The Inveterate Tinkerer
13. Experiments With Hele–Shaw Cells

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In this series of articles, the authors discuss various phenomena in fluid dynamics, which may be investigated via tabletop experiments using low-cost or home-made instruments. The thirteenth article in this series demonstrates some experiments with Hele–Shaw cells.

Procedure

A Hele–Shaw cell [1–3] comprises two parallel plates (usually made of glass or plexiglass) with a separation that is small relative to a linear dimension of the plates, having a fluid confined within the gap. Fluid flow in this setup may be visualised using a dye, as the liquid is made to flow through the cell from end to end. The setup has traditionally been used to represent the velocity field of steady, two-dimensional, inviscid, incompressible, potential flow (i.e., with zero vorticity) around an obstacle placed within the gap. At first glance, this may appear paradoxical, since viscous effects are significant in the low Reynolds number flow within the gap. The reader is urged to refer [4] for a discussion of this matter.

Suggestions for Further Work

Experiment 1: Radial Saffman–Taylor Instability in a Hele–Shaw Cell

Keywords
Hele–Shaw cells, Saffman–Taylor instability, Darcy's law, Newtonian fluid, dynamic viscosity.

Saffman and Taylor [5] exploited the mathematical analogy between two-dimensional flow in a homogeneous, isotropic porous medium obeying Darcy's law and the flow of steady, incompressible Newtonian fluid in a Hele–Shaw cell at low Reynolds numbers. Darcy's law [4] states that the velocity of a Newtonian fluid flowing through a porous medium in a Hele–Shaw cell averaged

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over a volume large compared to the individual pore volume and small compared to the volume of the cell is proportional to the pressure gradient imposed across the cell. The constant of proportionality in this expression is the ratio of the permeability (which depends on the geometry of the porous medium, fluid-solid interaction, etc.) and the dynamic viscosity of the fluid.

The authors [5] considered the instability of the horizontal interface (moving with uniform velocity) between two incompressible, isothermal, immiscible, Newtonian fluids having differing dynamic viscosities (and densities) in a vertical Hele–Shaw cell subject to a gravitational force or an imposed pressure gradient. They showed that the interface between the fluids is (linearly) unstable if the less viscous fluid penetrates the more viscous fluid with a front velocity that exceeds a critical value which depends on the viscosity (and density) difference of the two fluids and the acceleration due to gravity. Experimentally, the perturbed interface shows large fingers of the less viscous fluid which protrude across the moving interface into the more viscous fluid. An extensive body of literature is available on the Saffman–Taylor problem, and the reader may wish to read the review by Saffman [6].

A Hele–Shaw cell for radial flow of fluid within the gap was fabricated using two plexiglass plates of dimension $23\times18\times0.5$ cm^3. A through-hole of diameter 1 mm was drilled using a PCB drill bit at the centre of the top plate and a single layer of transparent cellulose tape was stuck along the edges of the bottom plate, which served as a spacer of thickness ~0.1 mm. The sharp end of a (hollow) syringe needle was twisted off using pliers and the needle hole widened using a metal file to obtain a tube of length 11 mm with outer diameter 1 mm. The metal tube was inserted into the through-hole in the top plate and glued into place using an epoxy adhesive (Fevikwik, Pidilite) taking care to prevent the glue from entering the tube. A flexible plastic tube of length 45 cm and an outer diameter of 1.5 mm was inserted into the metal tube with the other end connected to a 20 ml syringe (see *Figure* 1).

A small volume of glycerol was poured onto the bottom plate.

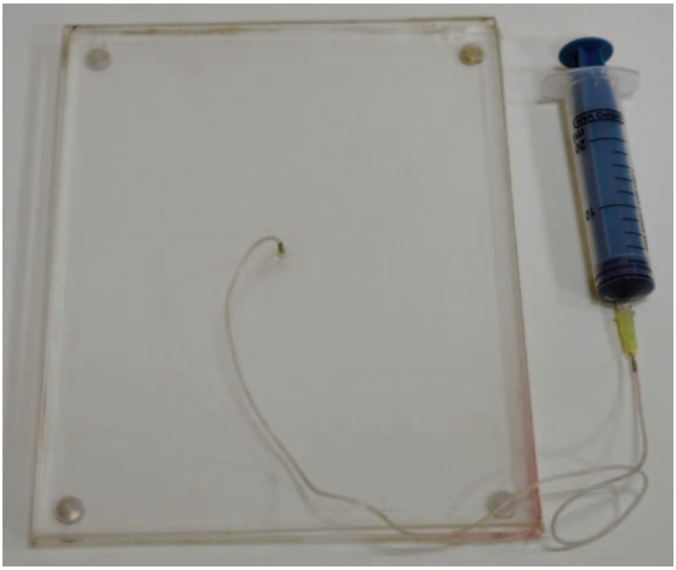

Figure 1. Hele–Shaw cell setup for radial Saffman–Taylor instability.

Note that the dynamic viscosity of glycerol is ~1500 times larger than that of water. The syringe and the fine plastic tube were filled with dyed water right up to the end of the metal tube attached to the top plate. This step is essential to avoid introducing air in the plastic tube from entering the glycerol layer in the form of bubbles. The top plate was placed on the glycerol layer and pressed downwards taking care to avoid trapping air bubbles in the glycerol layer. One may use C-clamps to maintain the gap and prevent the plates from sliding past one another. A white backdrop below the cell helps visualise the flow. When the plunger of the syringe is pressed, characteristic fingering patterns are produced as the dyed water spreads radially within the gap, the interface between the two liquids exhibit lobes that bifurcate repeatedly. See *Figure* 2(a) and the video: `youtube.com/watch?v=9Flsy0sc_cg`

The experiment discussed above was also carried out with a dilute aqueous solution of gelatine in the gap, a curious fractal [7] interface was observed after a momentary delay as the dyed water burst through the gelatine network. See *Figure* 2(b) and the video: `youtube.com/watchv=GAVzuk5c2qM`

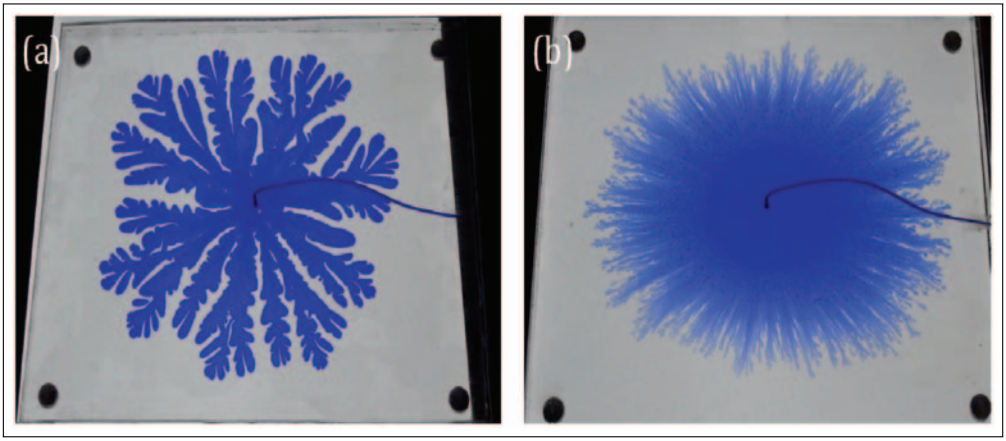

Figure 2. Saffman–Taylor instability in a Hele–Shaw cell for **(a)** dyed water driven through glycerol and **(b)** dyed water driven through aqueous gelatine solution.

Experiment 2: Visualisation of Foam Flowing in a Hele–Shaw Cell

We fabricated a Hele–Shaw cell for visualising the flow of foam around an obstacle. A semi-circular trough (diameter = 1 cm) was carved through the centre of a wooden block of dimension $31 \times 25 \times 2\ cm^3$. The Hele–Shaw cell comprised two plexiglass plates of dimension $20 \times 15 \times 0.25\ cm^3$ with a five Rupee coin (stuck using double-sided tape) as spacer. Strips of double-sided tape with the same thickness as the coin were used to stick the plates together at the edges, as shown in *Figure* 3(a). A hollow plastic tube of length 35 cm with an outer diameter of 3.5 mm was stuck using double-sided tape to the base of the cell with its opening facing the gap, and the other end of the tube attached to a plastic microtip. The trough was filled with soap solution and the cell positioned vertically with the plastic tube submerged within the soap solution, while air was blown in through the microtip. The foam flows upwards and around the coin as seen in *Figure* 3(b) (with a black backdrop behind the cell) and in the video: `youtube.com/watch?v=2XxRCyLfwEA`

Experiment 3: Meandering Rivulets in a Hele–Shaw Cell

Two glass plates of dimension $45.5 \times 28 \times 0.5\ cm^3$ were stuck together using short strips of heavy-duty double-sided duct tape (Gorilla Tape, Gorilla Glue Inc.) which acted as spacers of width

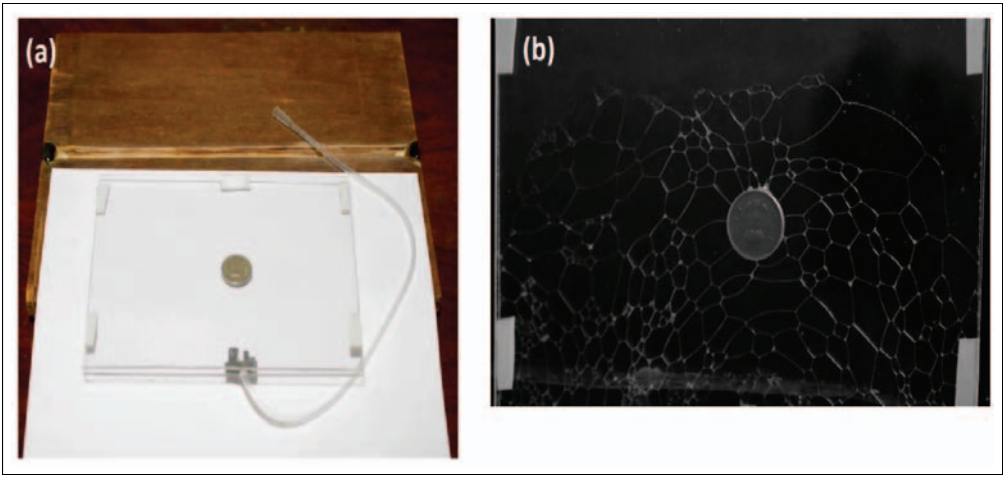

Figure 3. **(a)** A Hele–Shaw cell for visualising foam flowing around an obstacle. **(b)** Foam from a soap solution flowing around a coin.

1 mm. The edges of the plates were covered with strips of brown sealing tape, and the setup was mounted vertically using a laboratory ring stand with clamps. A white backdrop behind the glass plates helped visualise the flow. Dyed dilute aqueous soap solution was injected vertically downwards into the gap between the plates using a syringe. A vertical rivulet of liquid descends under gravity. On pushing down the syringe plunger, the straight rivulet exhibits damped sinusoidal waves (see *Figure* 4). At higher flowrates, we observed non-sinusoidal waves of large amplitude [8] which resemble distributary channels in a river delta, as seen in the video: `youtube.com/watch?v=qB2f4GS8pfA`

Experiment 4: Pattern Formation in a Lifted Hele–Shaw Cell

An aqueous suspension of poster color (Camlin, India) was painted onto two glass plates of dimensions $15 \times 10 \times 0.5\ cm^3$ using a brush. The plates were pressed firmly together and then separated by using a metal scraper.

Symmetric fractal [7] patterns were observed (see *Figure* 5) for a specific concentration (found empirically) of the suspension. See the video: `youtube.com/watch?v=1h8PrQqZfn8`

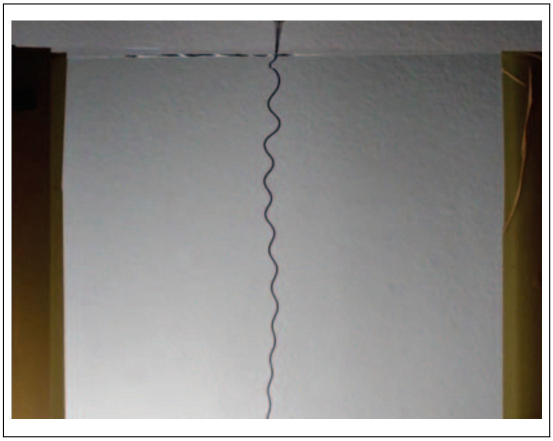

Figure 4. Damped sinusoidal rivulet of aqueous soap solution in a vertical Hele–Shaw cell.

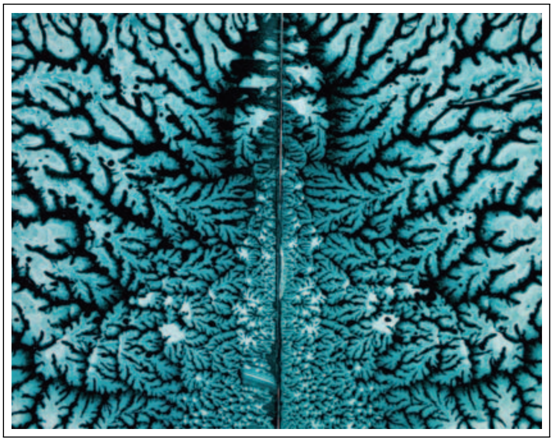

Figure 5. Fractal-like patterns formed on glass plates coated with a solution of poster paint, after separation.

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Suggested Reading

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[9] A novel labyrinth instability may be observed by sandwiching a drop of ferrofluid between two glass slides and placing a strong permanent magnet under the glass slide, as seen in the video: `youtube.com/watch?v=XMdOD8cr7oA`

[10] Injection of a gas into a wet granular material in a Hele–Shaw cell shows unusual fingering patterns, see the video: `youtube.com/watch?v=C7D-JXzdW6c`