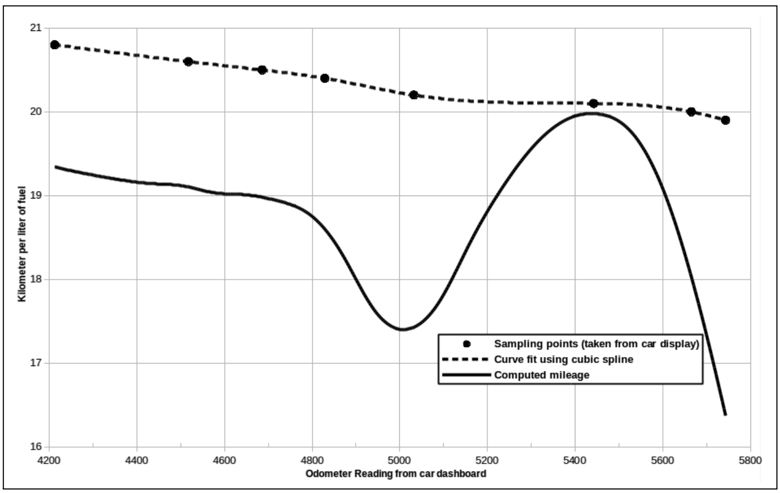

Figure 3. Inner city driving.

represent the data points of *Table* 2, the dotted line represents the cubic spline interpolation, and the solid line represents the computed function $i(x)$ using (4). As can be seen very clearly from the graphs, highway driving does improve the mileage figures significantly. This much was expected but we now have a lot more information about the fuel efficiency of the car. All at the cost of a few snaps from a mobile camera. Not a bad deal!

The reader is encouraged to read more on interpolation and curve fitting techniques as well as several kinds of means from various sources. *Wikipedia* would be a good start. The keywords of this article serve as good topic points.

Suggested Reading

Students can refer to any introductory textbook to understand harmonic mean and cubical spline interpolation.