Second Law, Landauer's Principle and Autonomous Information Machine

Shubhashis Rana and A M Jayannavar

Second law of thermodynamics can be apparently violated in systems whose dynamics depend on information acquired by measurement. However, when one considers measurement and erasure together with the system, it saves the second law. We consider the simple example of an information machine, where information is used as a resource to increase the machine's performance. The system is connected to two baths, a work source, and a moving tape which is used as an information reservoir. The performance of the device is autonomous. The system acts as an engine, erasure or refrigerator. Even combination of any two is possible. All these possibilities are allowed by the generalized second law.

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Introduction

Second law of thermodynamics is one of the strongest rules of nature. While its validity has been questioned many times, but it always hold on average. Challenging the validity of the second law dates back to nearly 150 years, when Maxwell proposed his famous thought experiment [1]. In his gedankenexperiment, he considered a box filled with gas. The box was divided into two sections by a partition. The partition had a small door whose dimension was comparable to the gas molecule. An imaginary demon monitors the gas particles and controls their passage through the door. When a fast (red) molecule comes from the left (*Figure* 1), the demon allows it pass to the right side by opening the door, while the demon closes the door if a slower (blue) molecule comes from the left. On the other hand, the demon only allows the transfer of slower (blue) molecules from the right side to the left. Hence, over a course of time, sorting takes place and the

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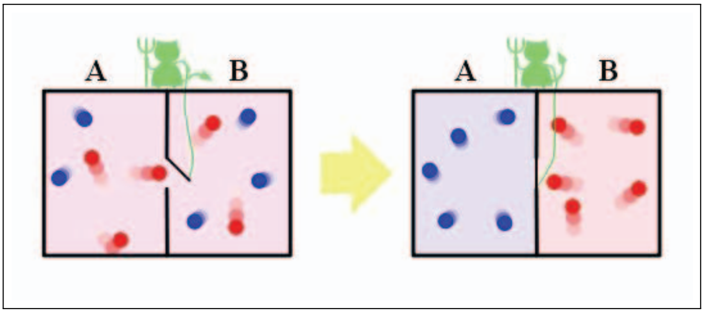
Figure 1. Schematic diagram of Maxwell's demon.



right part of the box contains faster molecules (red) compared to the left (as shown in *Figure* 1). Since the average kinetic energy of the gas particles determines the temperature of the gas, over time, the colder side become more colder and the hotter side become more hotter. This happens when the demon monitors the gas particles by only knowing the velocity of each individual gas molecule. This seems to be a violation of the second law because, second law forbids the transfer of heat from a cold body to a hot body without doing any work.

A Maxwell demon transfers heat from the colder region to the hotter region without doing any work! This is a violation of the second law of thermodynamics as described by J C Maxwell in 1871.

There is another famous example, given by Szilard in 1921, where a demon is involved and the second law is apparently broken [2]. He formulated an engine (*Figure* 2) that could extract energy from a single heat bath and convert it into useful work in the presence of information in a cyclic process. In his original derivation, he took a single gas particle confined in a box of volume V and the box was connected to a bath with temperature T. A demon is placed to monitor the system and extract energy.

First, the demon puts a partition quickly in the middle and separates the box into two equal parts. The gas molecule would be confined into any one of the parts. Then it measures in which part the molecule is in. Depending on the measurement, the partition is moved isothermally such that $k_B T \ln 2$ amount of work is extracted. Finally, the partition is removed and the system returns to its original state. For classical systems, insertion and removal of the partition do not need any energy. Hence, one can extract energy from the heat bath repeatedly which is surprising and the

Szilard engine can perform work taking heat from a single bath cyclically. This is again an apparent violation of the second law.

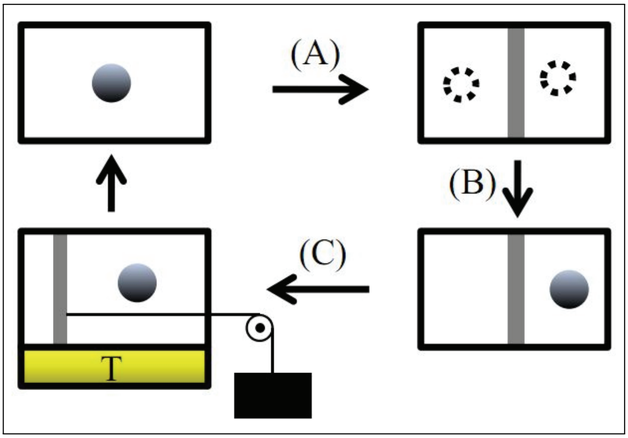


Figure 2. Schematic diagram of Szilard engine. The process includes 4 steps: A) Insertion of the partition, B) Measurement in which side the particle is in, C) Feedback depending on the measurement outcome and finally D) Removal of the partition.

system is termed as a 'perpetual machine'. However, it is unfair to treat a single particle kicking randomly to the wall of the box as an ideal gas. But, any system that undergoes a phase-space splitting can be used as a working system to harness energy. In both the cases, the demon uses its acquired information smartly and decreases the entropy of the system without performing any work by himself. It can be shown that one can perform measurement without involving any energy cost. The paradox of this apparent violation of the second law can be resolved if one includes memory of the demon as a part of the whole system. The memory which records the state of the system, has to be reset to its initial state. This process is called 'erasure'. It is a irreversible process because there is no one to one correspondence between the input and output of this process. According to Landauer's principle, any logically irreversible transformation of classical information is necessarily accompanied by dissipation of at least $k_B T ln2$ of heat per lost bit [3]. This leads to an entropy increase of the bath at least by $k_B ln2$ per bit. Entropy cost for resetting the demon's memory is always larger than the initial entropy reduction, thereby, safeguarding the second law. The importance of Szilard's engine and Maxwell's demon is that they brought information to the same footing as entropy. This will be discussed below in more detail.

1. The Second Law in Non-Equilibrium Regime

In information theory, Shannon entropy of a random variable X with probability density at any time $\rho(x,t)$, is defined as $\mathcal{H}(X,t) = -\mathbf{Tr}\rho(x,t)\ln\rho(x,t)$. Shannon entropy denotes the uncertainty of the random variable and measures the amount of information required to describe the random variable. When X represents the microscopic state of a physical system, one can define the non-equilibrium entropy as:

$$S(t) = k_B\mathcal{H}(X,t) = -k_B\mathbf{Tr}\rho(x,t)\ln\rho(x,t), \tag{1}$$

where k_B is the Boltzmann constant. Recent developments in stochastic thermodynamics reveal that Shannon entropy has a clear meaning in certain situations and it determines the energetics of non-equilibrium processes for systems connected to one or more thermodynamic reservoirs. Let us consider a system connected to a bath of temperature T. The system is evolved by a protocol $\lambda(t)$, and $H(\lambda(t))$ denotes the corresponding time dependent Hamiltonian. Then the system energy at any time is given by:

$$E(t) = \mathbf{Tr}\rho(t)H(t). \tag{2}$$

Corresponding non-equilibrium free energy is defined as [4]:

$$\mathcal{F}(t) = E(t) - TS(t). \tag{3}$$

Here T denotes the temperature of the bath to which the system is connected. Suppose the protocol is kept fixed at a particular value of $\lambda(t)$, then the system will reach the equilibrium distribution:

$$\rho^{eq}(t) = \frac{1}{Z(\lambda(t))}\exp^{-\beta H(\lambda(t))}, \tag{4}$$

At equilibrium, the non equilibrium free energy coincides with the equilibrium free energy.

where $\beta = 1/k_BT$ is the inverse bath temperature and the partition function $Z(\lambda(t)) = \mathbf{Tr}\exp^{-\beta H(\lambda(t))}$. Note that equilibrium distribution should not depend on time. Here the time in the bracket just denotes the value of the protocol $\lambda(t)$ at which the equilibration is done. Now, if we introduce $\rho^{eq}(t)$ in (1) then the non-equilibrium entropy will coincide with the corresponding equilibrium entropy, and one can recover the usual relation $F(t) = \mathbf{Tr}\rho^{eq}(t)H(t) - TS$ with $F(t)$ as equilibrium free energy, when the protocol is fixed at the value of $\lambda(t)$.

When a system is driven from one non-equilibrium state to another non-equilibrium state by a protocol, then the average work done on the system is always greater than the corresponding non-equilibrium free energy change. This is the generalized second law of thermodynamics.

Suppose the system is initially in a non-equilibrium state $\rho(0)$. Then it is evolved by an external protocol $\lambda(t)$ upto time τ, and the system reaches another non-equilibrium state $\rho(\tau)$. W and Q represents the work done on the system and the heat transferred to the bath during this process.

Then, according to the first law of thermodynamics, the internal energy change of the system during this evolution becomes:

$$\Delta E = W - Q. \tag{5}$$

The change in non-equilibrium system entropy

$$\Delta S = \Delta S_{tot} - \Delta S_B, \tag{6}$$

consists of two terms. Total entropy production $\Delta S_{tot} \geq 0$ is strictly a positive quantity. The second term is the entropy flow to the bath and is defined as $\Delta S_B = Q/T$. Using the first law and the definition of non-equilibrium free energy, one can get the second law for non-equilibrium regime:

$$T\Delta S_{tot} = W - \Delta\mathcal{F} \geq 0, \tag{7}$$

where $\Delta\mathcal{F} = \mathcal{F}(\tau) - \mathcal{F}(0)$. This means that the extractable work $-W$ is always bounded by decrease of non-equilibrium free energy difference $-\Delta\mathcal{F}$. This is a more general equation. Now, if the system starts from equilibrium and reaches another equilibrium point after evolution, then the above equation will be reduced to the usual equation:

$$W_{diss} = W - \Delta F \geq 0, \tag{8}$$

where W_{diss} denotes the dissipated work and $\Delta F = F(\tau) - F(0)$ is the equilibrium free energy change.

2. Relative Entropy and Mutual Information

Relative entropy determines the error to take one distribution instead of actual distribution. It vanishes if one takes actual distribution.

In information theory, the relative entropy or Kullback–Leibler distance, measures the distance between two distributions and is defined as:

$$D(p||q) = \sum_x p(x) \ln \frac{p(x)}{q(x)}. \tag{9}$$

This distance measures the distinguishability between two distributions.

However, it is not symmetric. If the two distributions coincide, $p(x) = q(x)$ then relative entropy vanishes. Physically, $\mathcal{H}(p) + D(p||q)$ represents the number of bits required on average to describe the random variable $p(x)$ in terms of $q(x)$. Using the property, $\ln(x) \leq x - 1$ for $x \geq 0$ we have:

$$\begin{aligned} D(p||q) &= \sum_x p(x) \ln \frac{p(x)}{q(x)} \\ &= -\sum_x p(x) \ln \frac{q(x)}{p(x)} \\ &\geq \sum_x p(x) \left[1 - \frac{q(x)}{p(x)}\right] = \sum_x [p(x) - q(x)] = 1 - 1 = 0, \end{aligned} \tag{10}$$

which implies that relative entropy is always positive. The mutual information (*Figure* 3) between the two random variables U and V is defined as:

$$I(U;V) = \sum_{u,v} \rho(u,v) \ln \frac{\rho(u,v)}{\rho(u)\rho(v)} = \mathcal{H}(U) + \mathcal{H}(V) - \mathcal{H}(U,V). \tag{11}$$

Mutual information is basically the relative entropy between the joint distribution of the two random variables with their product distribution. Note that unlike relative entropy, mutual information is symmetric, *i.e*, $I(U;V) = I(V;U)$. $I(U;V)$ is always positive and vanishes only when these two random variables U and V are statistically independent, *i.e.*, there exists no correlation between them. One can always rewrite $I(U;V)$ as:

Mutual information measures the correlation between two random distributions. If the two distributions are independent, then the mutual information becomes zero.

$$I(U;V) = \mathcal{H}(U) - \mathcal{H}(U|V) = \mathcal{H}(V) - \mathcal{H}(V|U), \tag{12}$$

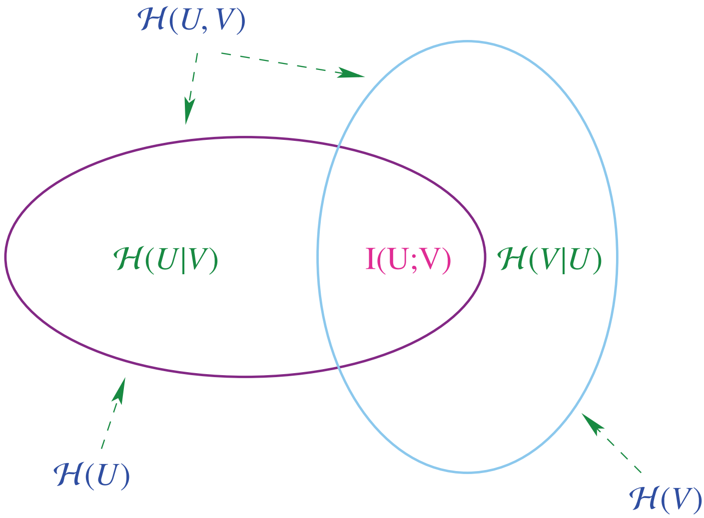


Figure 3. A Simple Diagram to Describe Mutual Information $I(U;V)$.

where $\mathcal{H}(U|V) = -\sum_{u,v}\rho(v)\rho(u|v)\ln\rho(u|v)$ denotes the conditional entropy and quantifies the uncertainty of U for given V. Now, $\mathcal{H}(U)$ represents the uncertainty of U. Hence, mutual information denotes the reduction of uncertainty of one random variable due to the knowledge of another.

3. Measurement and Entropy

We will now discuss the effect of measurement on a system with the probability density $\rho(x)$. Suppose a measurement is performed and m is the outcome like left or right in the Szilard engine. The probability density will be updated to $\rho(x|m)$ after this measurement. If the system remains initially at equilibrium, $\rho(x|m)$ will not be canonical after the measurement. Thus, measurement leads the system to a non-equilibrium state, although there is no energy cost. The change in non-equilibrium entropy due to measurement is $S(\rho(x|m) - S(\rho(x))$. Here, $S(\rho(x|m)) = -k_B\sum_x\rho(x|m)\ln\rho(x|m)$ and $S(\rho(x)) = -k_B\sum_x\rho(x)\ln\rho(x)$. Taking the average over all the possible outcomes with probability p_m, the non-equilibrium entropy change becomes:

$$\begin{aligned}\Delta S_{meas} &= -k_B \sum_{x,m} p_m \rho(x|m) \ln \rho(x|m) + k_B \sum_{x.m} p_m \rho(x) \ln \rho(x) \\ &= k_B(\mathcal{H}(X|M) - \mathcal{H}(X)) \\ &= -k_B I(X;M). \end{aligned} \tag{13}$$

Note that in measurement process, neither the Hamiltonian nor the micro-state of the system is affected. That means the average energy of the system does not change due to measurement. Hence, the non-equilibrium free energy change will become:

$$\begin{aligned}\Delta \mathcal{F}_{meas} &= \sum_m p_m \mathcal{F}(\rho(x|m);\mathcal{H}) - \mathcal{F}(\rho(x);\mathcal{H}) \\ &= -T\Delta S_{meas} = k_B T I(X;M). \end{aligned} \tag{14}$$

As, $I(X;M) \geq 0$ there is always an increase in non-equilibrium free energy, which can be eventually used to extract work in an isothermal process at later times. This explains the functioning of a Szilard engine in a cyclic process.

4. Landauer's Principle and Memory

Information seems to be an abstract quantity at first glance. However, it is not. When a measurement is performed, the obtained information is generally stored in a piece of paper or a hard-disk, *etc*. Landauer shed new light on these perspectives in his famous article 'Information is Physical' [6]. Any physical system with multiple, distinguishable, metastable states can be used to store the information. However, these states should have long enough lifetimes and should not be affected by the environmental fluctuations or any external constraints. Only then can a system act reliably as memory in desired time. It means ergodicity must be broken or effectively broken for timescale for which the memory is reliable. The total phase space Γ is split into several ergodic regions Γ_m for each memory outcome m. Magnetization in a small ferromagnetic domain of a standard magnetic memory or high energy barriers separating microscopic degrees of freedom in a single electron memory are few examples in this regard.

A memory is a device which stores information. This means that its state does not evolve with time. Only during writing and erasing, when it is connected to another device, its state can change. Hence it can be termed as an information reservoir. Examples include CD, hard disk, data chip, *etc*.

Let p_m denote the probability to be in the ergodic region Γ_m of the memory. Now, if the memory is in local equilibrium, one can take E_m and S_m as the average energy and non-equilibrium entropy of the corresponding ergodic region. The non-equilibrium free energy of the memory can be written as [5]:

Work is always needed to erase information. The minimum amount of work needed to erase one bit of information is given by $k_B T \ln 2$.

$$\mathcal{F}(M) = \sum_m p_m F_m - k_B T \mathcal{H}(M), \tag{15}$$

where, $F_m = E_m - T S_m$, and $\mathcal{H}(M) = -\sum_m p_m \ln p_m$ is the Shannon entropy of the informational states. Note that the total entropy of the memory is the sum of Shannon entropy $\mathcal{H}(M)$ and the individual internal entropies S_m. Now, after manipulation of the memory, we assume that the Hamiltonian of the system reaches its initial Hamiltonian. Then, we need to be concerned only about the expression of p_m for the particular memory state. Suppose, during measurement, the state changes from M' with probability p'_m to the state M with probability p_m. The change in free energy for the memory in this case will be $\Delta\mathcal{F}^{mem}_{meas} = \mathcal{F}(M) - \mathcal{F}(M')$. Similarly, for resetting the memory to its original state M', the free energy change will be $\Delta\mathcal{F}^{mem}_{reset} = \mathcal{F}(M') - \mathcal{F}(M)$, and work done to reset the memory must follow:

$$W_{reset} \geq \Delta\mathcal{F}^{mem}_{reset}. \tag{16}$$

In this respect, if one takes symmetric memory $F_0 = F_1 = F_2 = \ldots$, the free energy change will be reduced to only the Shannon entropy change $\Delta\mathcal{F}^{mem}_{reset} = -k_B T(\mathcal{H}(M') - \mathcal{H}(M))$. Now, if we reset the state to a standard state such that $p'_0 = 1$ and all other $p'_m = 0$ then $\mathcal{H}(M') = 0$. This is the 'restore to zero process', and one gets $\Delta\mathcal{F}^{mem}_{reset} = k_B T \mathcal{H}(M)$. If the memory consists of only two states, and for random bit $p_0 = p_1 = 1/2$, one obtains the celebrated Landauer's limit:

An information reservoir is like a memory device which stores information. In an information machine, this information can be used as a resource to drive the system and change its performance. In 2012, Jarzynski gave the first example of this and showed that work can be extracted from a single heat bath in an autonomous cyclic process.

$$W_{reset} \geq k_B T \ln 2, \tag{17}$$

while equality holds for reversible processes. This is the famous Landauer's principle which states that the minimum work needed to erase one bit of information is $k_B T \ln 2$. The opposite of the restore to zero process increases disorder in the states of memory. As a result, one can easily extract work out of this process. This trick is used in the recently proposed autonomous information engine model where information is used as a fuel to extract thermodynamic work in a isothermal process.

Hence, an ordered state of memory register can be treated as a reservoir of information which can be taken to be on the same footing as other thermodynamic reservoirs like thermal or chemical bath.

5. An Autonomous Information Machine: Working as Engine, Refrigerator, and Erasure

In this section, we have described a model where information is used as a fuel to extract work [7]. The model consists of a system (demon), a work source, and an information reservoir (*Figure* 4). The system can interact with two thermal reservoirs, hot and cold. A mass that can be lowered by gravity or pulled up against gravity is used as a work source. A stream of bits written on a tape is used as an information reservoir. The information can be written/ removed during the operation. The system can exchange information with the information reservoir but not energy, although, it can exchange energy with the work source and the thermal bath. The dynamics is taken to be autonomous, which means that there is no external control. The system reaches unique steady state, depending on its parameters. The main motivation of this study is to find the effect of thermal bias along with the work source and information source. Recently, D Mandal and C Jarzynski has given a simple model where the system is connected to a single bath, and works as erasure and engine [8]. In another model [9] it acts as refrigerator and erasure. In the present model, a single system acts as engine, refrigerator, and erasure simultaneously. The system can perform as an engine by extracting energy out of the thermal reservoirs, as a refrigerator by transferring heat from cold to hot bath, and as an erasure by removing information content in the tape. We will explain these definitions in more detail

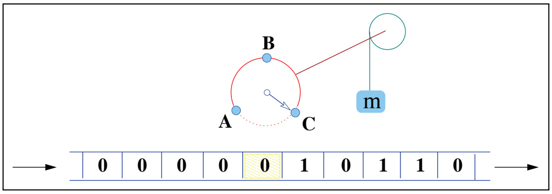


Figure 4. A schematic diagram of our model. The demon is a three state system which is coupled with an external load and two thermal reservoirs (not shown). A sequence of bits (tape) passes from left to right at a constant speed. The nearest bit interacts with the demon. For positive load, *i.e*, $w > 0$, the mass is lifted at an amount Δh for every transition C $\rightarrow$ A, while for every transition A $\rightarrow$ C the mass is lowered by the same amount. However, at $w < 0$ the mass is connected to right side of the small circle, so the transition C $\rightarrow$ A lowers the mass, and the transition A $\rightarrow$ C lifts it up.

in the later section. It is observed even more surprisingly that the system can perform any two of these behaviors simultaneously! However, these behaviors are consistent with the second law of thermodynamics. Moreover, the efficiency of the engine and the coefficient of performance of the refrigerator can go beyond the Carnot limit.

5.1 *The Model*

Let us take a three state system (A, B, C), and these states are non-degenerate (*Figure* 5). The difference between the two successive energy levels is taken to be the same (E_1). Transition can occur between A to B, B to C, and *vice versa* by exchanging heat from the cold bath spontaneously with temperature T_c, and internal energy of the system gets changed.

However, the transition between A to C and *vice versa* is restricted. It depends on the value of the interacting bit written on the tape. As the bit has two states 0 and 1, the combined system has 6 states. The bit state is changed from 0 to 1 when the transition occur from C0 to A1 (clockwise rotation) and *vice versa* (anticlockwise rotation). However, transition between C1 and A0 is not allowed. Note that the bit state does not change when transitions occur between A and B, B and C. During the transition from C0 to A1 (clockwise rotation), energy is absorbed from the hot bath at temperature T_h by an amount E while the system performs w amount of work by pulling a mass m to a height Δh by a frictionless pulley in a gravitational force field g ($w = mg\Delta h$). Note that during this transition, the internal energy of the demon is increased by $2E_1$. Using first law one can write:

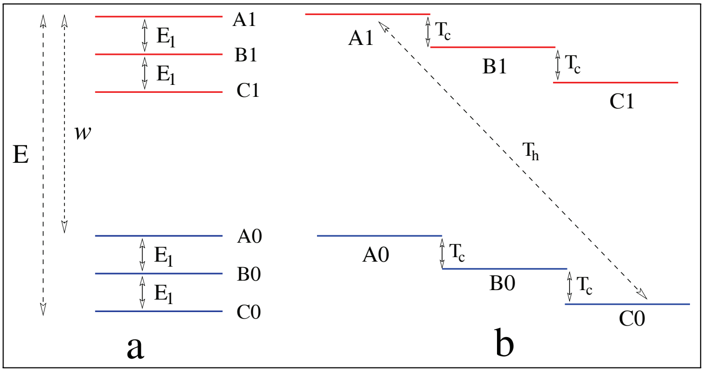


Figure 5. Possible six states depending on the demon and the bit. **(a)** The difference between the energy levels between A and B is E_1. Similarly the difference between B and C is also E_1. The energy absorbed/released during any transition between C0 and A1 is denoted as E such that $E = w + 2E_1$. **(b)** All the allowed transitions and corresponding bath where energy is exchanged.

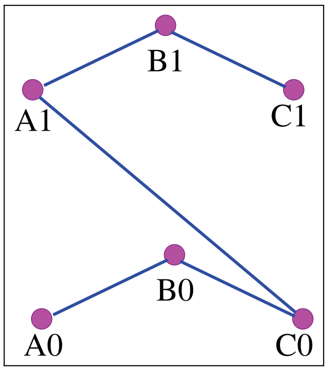


Figure 6. All the allowed transitions form a liner chain during each operation interval τ.

$$E = w + E_1. \tag{18}$$

All the allowed transitions are shown in *Figure* 5 and they form a linear chain *Figure* 6. The weight parameter ε is defined by:

$$\varepsilon = \tanh\left(\frac{E}{2T_h}\right), \tag{19}$$

and is bounded by $-1 < \varepsilon < 1$. Assuming the tape is moving with a speed $\frac{1}{\tau}$ and interaction happens only with the nearest bit (one bit at a time), during time τ, the system (demon) evolves along with the bit. Then next bit arrives and the earlier bit moves forward containing the information. Note that only during the interaction with the system the bit may flip. If τ is small, system hardy evolves. However, if τ is large, system gets enough opportunity to evolve along with the bit. Let $p(0)$ and $p(1)$ denote the probability of 0 and 1 in the incoming bit stream: $p(0)+p(1) = 1$. Then one can define:

$$\delta = p(0) - p(1), \tag{20}$$

as the probability of excess number of 0 in the incoming bit stream. The Shannon entropy of the incoming bit stream is defined as:

$$S = -p(0)\ln p(0) - p(1)\ln p(1). \tag{21}$$

This Shannon entropy measures the amount of disorder present among the bits. However, we have ignored the correlation between the successive bits. Shannon entropy also quantifies the amount of information present in the bit stream. More disorder is equivalent to more information. Note that if every bit is zero, the Shannon entropy is also zero. This is again true when every state is 1. Shannon entropy becomes maximum when both 0 and 1 states are equally probable and its value then becomes $S_{max} = ln2$. Now, if $p'(0)$ and $p'(1)$ represents the probability of 0 and 1 in the outgoing bit stream respectively, then, corresponding Shannon entropy will be:

$$S' = -p'(0)\ln p'(0) - p'(1)\ln p'(1). \tag{22}$$

The change of Shannon entropy of the bits, written on the tape, will be simply $\Delta S = S' - S$. Now, if $\Delta S > 0$, it means that some information is written on the tape or the bit stream has become more disordered. On the other if $\Delta S < 0$, it implies that during the interaction with the demon, some information has been erased and it acts as an erasure.

If ϕ represents average number of clockwise (CW) rotations then,

$$\phi = p'(1) - p(1) = (\delta - \delta')/2, \tag{23}$$

where $\delta' = p'(0) - p'(1)$. During each CW rotation, E energy is absorbed from the hot bath while the system performs w amount of work. Hence, the average heat dissipated to the hot bath will be given by:

$$Q_h = -\phi E, \qquad (24)$$

and average work done on the system will be:

$$W = -\phi w. \qquad (25)$$

Using the first law, on can readily write the average heat dissipated to the cold bath as:

$$Q_c = 2\phi E_1. \qquad (26)$$

Since the system operates in a steady state, the entropy production of the system (demon) will be zero. Then the total entropy production will be:

$$\Delta S_{tot} = \Delta S + \Delta S_B, \qquad (27)$$

where $\Delta S_B = \frac{Q_h}{T_h} + \frac{Q_c}{T_c}$ denotes the bath entropy production. In next section, the model is analyzed with the help of numerical simulation.

5.2 *Results and Discussions*

This problem may become simpler by taking $E_1 = 0$ $(w = E)$. This implies that all the three states of the system are degenerate. Due to presence of bits, the joint state $A1$, $B1$, $C1$ has energy E while $A0$, $B0$, $C0$ has zero energy. The transition between $C0$ and $A1$ takes place with the interaction of the bath T_h, and $E = w$ energy will be exchanged. Then this problem will be reduced to the original Mandal–Jarzynski model [8]. The temperature of the bath is set at $T_h = 1.0$. The interaction time with the demon with each bit is kept fixed at $\tau = 1.0$. Then one can obtain the phase diagram numerically as shown in *Figure* 7 by simply varying $-1 < \varepsilon < 1$ and $-1 < \delta < 1$. The system acts as an engine (red plus region) by extracting heat from a single heat bath and converting it completely into output work. This is quite surprising. However, the entropy of the outgoing bits will be more compared

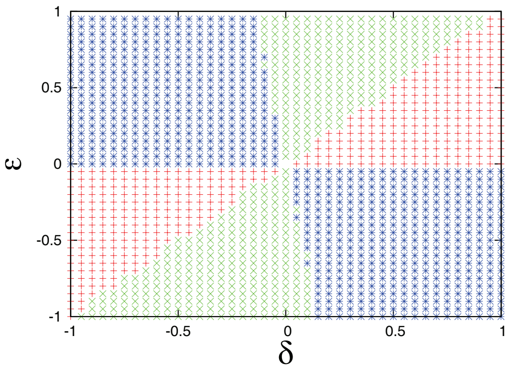


Figure 7. Phase diagram when the demon is connected to a single bath with temperature $T_h = 1.0$ and cycle time $\tau = 1.0$. There are thee regions in the phase diagram namely i) Engine (red plus), ii) Erasure (green cross), iii) Dud (blue star).

to the incoming bits ($\Delta S > 0$). Hence, at the expense of information as fuel, the system performs as an engine. In the green region, the system acts as an erasure, where it erases information ($\Delta S < 0$) written on the tape, while work is done on the system ($W > 0$). In other region of the phase space (blue region), it neither performs as an engine ($W > 0$) nor as an erasure ($\Delta S > 0$) and is denoted by dud.

Next, we move to the earlier problem. The temperature of the hot and cold bath is set at $T_h = 1.0$ and $T_c = 0.5$ respectively. The separation between two successive energy levels is taken at $E_1 = 0.5$ and $\tau = 1.0$. Then, one can obtain the phase diagram by simply varying $-1 < \varepsilon < 1$ and $-1 < \delta < 1$. It is important to mention that the system experiences two forces (randomization of bits and pull of gravity) besides the thermal bias between two baths. The phase diagram is obtained due to the interplay of these three forces.

The phase diagram (*Figure* 8) consists of 7 regions.

i) In the red plus region, the system performs as an engine by extracting work ($W < 0$) on average while writing information on the tape ($\Delta S > 0$). The efficiency of an engine $\eta = \frac{W}{Q_h}$ can exceed Carnot limit ($\eta > 0.5$ for this case).

ii) The system acts as an erasure ($\Delta S < 0$) in the green cross region, while work is done in the system ($W > 0$).

iii) In the pink open box region, it performs as a refrigerator ($Q_c <$

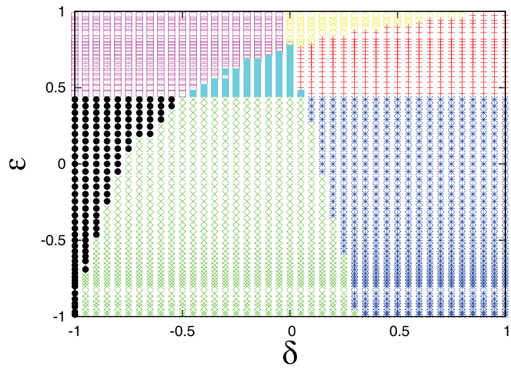


Figure 8. Phase diagram when the demon is connected with two baths with temperature $T_h = 1.0$ and $T_c = 0.5$. The other parameters are set at E_1=0.5 and $\tau = 1.0$. The phase diagram consist of seven regions namely: i) Only engine (red plus), ii) Only erasure (green cross), iii) Dud (blue star), iv) Only refrigerator (pink open box), v) Erasure and refrigerator(yellow open circle), vi)Erasure and engine (cyan full box), vii) Engine and refrigerator (black full circle).

0) by transferring heat from the cold bath while $W > 0$ and ($\Delta S > 0$).

Apart from these three regions, there are another three regions where a combination of any two is possible. Such as:

iv) In the yellow open box region, the system simultaneously acts as an erasure ($\Delta S < 0$) and a refrigerator ($Q_c < 0$). However, in this regime, work is done on the system on average.

v) In the triangular area in the middle (cyan full box), thermal bias plays a dominate role in such a way that the demon simultaneously performs as an erasure ($\Delta S < 0$) and as an engine ($W < 0$) by extracting work on average.

vi) In the black full circle region, the demon transfers heat from the cold bath ($Q_c < 0$) as well as it extracts work on average ($W < 0$). However on average, information is written on the tape ($\Delta S > 0$). The coefficient of performance of the refrigerator $\sigma = \frac{-Q_c}{W}$ can take value beyond Carnot limit ($\sigma > 1$ for this case) in this region and in some parts of the pink box region.

vii) Finally in the blue star region, the system does not act as an engine, erasure or refrigerator and we call it dud.

It is observed $\Delta S_{tot} > 0$ throughout the phase space. Hence, all the regions are consistent with the generalized second law of thermodynamics.

6. Conclusions

Information and thermodynamics are treated in a single framework. The generalized second law is proved when the system starts and ends in a non-equilibrium state. We have explained the performance of Szilard engine which seems to violate the second law. We have shown that when measurement is performed, although the energy of the system is not changed, the non-equilibrium free energy apparently increases. Hence, one can extract energy in a cyclic process from a single heat bath using information acquired by measurement. However, erasure of stored information requires work to be done on the system. A memory device can be used as an information reservoir which can be used to increase the performance of a device. We have formulated a simple model of autonomous information engine. We have shown that the system can act as an engine, a refrigerator, or as an erasure. Even combination of any two is possible in some parameter spaces. The efficiency of the engine can be greater than Carnot limit. The coefficient of performance of refrigerator also goes beyond the Carnot limit. Our findings are consistent with the generalized second law of thermodynamics along with information.

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