Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

The Inveterate Tinkerer
2. Instability of Kolmogorov Flow

Aditi Kambli and
Chirag Kalelkar*
Department of Ocean
Engineering and
Naval Architecture
IIT Kharagpur
Email:aditi1997kambli@gmail.com
* Department of Mechanical
Engineering,
IIT Kharagpur, Kharagpur
West Bengal 721 302
Email:kalelkar@mech.iitkgp.ernet.in

In this series of articles, the authors discuss various phenomena in fluid dynamics, which may be investigated via table-top experiments using low-cost or home-made instruments. The second article in this series is about a simple set-up for demonstrating the instability of Kolmogorov Flow.

Background

Few exact solutions are known for the equations of motion of an incompressible Newtonian fluid *viz.* the Navier–Stokes equations [1]. A notable exception is the so-called Kolmogorov flow [see 2–4], named after the mathematician A N Kolmogorov. A special case of Kolmogorov flow is a steady, incompressible, two-dimensional, unidirectional shear flow with a sinusoidal velocity profile having velocity components (in the xy-plane):

$$(u = u_0 \sin(y), v = 0),$$

with constant pressure and periodic boundary conditions, which in viscous fluids is maintained by an external sinusoidal force per unit mass:

$$(F_x = F_{x,0} \sin(y), F_y = 0).$$

Keywords
Kolmogorov flow, instability, turbulence, shear flow, vortices.

For the instability of Kolmogorov flow, destabilising effects of shear due to sinusoidal electromagnetic forcing overcome the stabilising effects of viscosity and thermal diffusivity.

Kolmogorov suggested that the simplicity of the flow profile may serve as a theoretical test bed for studies of fluid instabilities and transition to turbulence. The reader may wish to read [1], for an elementary discussion of fluid instabilities. For the instability of Kolmogorov flow, destabilising effects of shear due to sinusoidal electromagnetic forcing overcome the stabilising effects of viscosity and thermal diffusivity.

Materials Required

Black plexiglas tray, perforated plexiglas sheet, neodymium-iron-boron disk magnets (NdFeB, grade N45 or higher), spherical polyamide beads (mean diameter = 55 um, density = 1.016 g/ml, Model: 10090, TSI Instruments), deionized water, glycerol, hydrated copper sulphate ($CuSO_4.5H_2O$), silicone spray, soap solution, DC power supply, copper wire, copper strips, falcon centrifuge tubes, vortex shaker, crocodile clips.

Method

Prepare a 10% by weight solution of hydrated copper sulphate in deionized water. Take 5 g of hydrated copper sulphate and add to 50 ml deionized water. Add 10 ml glycerol to 40 ml of the above solution to increase the viscosity. Use a vortex shaker to mix the contents in a 50 ml falcon centrifuge tube. Add ~3 drops of soap solution to the falcon, and gently shake to avoid bubble formation.

The black plexiglas tray is a square with sides 16 cm in length, and a wall height of 7 mm. The perforated plastic sheet is a square with sides 16 cm in length, containing 121 equally spaced holes for placing an 11×11 array of magnets within the holes (see *Figure* 1). Place the magnets in columns with opposing polarity, i.e., a column with '+' (North pole facing upwards) followed by a column with '-' (South pole facing upwards) and stick them with carton sealing tape. Using a smaller-sized tray is not advisable, as the results are affected by motion of the liquid meniscus near

Figure 1. Black plexiglas tray (left) and plexiglas perforated sheet (right) with an 11×11 array of neodymium alloy disk magnets.

the walls.

Use double-sided tape to stick the black plastic tray atop the lattice of magnets placed within the perforated sheet. It is important to ensure that the tray (and the perforated sheet) is levelled with respect to the support on which it is placed. Apply a thin film of oil on the tray using silicone spray, and wipe it with a microfibre cloth (avoid using tissue paper, as it tends to leave fragments of paper). The spray lubricates the bottom of the tray, which reduces viscous drag during fluid motion. Take two thin copper strips of length ~16 cm each, and use crocodile clips to connect to a DC power supply.

Pour the copper sulphate solution onto the tray ensuring that all parts of the tray bottom is equally wetted (the height of the liquid is ~5 mm). Sprinkle a small quantity of polyamide beads onto the surface of the liquid. Note that the soap solution was added during the preparation of liquid to reduce surface tension, and prevent clumping of the beads on the surface of the liquid. It is recommended that an LED light source (covered with parchment paper, which acts as a diffuser) be used to illuminate the surface of the liquid (see *Figure* 2). Switch off other external sources of light.

Soap solution added during the preparation of liquid reduces surface tension, and prevents clumping of the beads on the surface of the liquid.

Start the experiment by applying a DC current of 0.05 A using the power supply. You will observe that the surface of the liquid shows a striped pattern with opposing direction of flow in neighbouring stripes (see *Figure* 3). The particles are convected

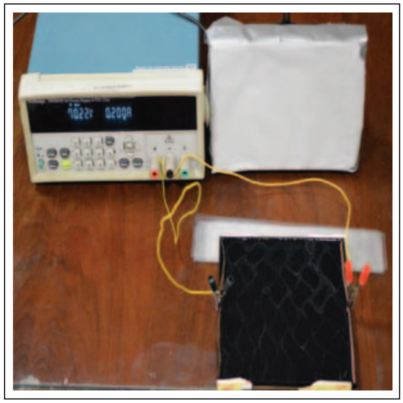

Figure 2. Experimental set-up showing DC power supply, LED light source, and the tray containing 10% hydrated copper sulphate solution.

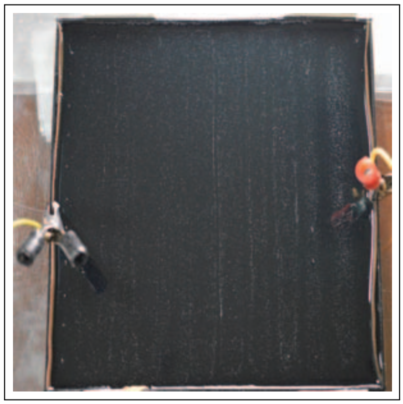

Figure 3. Base state of Kolmogorov flow at a DC current = 0.05 A.

along a stripe, and upon reaching the walls, move into a neighbouring stripe or travel back along the same stripe. This is the base state of the flow, under a magnetic force which is stationary in time and is spatially periodic. If the DC current is increased to ~0.2 A, the stripe-like patterns are transformed into a pattern of counter-rotating vortices (see *Figure* 4). Particles which are located within a vortex usually remain trapped, but occasionally will convect into a neighbouring vortex. Note that the span of each vortex encompasses several magnets, and is not necessarily centred about a specific magnet. This is the primary instability of Kolmogorov flow.

We carried out several runs and recorded movies of the particle motion in a rectangular section at the centre of the tray, using a

Figure 4. Primary instability of Kolmogorov flow at a DC current = 0.2 A.

DSLR camera with a macro lens. The movies were converted into a sequence of images and analysed using an open-source particle-tracking code available at:
`web.stanford.edu/~nto/software_tracking.shtml`

In *Figure* 5 and *Figure* 6 we show a scatter plot of the magnitude of the vorticity (which is equal to the curl of the velocity) for both the base state and the primary instability. The flow is assumed to be two-dimensional, therefore the vorticity vector is perpendicular to the flow-field. The magnitude and sign of the vorticity component is indicated by the color-bar. In *Figure* 5, we note the presence of vortices located between two neighbouring stripes, which may be attributed to the classical Kelvin–Helmholtz instability of the interface between two fluids sheared past one another [1]. In *Figure* 6, we note the reversed sign of vorticity (which is relatively larger in magnitude compared to the base state) for neighbouring vortices.

A close-up view of the Kolmogorov flow instability may be seen at: `youtube.com/watch?v=8ulmm8HOYfc`

Scan the QR code to view the video.

Suggestions for Further Work

1. Change the concentration of the hydrated copper sulphate solution in discrete steps, and find changes in DC current at which the primary instability is observed.

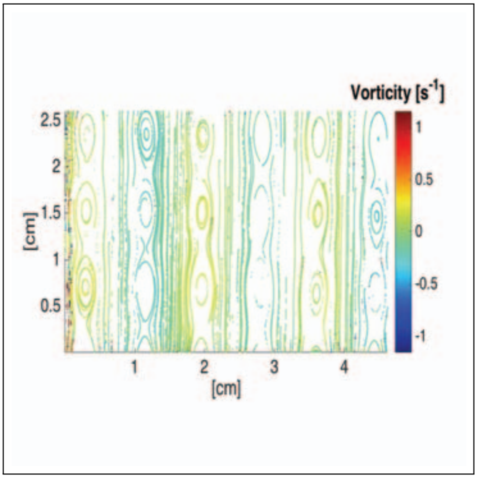

Figure 5. Scatter plot of the magnitude of vorticity at a DC current = 0.003 A.

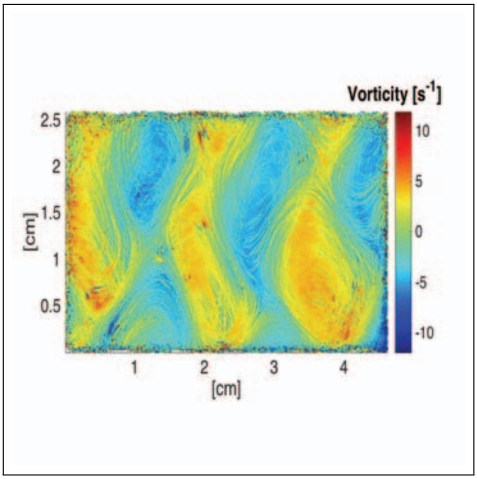

Figure 6. Scatter plot of the magnitude of vorticity at a DC current = 0.1 A.

2. Add a pinch of polymer such as polyethylene oxide/polyacrylamide to the solution, and stir thoroughly using a magnetic stirrer. At what value of the DC current do you observe the primary instability? Do you see any change in the shape of the vortices?

3. The magnets were arranged in a checkerboard configuration, i.e., each column having the arrangement '+ - + - + -', and so on. On repeating the experiment, no instabilities were observed (upto the highest DC current values we explored). Instead, we found a

state with counter-rotating vortices located above individual magnet positions. Try and account for this different result.

4. Increase the DC current until the primary instability pattern breaks up and the fluid becomes turbulent. Particle trajectories are observed to vary both spatially as well as temporally, without any distinct vortex structures visible.

5. Using dimensional analysis, define a suitable Reynolds number for this system, which incorporates the magnetic field, the DC current, the dynamic viscosity, the mass density of the fluid, and the dimensions of the tray.

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Suggested Reading

[1] E Guyon, J Hulin, L Petit, and C Mitescu, *Physical Hydrodynamics*, Oxford University Press, 2nd ed., 2015.

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