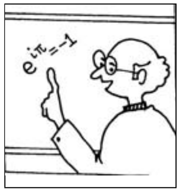
Classroom



In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

The Inveterate Tinkerer
10. Analog Computing With Soap Films

Chirag Kalelkar
Department of Mechanical Engineering,
IIT Kharagpur, Kharagpur
West Bengal 721 302, India.
kalelkar@mech.iitkgp.ernet.in

In this series of articles, the authors discuss various phenomena in fluid dynamics, which may be investigated *via* table-top experiments using low-cost or home-made instruments. The tenth article in this series demonstrates the use of the area-minimization property of soap films in the computation of analog solutions to mathematical problems.

Procedure

Soap films constrained by fixed boundaries adopt configurations that minimize the surface area [1–3]. This property is a consequence of the principle that a system in thermodynamic equilibrium minimizes its Helmholtz free energy at constant temperature and volume (or surface area) [4]. In fact, the spherical shape of an isolated soap bubble (in zero gravity) may be attributed to the fact that a sphere has the least surface area for a fixed volume of air trapped inside. For a soap film, this free energy is proportional to the area of the film, and one may utilize the property to compute analog solutions to mathematical problems which require the minimization of surface area, although the equilibrium configuration need not correspond to *global* minima. By gently blowing on a soap film constrained by fixed boundaries, one may perturb the film so that it jumps between local minima states. Three soap films meet along an edge (called a 'Plateau border' [2]) at angles

Keywords
Plateau border, catenoid, motorway problem, analog computer.

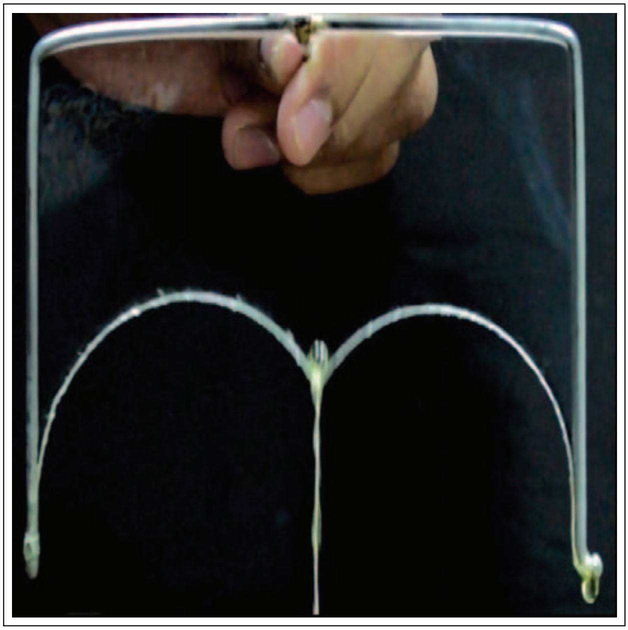


Figure 1. Minimum area configuration for a soap film in a U-frame, bounded by a cotton thread pulled at the center.

Soap films can also be used to compute minimum surfaces to solve three-dimensional problems by constructing frames which are shaped like a cube, an octahedron, and a helix joined to its central axis.

of 120^o, and Plateau borders meet in fours at a vertex, at tetrahedral angles of 109.47^o. In hydrostatic equilibrium, the difference in pressure across the curved soap film is proportional to the mean curvature (defined as $1/R_1 + 1/R_2$, where R_1 and R_2 are the principal radii of curvature) of the film [1]. The difference in pressure is constant at any point on the surface, therefore the mean curvature of the film is a constant.

A simple demonstration of the area-minimization property of soap films may be performed by constructing a U-frame from bicycle spokes and tying a cotton thread across the open end. On immersing the frame in a bath of soap solution (we used a dilute aqueous solution of Fairy liquid detergent, Procter and Gamble) and withdrawing the thread, the thread adopts a semicircular shape which minimizes the area of the film. If the thread is pulled outwards at the center, the thread adopts the shape shown in *Figure* 1. Also see the video: `youtube.com/watch?v=nKKS3qQGR8w`

Soap films can also be used to compute minimum surfaces to solve three-dimensional problems by constructing frames which

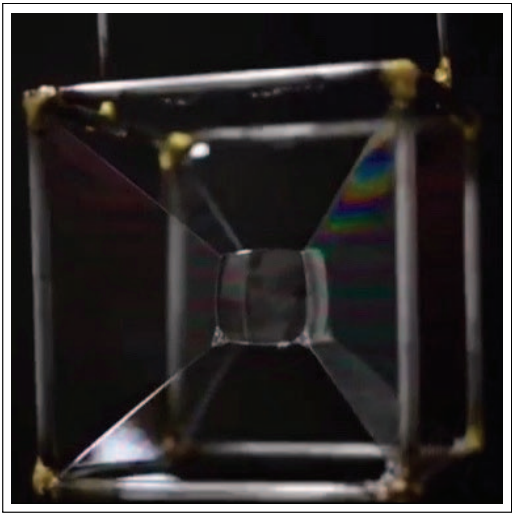


Figure 2. An air bubble trapped within a soap film in a cubical frame.

are shaped like a cube, an octahedron, and a helix joined to its central axis, using bicycle spokes which were cut into sections and welded together or twisted. See the video:
`youtube.com/watch?v=E_KH3FNtOIw`

Note that for the cubical frame, the soap film surfaces do not meet at a point in the center of the cube, but at a 'square-shaped' film near the center of the cube. In the case of a helix joined to its central axis, a soap film helicoid is obtained. One may trap an air bubble in the center of the framework by first immersing it in the soap solution and then partially immersing the wetted structure into a thin layer of the same solution in a tray (see *Figure* 2). The trapped air bubble shares the symmetry of the enclosing framework.

Experiment 1: Catenoidal Soap Films

Two coaxial metal hoops placed in contact with each other are immersed in soap solution and withdrawn. The membrane within the hoop is punctured, and the hoops are slowly separated from each other along their axis. The shape of the soap film bridge formed is called a 'catenoid' (see *Figure* 3), which is the surface of revolution of a catenary (see: `mathworld.wolfram.com/Catenoid.html`

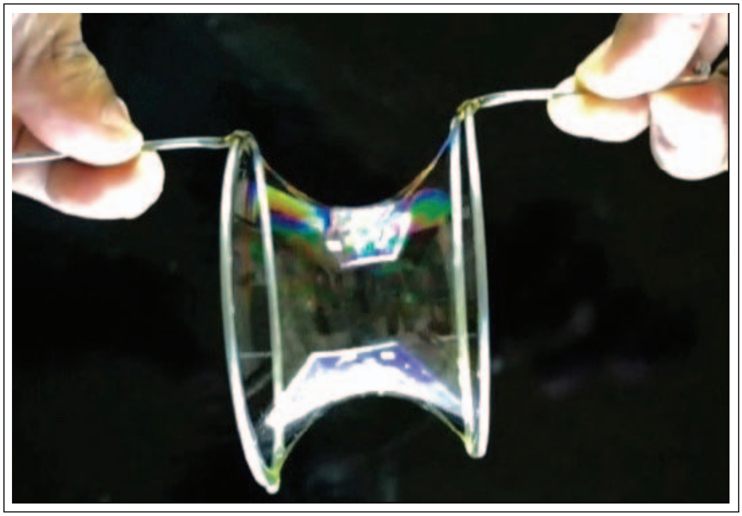


Figure 3. A catenoidal soap film bridge between two metal hoops.

for a discussion of properties of this surface). See the video: `youtube.com/watch?v=AcxcgdcUNzA`

The instability of a liquid bridge was discussed by Rayleigh [5], who showed that the bridge is (linearly) unstable when the ratio of its length and the hoop radius exceeds 2π. The actual process of breakup is dynamically complex [6] as seen in *Figure* 4, wherein a soap bubble is shown to be linked by thin filaments to planar discs on the hoops and eventually pinches off. See the video taken using a high-speed camera (Phantom M110) with a macro lens, and backlit using an LED light source: `youtube.com/watch?v=BUyU0dVIELs`

In the above experiment, if the membrane within the hoops is not punctured, one obtains a planar film at the center of the liquid bridge, see the video showing the breakup of this capillary bridge: `youtube.com/watch?v=zCvsZ6HEWaM`

Experiment 2: The Motorway Problem

A soap-film model of the motorway problem uses parallel Plexiglas plates separated by pins and dipped in a soap solution.

Consider towns linked by a motorway, with the constraint that the total length of the motorway must be of the smallest length ensuring that every town is linked to every other town. The problem reduces to finding a (possibly non-unique) motorway configuration that minimizes the overall length. A soap-film model of the

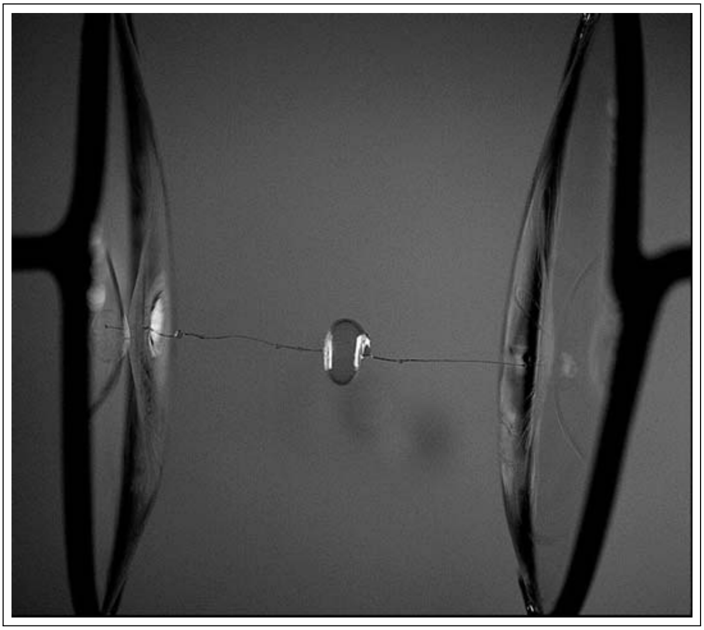


Figure 4. A soap bubble linked by filaments to planar discs, during the breakup of a catenoidal bridge.

Figure 5. An analog solution to the motorway problem for a regular hexagonal configuration (see the text for discussion).

motorway problem [1] uses parallel Plexiglas plates of dimension $10 \times 10 \times 0.2$ cm^3 which are separated by pins perpendicular to the plates which represent, to scale, the positions of the towns. The ends of the pins are glued to the plates using an epoxy adhesive

(Fevikwik, Pidilite). The plates are dipped in the soap solution and placed on a transparency projector. After a delay of a few seconds, the perpendicular soap film bounded by the plates occupies minimum surface area resembling a tape of constant width, with an area proportional to the length of the film. In *Figure* 5, a non-trivial configuration is shown for the case of pins that are placed at the vertices of a regular hexagon. Other possible configurations for pins placed at the vertices of a square and a regular pentagon may be seen in the video: `youtube.com/watch?v=BVbIRM01UTs`

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Suggested Reading

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