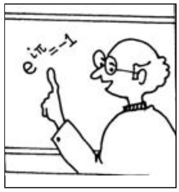
Classroom



In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

The Inveterate Tinkerer
9. Rayleigh–Taylor Instability

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In this series of articles, the authors discuss various phenomena in fluid dynamics, which may be investigated *via* tabletop experiments using low-cost or home-made instruments. The ninth article in this series explores and demonstrates Rayleigh–Taylor instability.

Procedure

Consider the ceiling of a room which is uniformly coated with a 'flat' layer of water. If the layer of water is in hydrostatic equilibrium, one may erroneously conclude that the atmospheric pressure ($\sim 10^5$ N/m^2) can support a column of water of thickness $\sim$10 m! However, this scenario is not realised in practice since air pressure cannot prevent small initial irregularities on the interface (possibly due to thermal motion of water molecules) from increasing in time, which may induce the Rayleigh–Taylor (RT) instability [1–4]. The water moves downwards in the form of 'spikes', and the air pushes upwards through the water layer in the form of 'mushroom-shaped' columns. The RT instability is driven by an interplay between gravity, which destabilises the interface and surface tension forces which act as restoring forces, tending to flatten the interface.

Keywords
Rayleigh–Taylor instability, Bond number, surface tension.

Rayleigh [1] considered an idealised system with a base state comprising two horizontal, parallel, unbounded fluids in a grav-

Bond number is a ratio of the gravitational force (per unit volume) and the difference in pressure across the curved fluid interface.

itational field, which are inviscid, incompressible, and immiscible. The upper fluid is denser than the fluid below the interface. If the base state of the fluids are subject to infinitesimal perturbations, the interface is unstable for perturbation wavelengths which are larger than 2π times a certain length scale which is defined below. The relevant dimensionless quantity is the Bond number (Bo), which is a ratio of the gravitational force (per unit volume) and the difference in pressure across the curved fluid interface, defined as:

$$Bo \equiv \frac{\Delta\rho g L^2}{\gamma} .$$

Here, $\Delta\rho$ is the difference in densities of the two fluids, g is the acceleration due to gravity, L is a relevant length scale for the fluids, and γ is the interfacial tension between the liquids. For Bo =1, one obtains the capillary length:

$$L = L_\mathrm{c} \equiv \sqrt{\gamma/(\Delta\rho g)} ,$$

which has a value of 2.7 mm for an air-water interface. Perturbation wavelengths shorter than $2\pi L_\mathrm{c}$ are stabilised by surface tension forces. On considering the effects of viscosity, the range of unstable wavelengths is found to remain unchanged. Taylor [2] generalised Rayleigh's argument to include systems wherein, both superposed fluids are accelerated perpendicular to their interface. For the occurrence of RT instability, the acceleration relative to gravity must be directed from the less dense fluid to the more dense fluid (see [5] for an experimental realisation). For an axisymmetric interface, the interface exhibits a coronet-like pattern. However, the formation of such patterns is a consequence of finite perturbation of the interface, a situation which was not considered by Rayleigh and Taylor.

A vivid demonstration of the RT instability may be obtained *via* the following experiment:

A glass aquarium ($45 \times 45 \times 20\, cm^3$) was filled with 6 L of water (dyed with blue ink) and a similar volume of heptane. Heptane has a lower density and interfacial tension (with respect to air) as

compared to water. A test tube is lowered horizontally into the water layer, with the inlet closed by pressing against the thumb. When the test tube is fully submerged within the water layer, we allow water to enter the test tube by removing the thumb. The test tube is then rotated into a vertical position with the inlet supported at the bottom of the aquarium. If the test tube is lifted to the heptane layer, the water in the test tube does not fall out despite the higher density of water *vis-a-vis* heptane ! The water remains inside the test tube if the test tube is shaken. However, if the test tube is lifted further, the water pours out as soon as the inlet breaches the air-heptane interface. See the video:
`youtube.com/watch?v=kT_sMVVzlXE`

A water drop impacting a shallow pool of water in a petri dish shows quite clearly, the sequence of events which lead to the formation of spikes and mushroom-shaped irregularities.

A remarkably simple experiment may be used to demonstrate the RT instability:

Pour a thin layer of silicone oil on a glass petri dish placed on black paper. After the oil layer comes to rest, turn the petri dish upside down. The oil layer exhibits periodic fingering patterns. A detailed analysis of the experiment may be found in [6]. See the video: youtube.com/watch?v=iMW0sAKgMqM

Suggestions for Further Work

1. ***Water Drop and a Plastic Tube Impacting a Pool of Water***

A water drop impacting a shallow pool of water in a petri dish shows quite clearly, the sequence of events which lead to the formation of spikes and mushroom-shaped irregularities. See *Figure* 1 and a video filmed with a high-speed camera (Phantom M110) and macro lens, backlit using an LED light source:
`youtube.com/watch?v=-PpMRUkGL7Y`

The conical bottom of a 15 ml plastic centrifuge tube was cut off using a knife and the tube was filled with epoxy adhesive (M-Seal, Pidilite). The tube was dropped vertically onto a pool of water, and the impact filmed using a high-speed camera. The water sheath thrown upwards upon impact also exhibits a coronet-like pattern. See *Figure* 2 and the video:
`youtube.com/watch?v=M1Fiq7s8vs4`

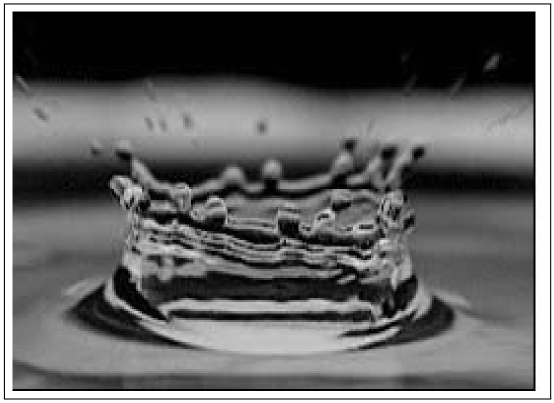

Figure 1. Water drop impacting a pool of water. Note the coronet-shaped interface.

Figure 2. A plastic cylinder impacting a pool of water. The sheath of water thrown upwards is shaped like a coronet.

2. ***Water Drop Impacting a Cylindrical Lens***

A cylindrical glass lens (diameter = 0.5 cm, height = 1 cm) was stuck onto a glass tray using double-sided tape. A drop of water was made to impact the top of the lens from a height of ~30 cm. Upon impact, the water drop expands radially, forming a sheet with a coronet-like pattern, which disintegrates into smaller droplets. See *Figure* 3 and the video:

`youtube.com/watch?v=r0Bnpnv-1p0`

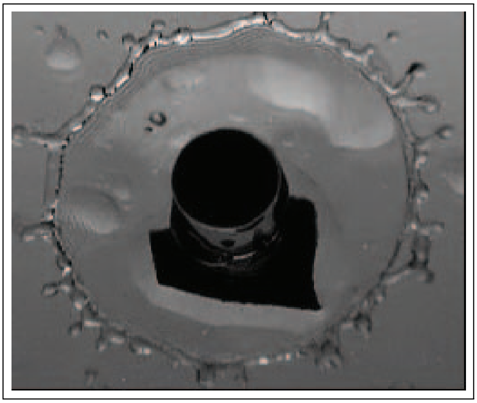

Figure 3. Water drop impacting the top of a cylindrical lens. The edge of the water sheet is shaped like a coronet.

Acknowledgments

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Suggested Reading

[1] L Rayleigh, Investigation of the Character of the Equilibrium of an Incompressible Heavy Fluid of Variable Density, *Proc. Lond. Math Soc.*, Vol.14, p.8, 1883.

[2] G Taylor, The Instability of Liquid Surfaces When Accelerated in a Direction Perpendicular to their Planes. I, *Proc. Roy. Soc. Lond. A*, Vol.201, p.192, 1950.

[3] D Sharp, An Overview of the Rayleigh–Taylor Instability, *Physica D*, Vol.12, p.3, 1984.

[4] E Guyon, J Hulin, L Petit and C Mitescu, *Physical Hydrodynamics*, Oxford University Press, 2nd ed., 2015.

[5] D Lewis, The instability of liquid surfaces when accelerated in a direction perpendicular to their planes. II, *Proc. Roy. Soc. Lond. A*, Vol.202, p.18, 1950.

[6] M Fermigier, *et. al.*, Two dimensional patterns in Rayleigh–Taylor instability of a thin layer, *J. Fluid Mech.*, Vol.236, p. 349, 1992.

[7] The growth of mushroom-shaped irregularities may be seen at the interface between salt water overlying dyed fresh water in the video: `youtube.com/watch?v=NI85oC-3mJ0`

[8] View the the video 'Flow instabilities' which is part of the series produced by the National Committee for Fluid Mechanics Films (United States) available at `web.mit.edu/hml/ncfmf.html`