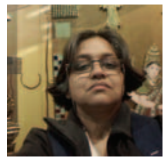
Gravity Defied
From Potato Asteroids to Magnetised Neutron Stars

4. Neutron Stars (Dead Stars of the Second Kind)

Sushan Konar

A star burns its nuclear fuel and balances gravitation by the pressure of the heated gas, during its active lifetime. After the exhaustion of the nuclear fuel, a low mass star finds peace as a 'white dwarf', where the pressure support against gravitation is provided by Fermi-degenerate electrons. However, for massive stars, the gravitational squeeze becomes so severe that in the final phase of evolution, the average density approximately equals the nuclear density. At such densities, most of the protons combine with electrons to convert themselves into neutrons. A 'neutron star', composed of such neutron-rich material, is host to some fascinating physics arising out of its amazingly compact state of matter (where a solar mass is packed inside a sphere of radius ∼ 10Km).



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1. The Beginnings

The discovery of a radio pulsar by Jocelyn Bell (*Figure* 1) in 1967, and its subsequent identification with a neutron star, one of the most exotic objects in the Universe, is a watershed moment for theoretical astrophysics when predictions made several decades previously were finally confirmed through observation. The story of neutron stars has always been marked with prescient ideas and serendipitous observations, which is not surprising given the unbelievably rich and complex physics these tiny stars pack within themselves.

Keywords
Neutron stars, radio pulsars, X-ray binaries, magnetic fields.

Along with the understanding of their nuclear energy source, arose the question about the end states of stars. The nuclear fuel is said to be exhausted either when a star is unable to reach the temper-

Figure 1. Jocelyn Bell, photographed in front of the Cambridge radio telescope in 1967. Bell was working with Anthony Hewish, for her PhD thesis on interplanetary scintillation, when she observed a unusually periodic signal coming from the sky. After discarding many plausible and implausible theories (including one involving the 'little green men') these signals were finally identified to be radio emissions coming from a highly magnetised neutron star.

The discovery of a radio pulsar by Jocelyn Bell in 1967 is a watershed moment for theoretical astrophysics.

ature required for the nuclear fusion of the next element or when ${}_{26}Fe^{56}$ is formed ending the fusion reaction chain. In 1930s, S Chandrasekhar showed that white dwarfs could be one such end state of stars where gravitation is balanced by the pressure of Fermi-degenerate electrons. The logical extension of this argument is the 'neutron stars' where the pressure comes from degenerate neutrons and from the forces of nuclear interaction making these the 'dead stars of the second kind'.

Interestingly, this is exactly what was suggested by Walter Baade and Fritz Zwicky in 1934, only two years after neutrons were discovered by James Chadwick (and almost three decades before neutron stars were actually observed)! The discovery that Andromeda is an external galaxy, enabled them to estimate the energy released from the nova observed in 1885 to be around $\sim 10^{52}$ erg[1].

They hypothesised that this must be associated with the formation of an object of radius $\sim$ 10 km, and such an object must be almost entirely made up of neutrons. In the mid 1930s, Lev Landau strengthened this argument by showing that beyond a density of 10^{11} g cm^{-3}, electrons would combine with protons to form

[1] The gravitational energy release associated with the formation of a neutron star is calculated to be,

$$E_G \simeq G\frac{M^2}{R} \simeq 10^{52} \text{ erg};$$

where it is assumed that $M = 1M_\odot$ and $R = 10$ km.

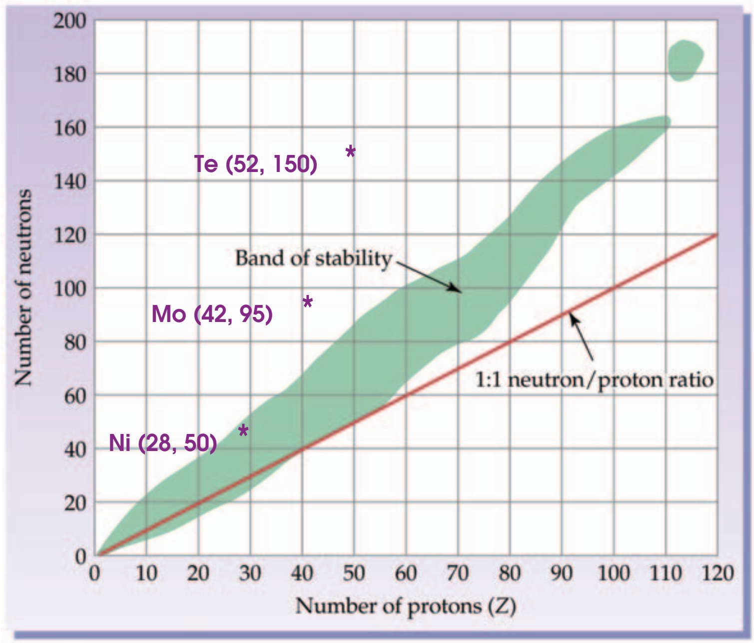

Figure 2. The neutron-proton ratio in terrestrial nucleons is shown with the stability band (greenish-coloured region). Picture courtesy: Mcmury Fay, *Chemistry*, Prentice–Hall, 2004. A few neutron-rich elements, expected to be found in neutrons star crusts are marked in magenta.

neutrons. To understand how prescient this suggestion was, one needs to remember that one of the end products of this process, known as β-capture[2], is a 'neutrino' which would be discovered only in 1956.

[2]β-capture reaction:

$$p^{+} + e^{-} \rightleftharpoons n + \nu\,.$$

In the 1940s, it was pointed out that the position of the Crab nebula coincided with that of the 1054 guest star, recorded by the Chinese court astronomers. The discovery of a radio pulsar at the centre of Crab nebula vindicated Baade and Zwicky's hypothesis completely. The current understanding from stellar evolution theories is that stars with main sequence masses of 8–20 $M_{\odot}$ end their lives in supernovae. Most of the stellar mass is thrown away in such a supernova explosion, and a compact neutron star ($M \sim 1.4 M_{\odot}$, $R \sim 10$ km) is left behind.

2. The Interior

One of the key theoretical concepts associated with neutron stars is the stability of neutrons inside it. Free neutrons decay[3] with

[3]Decay of free neutrons is known as β-decay and is the reverse process of β-capture, where a neutron decays into a proton, an electron, and an anti-neutrino.

$$n \rightleftharpoons p^{+} + e^{-} + \bar{\nu}\,.$$

a half-life of ∼15 minutes. This reaction is prohibited in the forward direction in a neutron star. As mentioned before, free neutrons exist only beyond a density of $\sim 10^{11}$ g cm^{-3}. At such densities, the electrons are degenerate with very large Fermi energies ($E_F \propto \rho^{1/3}$, because electrons are also relativistic at these densities). Since, all quantum levels with $E < E_F$ would be occupied, an electron produced through a β-decay process would require an energy larger than E_F. A neutron at rest is unable to supply such energies to the electron produced and the β-decay is therefore prohibited.

As in the case of a white dwarf, the structure of a neutron star is obtained by solving the hydrostatic pressure balance equation in conjunction with the equation of state. Except, a neutron star being far more compact (with $V_E/c \lesssim 0.5$), we would have to solve the TOV equation instead of the non-relativistic hydrostatic equation[4]. The main problem though, lies in finding the correct equation of state, that is identifying the correct state of matter.

[4]See S Konar, Gravity Defied: From Potato Asteroids to Magnetised Neutron Stars: 3. White Dwarfs (Dead Stars of the First Kind), *Resonance*, Vol.22, No.5, pp.475–484, 2017.

Neutron star matter encompasses a wide range of densities, from $\sim 10^6$ g cm^{-3} at the surface to several times the nuclear density in the stellar core (*Figure* 3). At a given density, the equilibrium composition is the one which is most stable against β-capture. Starting from ordinary Fe^{56} at the surface, the equilibrium nuclide becomes more and more neutron-rich (*Table* 1) as the density increases. Though neutron-rich, the material is expected to arrange itself into a crystalline solid with the electrons forming a Fermi-degenerate gas. At a density of $\sim 4 \times 10^{11}$ g cm^{-3} the neutrons are so numerous that their highest energy level, inside a nucleus, reach the continuum value and some of the neutrons become free of the nuclear binding ('drip' out of the nucleus). Therefore, from this 'drip' density onwards, the neutron star material is composed of a neutron-rich nuclei (again forming a crystalline solid), Fermi-degenerate electrons, and a gas of free neutrons. It is understood that this free neutrons actually exist in a superfluid state, giving rise to a number of interesting observational consequences. The solid crystalline phase extends up to the nuclear density and this

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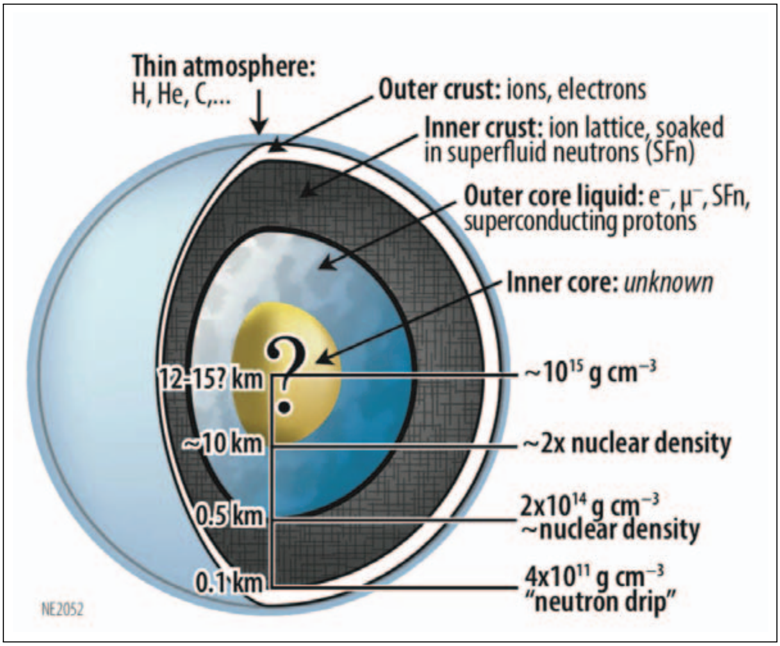

Figure 3. Composition of a neutron star. There is a huge uncertainty about the state of matter at highest densities, in the core of a neutron star (denoted by a question mark). Different assumptions about this result in different mass-radius relations. Accurate mass measurements can lift the degeneracy about core composition as can be seen from *Figure* 5. Picture courtesy - *https://heasarc.gsfc.nasa.gov/*

outer part of a neutron star is known as the 'crust'.

The nuclei completely dissolve into its constituent particles when nuclear density is reached in the inner part of the star known as the 'core'. It is believed that both neutrons and protons exist in separate superfluid phases (neutron forming a neutral superfluid, and protons forming a charged superconductor) inside the core of a neutron star. It is also understood that the rotation of a neutron star is supported by the creation of Onsager–Feynman vortices in the neutron superfluid, whereas the magnetic field is supported by Abrikosov fluxoids[5] generated in the proton superconductor. Both of these superfluid vortices are created when neutron star matter goes into the superfluid phase. It is understood that the superfluid transition temperature of the neutron star material is $\gtrsim 10^9$ K and the neutron star cools down to such temperatures (from $\sim 10^{11}$ K at the time of birth) within a very short time after its formation in a supernova explosion.

It is difficult to envisage the existence of superfluidity inside neutron stars except from a theoretical viewpoint. Fortunately, there

[5] Alexei Abrikosov (1928–2017) was instrumental in developing the theory of 'type-II superconductors', a phase of matter that was later found to be of great importance for modern technology. Protons are thought to form a type-II superconductor inside the core of a neutron star and can support magnetic fields in quantised Abrikosov fluxoids, each of which carry a flux quantum ϕ given by,

$$\phi = \frac{hc}{2e} = 2 \times 10^{-7}\ \text{G.cm}^2 .$$

Table 1. Neutron-rich nuclei expected to be found in the crust of a neutron star. The neutron to proton ratio increases with increasing density, as more and more protons convert into neutrons through β-capture. Stable terrestrial counterparts of these nuclei have also been shown for comparison. It can be seen from *Figure* 2 above that the nucleons expected in the neutron star crust have very different neutron to proton ratio compared to their terrestrial counterparts. The data is taken from Baym, Pethick and Sutherland, *ApJ*, 170, 299, 1971.

ρ g cm^{-3}	Z	A	Terrestrial Element
10	26	56	$^{26}Fe_{56}$
10^6	26	56	$^{26}Fe_{56}$
10^7	28	62	$^{28}Ni_{58}$
10^8	28	62	$^{28}Ni_{58}$
10^9	28	64	$^{28}Ni_{58}$
10^{10}	32	82	$^{32}Ge_{72}$
10^{11}	28	78	$^{28}Ni_{58}$
10^{12}	42	137	$^{42}Mo_{96}$
10^{13}	52	200	$^{52}Te_{128}$
5×10^{13}	74	375	$^{74}W_{184}$

are indirect evidences to support this conjecture. Many radio pulsars have been observed to undergo a period 'glitch'. The radiation from a radio pulsar comes at the expense of its rotational energy. As a result, a radio pulsar undergoes a secular spin-down. Some radio pulsars are observed to undergo a sudden spin-up, with a total or partial recovery to its original rate of spin-down (*Figure* 4). We can understand the origin of this sudden spin-up in the following way. The crust of a neutron star rotates like a rigid body. However, the superfluid component rotates by creating vortices, each of which carries a finite angular momentum. To slow down, the superfluid needs to expel a number of these vor-

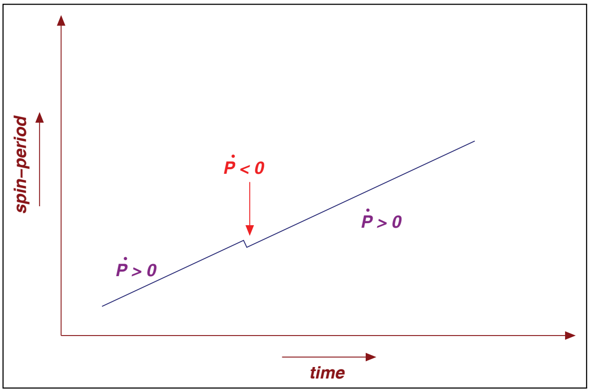

Figure 4. A schematic diagram showing a radio pulsar undergoing an episode of 'glitch' – a sudden spin-up.

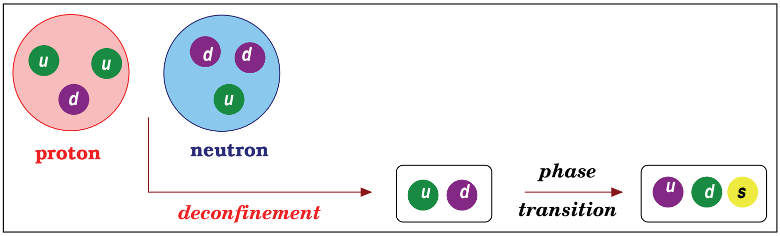

Figure 5. Inside the core of a neutron star, when the density is $\simeq 8\rho_{\text{nuclear}}$, the neutrons and protons are expected to undergo a deconfinement transition to a quark phase composed of 'up' and 'down' quarks. Apparently, the stable quark phase is achieved when some of these convert into 'strange' quarks and achieve a 1:1:1 ratio between all three quark flavors.

tices pinned to the crustal lattice through the neutron's interaction with the nucleons. So, even though the solid crust slows down, the superfluid component continues to rotate at a faster rate. As the difference in the rotation rate increases, so does the tension at the sites the vortices are pinned. When the difference in rotational energy overcomes the pinning energy, there is a sudden unpinning of the vortices and they are expelled from the superfluid. Evidently, this results in the solid crust gaining extra angular momentum which shows up as a sudden spin-up. Though the spin-up itself can be explained in terms of the superfluids (there exist other theories that do not involve superfluids), it is actually the post-glitch behaviour of certain pulsars that definitively indicates the presence of a superfluid inside a neutron star.

It is conjectured that various novel phases like the hyperon[6] matter, Bose–Einstein condensates of strange mesons, and quark matter can exist inside the core, giving rise to a plethora of possible equations of state. For example, EoS involving the strange quark matter (SQM), where up, down, and strange quarks exist in equal numbers (*Figure* 5), and expected to appear when the density is approximately 8 times that of the nuclear density, have been rather popular for a while. Fortunately, observed masses and radii of the neutron stars can directly constrain the composition and EoS of the interior. Technological advances has allowed for rather precise mass measurements in recent years. It can be seen from *Figure* 6 that mass measurement of the neutron star J1614-2230 has ruled out all EoS based on pure SQM.

[6]Hyperon is a baryon containing one or more strange quarks. These are highly unstable under terrestrial conditions and quickly decay into nucleons. At densities $\sim 2-3\rho_{\text{nul}}$, realisable in the core of a neutron star, the inverse reactions can take place producing hyperons.

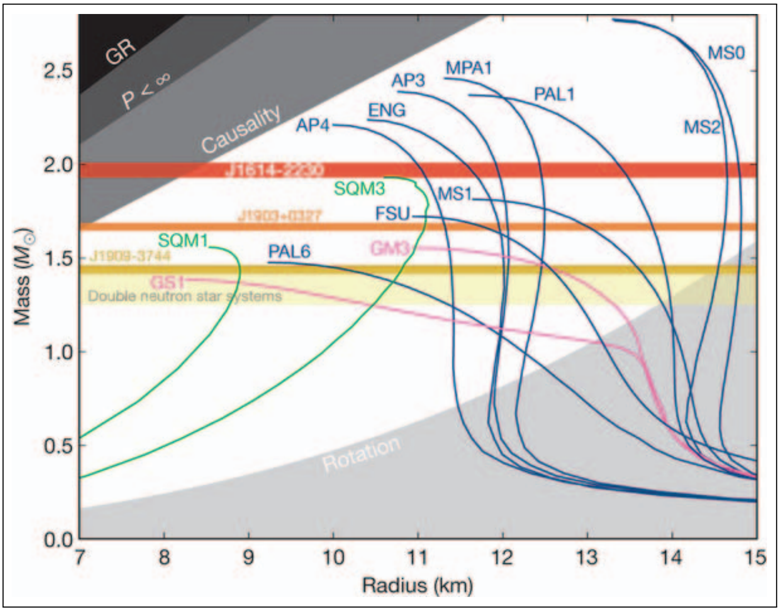

Figure 6. Mass-radius relations for non-rotating neutron stars for different equations state (blue – nucleons, pink – nucleons + exotic matter, green – strange quark matter). The horizontal bands correspond to actual observational mass measurements of a few neutron stars. An EoS line not intersecting an observed mass band is ruled out by that measurement. Most exotic matter EoS is ruled out by mass measurement of the neutron star J1614-2230. The grey regions show parameter space ruled out by other theoretical or observational constraints (GR – general relativity, P – spin period). Figure courtesy: Demorest *et al.*, *Nature*, 467, 1081, 2010.

3. The Observations

It's been half a century since the discovery of a neutron star as a radio pulsar, and ~3000 objects have been observed during the intervening period. As a result of these observations, the spin-period (P_s) and the surface magnetic field (B_s) have emerged to be the most significant intrinsic parameters of a neutron star. The population of neutron stars display a wide range of these parameters ($P_s \sim 10^{-3}-10^{2}$ s, $B_s \sim 10^{8}-10^{15}$ G) and have diverse observational characteristics. However, the population can be broadly classified into three categories, depending on the nature of energy generation in these stars.

Rotation Powered Pulsars (*RPP*)

The rotation powered radio pulsars are so named because of the pulsed radio emission received from them (*Figure* 7). This pulsed emission, characterized by its precise periodicity, is essentially 'magnetic dipolar' radiation that comes at the expense of the rotational energy. As a result the star slows down. By measuring this rate of slow down ($\dot{P}_s$) we can estimate the surface magnetic

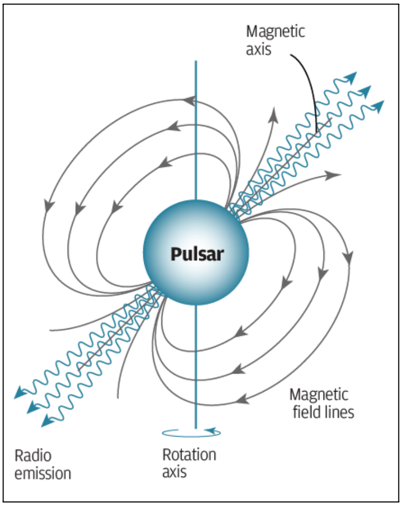

Figure 7. A neutron star in its radio pulsar phase. The misalignment between its rotation and magnetic axis makes the beam of radiation sweep through the space, giving it a lighthouse effect. Picture courtesy: *http://www.scmp.com/*

field to be :

$$B = (\frac{3Ic^3 P_s \dot{P}_s}{8\pi^2 R_s^6})^{1/2} \simeq 3.2 \times 10^{19} (P_s \dot{P}_s)^{1/2}\,\mathrm{G}\,,$$

where, I ($\sim 10^{45}$ g cm^{-2}) is the moment of inertia and R_s is the radius of the star. Of course, this simple estimate provides a measure of the long-range dipole field only, measures of higher multipoles (known to exist near the surface), etc., cannot be obtained from this.

From the measurement of $\dot{P}_s$ we can also obtain an approximate age of a radio pulsar. This is known as the 'characteristic age' ($\tau_{\rm ch}$) and is given by,

$$\tau_{\rm ch} = \frac{P_s}{2\dot{P}_s} \tag{1}$$

This estimate closely approximates the true age of a radio pulsar if:

a) The pulsar's initial spin-period has been much smaller compared to the current observed period.

b) There has been no decay of the magnetic field.

c) The energy loss is due entirely to magnetic dipolar radiation.

The class of radio pulsars can again be divided into two distinct groups:

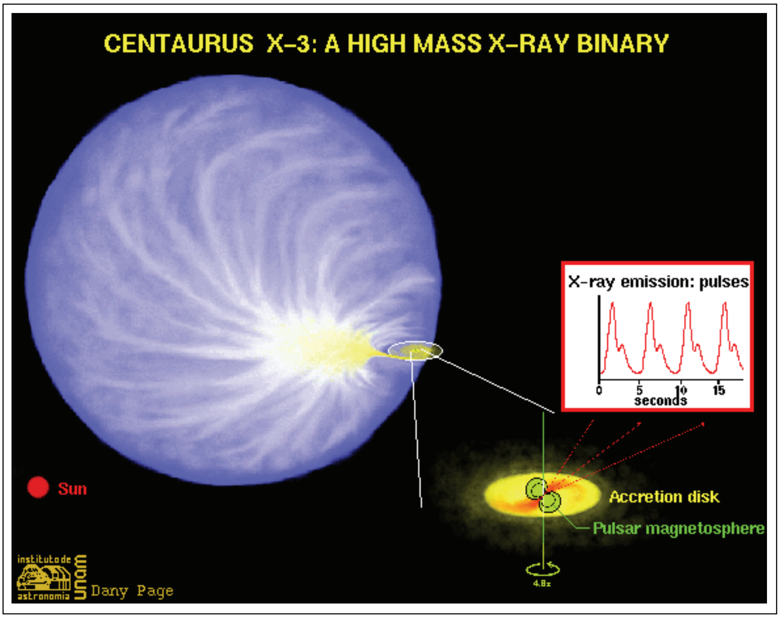

Figure 8. A neutron star in an HMXB. The accreting material is being channeled by the strong magnetic field and the resulting emission is in the form of X-ray pulses. Picture courtesy: Dany Page, UNAM, Mexico City.

(a) The classical radio pulsars (PSR) with $P \sim 1$ s, $B \sim 10^{11} - 10^{13.5}$ G

(b) The millisecond radio pulsars (MSRP) with $P \lesssim 20$ ms, $B \lesssim 10^{10}$ G, and having very different evolutionary histories[7].

[7] These are or have been members of binary systems and are believed to have been 'recycled' through accretion episodes reducing the value of both P_s and B_s in them.

Accretion Powered Pulsars (*APP*)

This class mainly refers to neutron stars in binaries undergoing active material accretion from the companion. Accretion onto the neutron star give rise to energetic radiations in APPs. Depending on the mass of the donor star, these are classified as high-mass X-ray binaries (HMXB) or low-mass X-ray binaries (LMXB).

Neutron stars in HMXBs typically have $B_p \sim 10^{12}$ G, and O or B type stars (*Figures* 8 and 9) as companions. The strong magnetic field channels the flow of material from the companion to the surface of the star, and the emission comes out in the form of periodic pulses. Consequently, neutron stars in HMXBs mostly show up as X-ray pulsars. Photons inside the hot accretion column on the surface of the neutron stars undergo resonant scattering from the electrons. This generates an absorption like feature at the resonance energies of the final emergent spectra, known as 'cyclotron

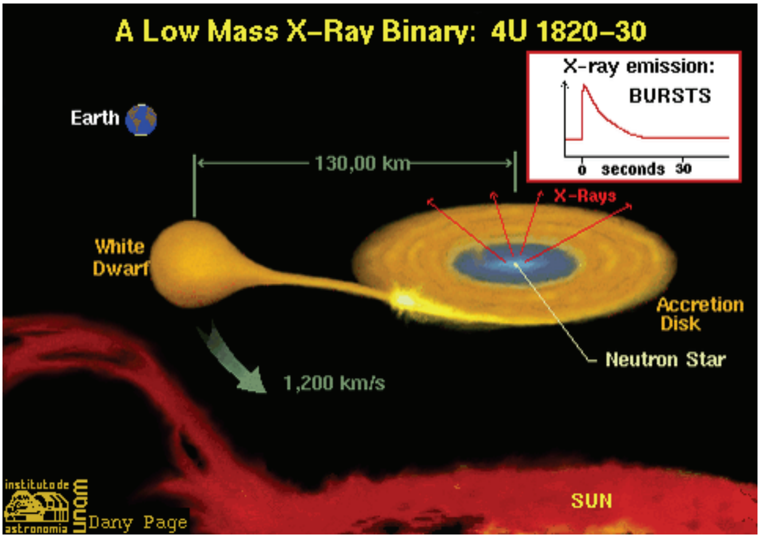

Figure 9. A neutron star in a low-mass X-ray binary. The accreted material accumulates on the stellar surface and ignites upon reaching appropriate temperatures. These sudden episodes of nuclear reaction is observed as X-ray bursts. Picture courtesy: Dany Page, UNAM, Mexico City.

resonance scattering features' (CRSF). These features are used to estimate the magnetic field strength of the neutron star. However, this estimate only gives the local field strength, and uncertainties about local geometry prevent us from obtaining the corresponding surface dipole field strength. So, field estimates obtained from radio pulsars and X-ray binaries, in reality, correspond to two altogether different physical quantities.

LMXBs, on the other hand, harbour neutron stars with magnetic fields significantly weakened ($B \lesssim 10^{11}$ G) through extended phases of accretion. Physical processes taking place in such accreting systems manifest as thermonuclear X-ray bursts, accretion-powered millisecond-period pulsations, kilohertz quasi-periodic oscillations, broad relativistic iron lines, and quiescent emissions. It is even more difficult to measure the magnetic field of neutron stars residing in LMXBs. However, recent work on accreting millisecond X-ray pulsars (where the accretion rate could be low enough for the neutron star's weak magnetic field to be able to channel such flow) have opened up a new direction in understanding the physics of LMXBs containing neutron stars.

Internal Energy Powered Neutron Stars (IENS)

A mixed bag of objects, these IENS are so called because the emission has its origin in some internal mechanism or energy source of the neutron stars. The following are some of the prime examples of such objects.

1. **Magnetars** are young, isolated neutron stars, and their emission is thought to be due to the decay of their super-strong magnetic fields.

2. The seven isolated neutron stars (**INS**), popularly known as the 'Magnificent Seven', have black body-like X-ray spectra ($T \sim 10^6$ K), relatively nearby and have long spin-periods ($P \sim -10$ s). They are probably like ordinary pulsars but a combination of strong magnetic field and spatial proximity make them visible in the X-rays.

4. Towards Unification

One of the important questions in neutron star research is to understand the connections (or absence thereof) between different observational classes. There are indications that there exist evolutionary links between these classes, and the magnetic field ranging from 10^8 G in MSRPs to 10^{15} G in magnetars, is instrumental in providing this link. It is expected that the evolution of magnetic field and associated phenomena chart those evolutionary pathways.

However, one of the major roadblocks encountered while developing a theory of field evolution is the uncertainty about the process of field generation, which determines the location of the currents that support the field. Currently, three plausible theories are in vogue, though each of them suffer from certain inherent inconsistencies.

The simplest theory considers the conservation of magnetic flux existing inside the core of the progenitor star through supernova explosion and core collapse. Flux conservation would ensure an increase in the field strength by a factor $(R_{\mathrm{progenitor}}/R_{\mathrm{NS}})^2$ ($\sim 10^{10}$) and magnetic fields of strength 10^{12}–10^{14} G can easily be formed. Evidently, magnetic field formed in this fashion would be located

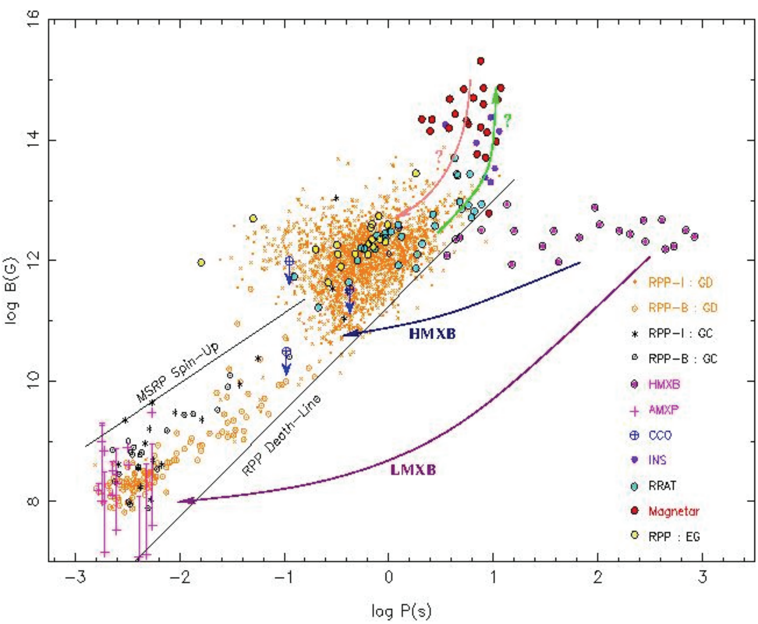

Figure 10. Different observational classes of neutron stars shown in the spin-period *vs.* surface magnetic field ($P_s - B_s$) plane. The blue and magenta arrows indicate 'recycling' of neutron stars in X-ray binaries. The orange and green arrows hint at possible evolutionary pathways between high magnetic radio pulsars and the magnetars. The 'RPP death-line' corresponds to a limiting combination of P_s and B_s, on the left hand side of which the radio pulsar mechanism stops working. The 'MSRP spin-up' line indicates the maximum value of P_s that can be achieved through binary processing for a given final value of B_s. ***Legends :*** I/B – isolated/binary, GC – globular cluster, GD – galactic disc, EG – extra-galactic objects. See Konar *et al.*, *JOAA*, 37, 36, 2016 to know about the data used here.

in the core of the neutron star. Another theory hold turbulent dynamo processes, active in the core of a neutron star in the early phases of its life, responsible for the generation of the magnetic field.

The magnetic field can also be generated as a consequence of thermal effects occurring in the outer crust during the early phases of the star's thermal evolution. The field can grow either in the liquid phase and then be convected into the solid regions, or it could grow in the solid crust itself. The coexistence of a heat flux and a seed magnetic field, in excess of 10^8 Gauss, in the liquid will cause the fluid to circulate which may lead to effective dynamo action which in turn would help the field grow stronger. The currents supporting this field would be located in the outer crust of a neutron star.

Whatever the origin, the evolution of magnetic field is either spontaneous or happens as a consequence of material accretion. Recent investigations, focusing on the magneto-rotational evolution of a neutron in the early phases, consider spontaneous evolution. This is important to understand the evolution of isolated neutron stars with strong magnetic fields and may answer the question whether there exists any evolutionary link between the magnetars and the strong magnetic field radio pulsars. On the other hand, accretion induced field evolution explains the processing of ordinary PSRs (with strong magnetic fields and long periods) that produce MSRPs with ultra-fast rotation and much weaker magnetic fields. All these pathways (established or conjectured) have been summarised in *Figure* 10.

It goes without saying that a lot of questions remain unanswered regarding these disparate observational classes of neutron stars. With the advent of finer technology and bigger and better telescopes, it is expected that we would be in a better position to answer many of them. However, detection of many more neutron stars would likely bring many new classes to the fore, giving rise to a new set of questions in the future.

Acknowledgment

Dipankar Bhattacharya, my formal 'guru' (thesis advisor), and G Srinivasan have not only taught me all that I have ever learned about neutron stars; but have also managed to transmit their passions for these exotic objects that challenge the very boundaries of known physics.

Suggested Reading

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