How Do Wings Generate Lift?

2. Myths, Approximate Theories and Why They All Work

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A cambered surface that is moving forward in a fluid generates lift. To explain this interesting fact in terms of simpler models, some preparatory concepts were discussed in the first part of this article. We also agreed on what is an acceptable explanation. Then some popular models were discussed. Some quantitative theories will be discussed in this concluding part. Finally we will tie up all these ideas together and connect them to the rigorous momentum theorem.

The triumphant vindication of bold theories – are these not the pride and justification of our life's work?

Sherlock Holmes, *The Valley of Fear*

Sir Arthur Conan Doyle

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1. Introduction

We start here with two quantitative theories before going to the momentum conservation principle in the next section.

1.1 *The Thin Airfoil Theory*

This elegant approximate theory takes us further than what was discussed in the last two sections of Part 1[1], and quantifies the ideas to get expressions for lift and moment that are remarkably accurate. The pressure distribution on the airfoil, apart from creating a lift force, leads to a nose-up or nose-down moment also. The moment plays an important role in the stability of the aircraft. The flow around flat and cambered thin plates shown in *Figure* 1 is analysed in the thin airfoil theory framework assuming flow to

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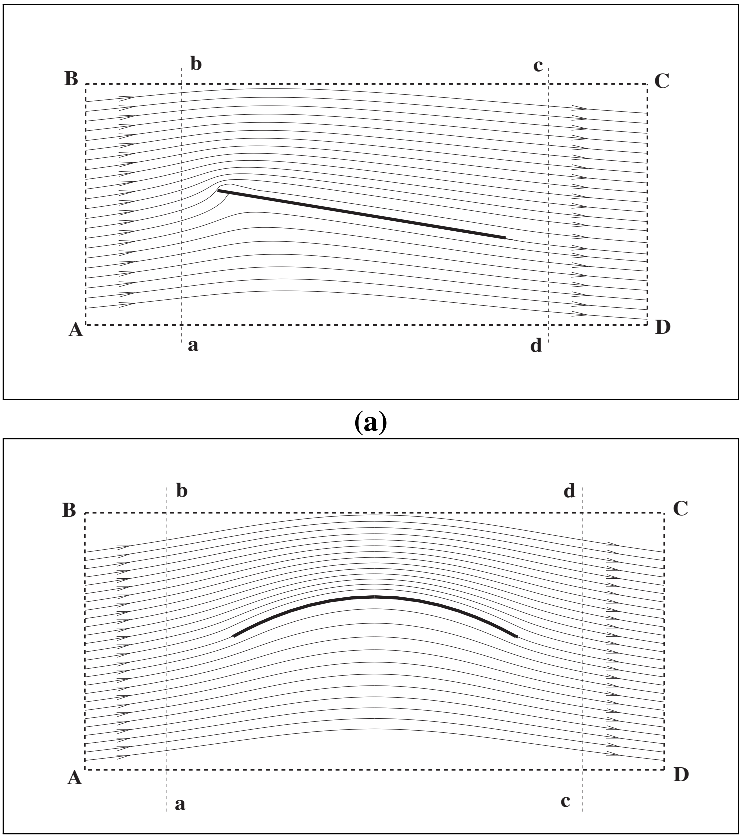
Keywords

Thin airfoil, camber, Kutta condition, momentum conservation, circulation.

Figure 1. Flow around **(a)** A flat plate at incidence, and **(b)** A cambered plate at zero incidence.



(b)

be inviscid. The two simple models discussed in the last two sections of Part 1 have indicated higher velocity and lower pressure on the upper side for the assumed streamline pattern. It is important to note that this lower pressure on the upper surface is only on an average. But in the present theory, it is quantified and its distribution on the airfoil surface is obtained.

Without going through the mathematical details (these can be found in [1, 2]) the key results may be summarised that the lift coefficient C_l for a flat plate is,

$$C_l = 2\pi\alpha, \tag{1}$$

and for a cambered plate it is,

$$C_l = 2\pi(\alpha + \alpha_0), \tag{2}$$

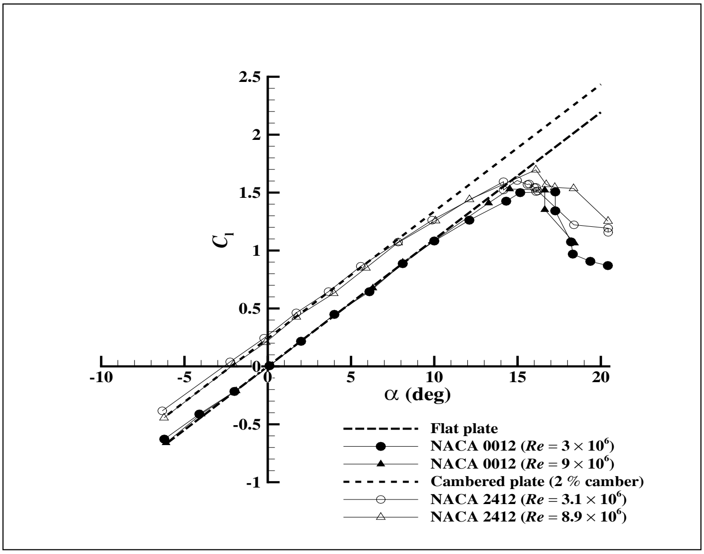


Figure 2. Lift coefficient C_l as a function of α for a flat plate and cambered plate compared with experimental data in [3]. Inviscid theoretical results are in good agreement with the experimental results for small α.

where α is the angle of attack in radians and α_0 is the angle such that at (- α_0) lift is zero (*Figure* 2). The experimental plots for two airfoils – NACA 0012 (12 % thick symmetric airfoil) and NACA 2412 (12 % thick airfoil, with 2 % camber at 40 % chord location) show remarkable agreement with these theoretical results if α is small (see *Box* 1). Further, this simple theory predicts that the centre of pressure for a flat plate is at a distance of quarter chord from the leading edge. The existence of aerodynamic centre is also predicted by this theory (see *Box* 2).

Brief comments on some details are given now to get an insight. In the thin airfoil theory, a more drastic assumption is made in the form of Kutta condition, as compared to the qualitative assumption of streamline patterns in the last two sections of Part 1. In the present theory, the inviscid equation of motion, represented by the Laplace equation, is solved for the flow around an airfoil kept in infinite medium. This requires the boundary conditions at infinity as well and we apply that there is no disturbance at infinity due to the airfoil, thus making the flow uniform there. The presence of airfoil forces normal component of velocity on its surface to be zero. But this prescription of boundary conditions

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Box 1. Streamline Pattern Around an Airfoil

The streamline patterns shown in the figure, if α is small (left frame), resemble what is assumed in the thin airfoil theory. Hence there is a good agreement between the results from approximate model and the experimental data in *Figure* 2. On the other hand, for sufficiently high α (right frame), we see a flow separation and reduction in lift as seen in *Figure* 2. The thin airfoil theory fails here quantitatively.

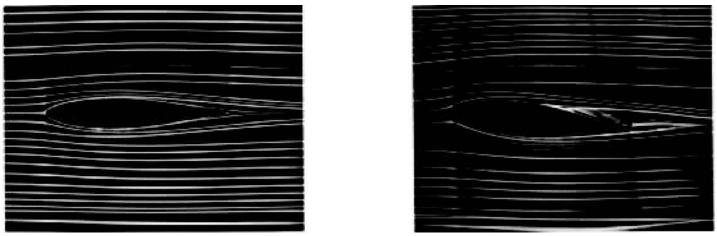


Figure A. Streamline patterns over an airfoil without separation and with separation observed experimentally [4].

Box 2. Aerodynamic Centre

For both the flat and cambered plates, the thin airfoil theory predicts that there exists a point such that moment about that point is constant – being independent of α, and the quarter chord point happens to be that point. This point is known as the 'aerodynamic centre'. Existence of such a point and correct prediction of its location have been confirmed by more elaborate experiments and viscous flow computations.

Because of the effect of viscosity, the flow cannot take a sharp turn at the trailing edge. The Kutta condition is applied by forcing the stagnation point at the trailing edge as observed experimentally.

at infinity and the airfoil surface is not sufficient to give a unique solution in the inviscid flow framework. Hence, two flows shown in *Figure* 3 are equally valid solutions of our model. The flow in frame (a) has zero circulation Γ and zero lift L. In frame (b) the Kutta condition is applied by forcing the stagnation point at the trailing edge as observed experimentally. Because of the effect of viscosity, the flow cannot take a sharp turn at the trailing edge as shown in frame (a). Thus solving the simpler inviscid equation and making the model definitive by borrowing additional information from viscous flow has led to wonderful results as shown

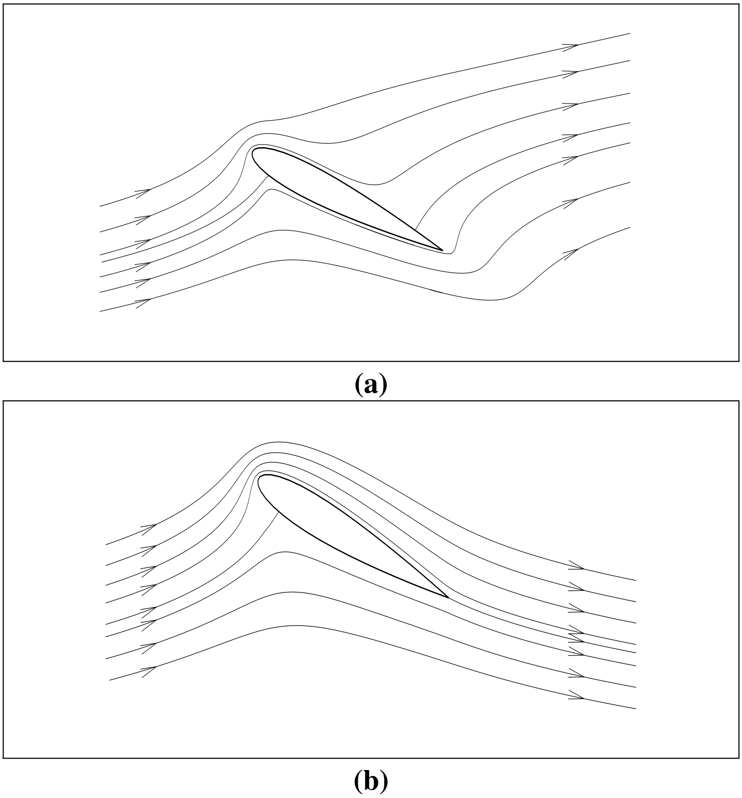


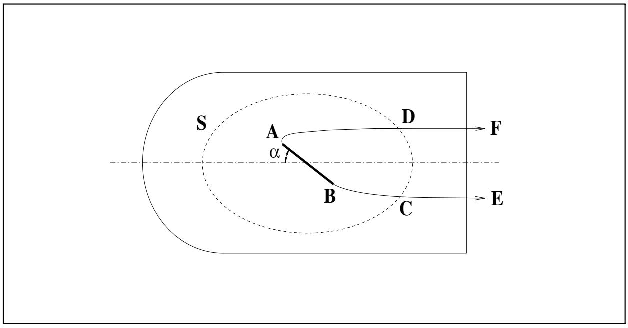
Figure 3. Two inviscid flow solutions around an airfoil; **(a)** Not satisfying the Kutta condition and with circulation $\Gamma = 0$, $L = 0$, and, **(b)** Satisfying the Kutta condition and having clockwise circulation Γ and upward lift L.

in (1) and (2) and *Figure* 2. The streamline pattern we get is consistent with the assumptions made in the last section of Part 1, and now the results are quantitative. This is true, despite the fact that the present inviscid model gives such an unacceptable result as zero drag. It may be mentioned that the range of *quantitative* applicability of the results is limited to small angles of attack as seen in *Figure* 2.

1.2 *Model with Free Streamlines and Dead-Water Region*

This model is something which one may imagine intuitively at first thought. But it appears so crude compared to the other approximate models we have considered. Still it is included here for historical reasons, and also to show that it has some reasons for its success to give the correct qualitative results. The flat plate AB (*Figure* 4) is at an angle of attack α and the two free streamlines

Figure 4. Flow around a flat plate with assumed free streamlines and dead-water region between them.



ADF and BCE divide the flow into two regions.

[2]The dead-water region extends to infinity and it is assumed to contain fluid at rest.

The dead-water region[2] FDABCE extends to infinity and it is assumed to contain fluid at rest. The flow outside this region has uniform velocity equal to the free stream velocity. Thus the two free streamlines are velocity discontinuities with positive and negative vorticity. We cannot apply Bernoulli's principle and conclude that pressure in the dead-water region is the stagnation pressure. In fact, no streamline enters from the uniform flow region into the dead-water region crossing the vortex sheet with velocity discontinuity. Quite interestingly even this simplistic model has been shown to give the correct relation between circulation and lift given by equation (3) below which we have considered earlier. To achieve this, however, the integration circuit around the plate to calculate circulation is chosen in a particular way.

$$\mathbf{L} = \rho \mathbf{u}_\infty \times \mathbf{\Gamma} \,. \tag{3}$$

This is done with some mathematical dexterity. See that the velocity discontinuity is not possible because of viscosity, and a uniform flow around the dead-water region is not possible due to kinematic reasons. But still it is possible to employ such a model usefully. It should be appreciated that if pressure is lower on the upper surface to generate lift, it leads to drag also.

Another model that attempted to explain the force on a body due to fluid motion goes back to Newton and is called Newton's corpuscular theory of fluid motion. In this model, air particles are assumed to impinge on the body and create a force due to momentum change analogous to the air molecules moving randomly, and creating pressure on a surface due to impingement. For a flat plate at an angle of attack α this model gives lift coefficient $C_l \approx 2\sin^2\alpha$, and drag coefficient $C_d \approx 2\sin^3\alpha$. It gives a non-zero value for C_d unlike the thin airfoil theory. But it is not surprising that $C_d = 0$ at zero incidence, since the effect of viscosity is not considered. Even though the model appears naive, it gives some reasonable values at hypersonic speeds [5].

2. Conservation of Momentum Principle and Putting Things Together

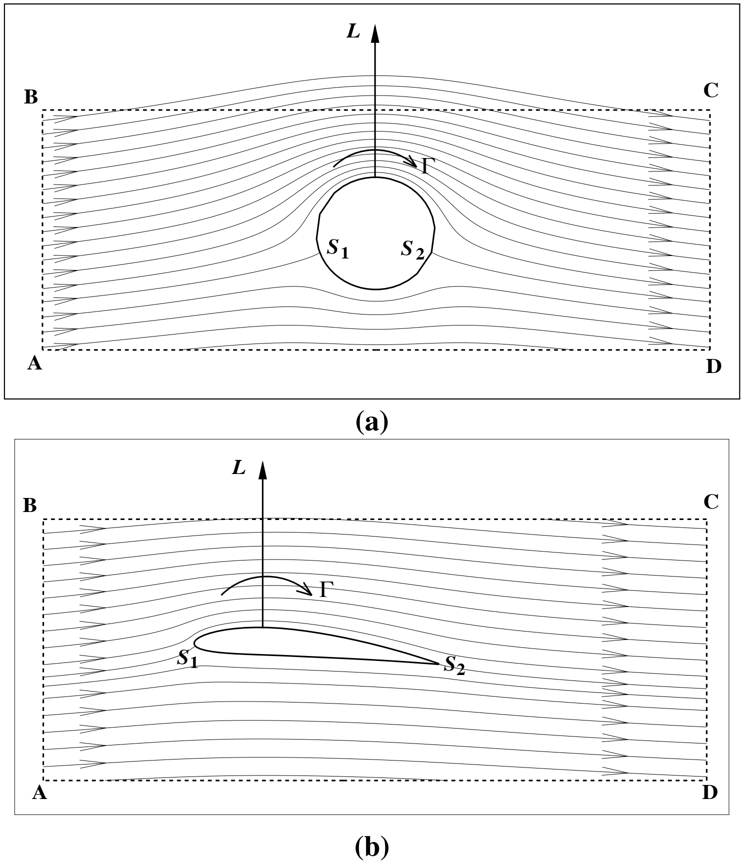
It is instructive to refer to the fundamental principle of conservation of momentum [1, 2] which will help us organise the ideas. First, the conservation of momentum principle – *The time rate of change of momentum of an identified mass of fluid is equal to the net force acting on this mass*. This is basically a restatement of Newton's second law of motion for a fluid. Often it is not convenient to use for a fluid since we would like to deal with a flow field or control volume *CV* that is fixed in space rather than the moving fluid mass that needs to be identified. For this situation the momentum theorem provides the required relation.

The momentum theorem as applied to the control volume *CV* fixed in space may be stated as – *The time rate of increase of momentum within a fixed control volume CV is equal to the rate at which momentum is flowing into CV plus the net force acting on the fluid within the CV.*

This appears to be a deceptively simplistic or obvious extension of the momentum principle and turns out to be useful in the application of the principle to practical problems.

Now take another look at the flow around a cylinder with circulation in *Figure* 5(a). Imagine a very large *CV* ABCDA surrounding the cylinder. The streamlines at far upstream and far downstream are horizontal and have uniform velocity u_∞. Hence, the flow passing over the cylinder appears to be unaffected far away in all directions. Then how can we account for the upward lift L?

Figure 5. **(a)** Uniform inviscid flow around a cylinder with circulation. **(b)** Same as in (a), but around a cambered airfoil to show non-zero Γ and lift.



This is a natural and legitimate question because if we apply the momentum theorem naively (and hence erroneously) it leads to a wrong answer; flow being steady, there is no change in the momentum flux in the *CV*. There is no net *y*-momentum flux through the boundaries of the *CV* since the velocity is uniform being u_∞ everywhere on the outer boundary, and on the inner surface (cylinder) the normal velocity is zero. Now by the momentum theorem, the net force has to be zero on the *CV*. Uniform pressure on the boundary ABCD gives zero force and hence on the inner surface formed by the cylinder, the net force has to be zero, leading to zero lift!

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Now we have to identify where is the problem in this argument leading to a wrong result. It is convenient to start from the concept of circulation Γ which is related to **L** as given by (3). And we saw that a way to generate Γ is to spin the cylinder. Because of viscosity, the no-slip boundary condition comes into effect, but the flow outside the boundary layer remains almost irrotational. In case of the cambered airfoil shown in *Figures* 1(b) and 5(b) or the flat plate at incidence in *Figure* 1(a) the streamlines are affected by the plate and develop Γ once the Kutta condition is invoked.

Is the flow far away really unaffected? No; first on the downstream side, the wake contains the effect of viscosity but we do not have to account for it in the present inviscid model. Once the presence of circulation is accepted, we completely ignore the wake and the boundary layer since it is known that the quantitative relation between circulation and lift is still valid at least approximately.

A careful look at the momentum theorem is instructive now. The lift force L that is acting on the plates in *Figure* 1 (or cylinder in *Figure* 5(a)) has an equal downward reaction on the air inside the *CV*. The vertical momentum flux entering the *CV* far upstream of the airfoil through face AB and that leaving it through far downstream face CD are zero. Notice the inclined flow at the planes ab and dc near the leading and trailing edges in *Figure* 1 which leads to a net vertical momentum flux. But if we push the boundaries AB and CD far away from the airfoil, the streamlines get horizontal and there is no net transport of vertical momentum flux across the *CV* boundary.

But something interesting happens with the pressure on the horizontal boundaries AD and BC of the *CV* in *Figure* 1. On the edge BC, pressure is slightly lower than free stream pressure p_∞ and higher on the edge AD. But if we take these edges far away from the airfoil, the pressure should tend to p_∞. In fact, this argument has been used in this paper all along. (See *Figure* 7 of the first part of this article.) But it so happens (it can be proved) that as the control volume *CV* is made larger, even though the pressure on the boundary tends to p_∞, the product of incremental pressure and the area (which tends to infinity) remains finite. Pressure dis-

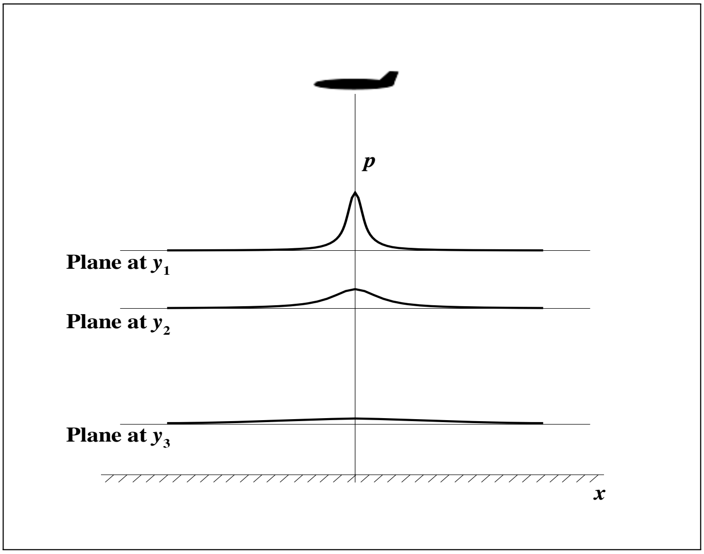

Figure 6. Schematic representation of pressure distribution on three horizontal planes located at three depths y_1, y_2, y_3, all far away from the aircraft. The pressure peak decreases as the plane goes down away from the aircraft and finally becomes zero for a large distance. However, the area under the curve remains finite and equal to the lift L. Far away from the aircraft, the curves becomes symmetrical as shown. The peaks in pressure occur below the aircraft as a consequence of incompressibility assumption.

tributions shown in *Figure* 6 at three horizontal planes, below the airfoil make this point clear. Also, there is no vertical force on the vertical faces AB and CD (*Figure* 1). Further, the vertical forces on AD and BC are upwards and add up to lift L. In our earlier naive application of the momentum theorem, these pressure forces were taken to be zero since pressure tended to p_∞. But now we realise that even though the incremental pressure on these two faces AD and BC tends to zero, its product with the area tends to a finite limit since area tends to infinity when these faces are taken far away. Further, the vertical force on the upper face BC is also transferred to the lower face AD by effectively decreasing the hydrostatic pressure. Thus even though air lifts an aircraft, the weight is transferred to the lowest face AD, and hence the ground finally supports the aircraft even when it is flying. This answer should comfort us. The readers are referred to Prandtl and Tietjens [6] for an elegant and elaborate discussion.

Thus, even though air lifts an aircraft, the weight is transferred to the lowest face AD and hence the ground finally supports the aircraft even when it is flying. This answer should comfort us.

A doubt that may arise here is on the role of viscosity. Is it required at all, like in the case of a cylinder with circulation? Yes, it is required as mentioned in Section 1.1 on thin airfoil theory. The inviscid flow solution is not unique, and to make it unique the Kutta condition is used. This is the consequence of fluid being viscous. Thus the streamline pattern is assumed in a qualitative way. That gives the correct results for lift. In case of a rotating cylinder or a cambered plate as shown in *Figure* 6, there will not be circulation without viscosity, and hence there will not be any lift.

A question may be posed now ! Referring to *Figure* 1, there is upwash in front of the airfoil (plane ab) and also downwash behind (plane dc). The airfoil is only locally modifying the streamlines. It is not diverting the air down but is only pushing it down to get the lift. Hence no work is done. Then why do we need to spend energy to get lift? The answer for this question is simple now from what has been discussed. Yes, no energy is needed in inviscid flow, except that to enforce the Kutta condition, viscosity is needed which is associated with drag. Hence to maintain forward velocity a thrust is needed to overcome the drag. This involves work. In case of the rotating cylinder in *Figure* 5(a), to set the fluid in motion and hold, again work has to be done to overcome the viscous drag. Yes, we cannot get a free ride.

Thus we have seen how simpler models that are popular, have strong theoretical basis and are able to satisfy the curiosity of the general public. Often, they are criticised for not being accurate and are unable to explain many related phenomena. The final correct result is alleged to be fortuitous like one concluding 16 / 64 = 1 / 4, obtained by cancelling 6 from both the numerator and denominator. The arguments given in this article justify that it is a wrong example. On the other hand, the correct example may be like putting 31 / 61 $\approx$ 30 / 60 = 3 / 6 = 1 / 2. The digit 1 is not cancelled here from the numerator and denominator but is subtracted knowing very well why it can be done so. As in this example, the agreement between the results from approximate theories with more exact results is not accidentally correct but has a strong theoretical basis.

3. Summary

There are simple explanations in the popular literature that describe how lift is generated by a wing. A flat plate at incidence or a cambered plate, or a rotating cylinder if moving forward generate lift. The key idea in all explanations or simple models is to conclude how a lower pressure is generated on the top surface as compared to the lower surface. A longer particle path on the top surface and hence higher velocity, and consequently lower pressure due to Bernoulli's equation is a popular model. Curved streamlines leading to radial pressure gradient results in lower pressure on the top surface. Both the models assume the shape of the streamlines and even though they are inviscid models the role of viscosity is assumed in a subtle manner. These models are only qualitative and conclude that the pressure on the upper surface is lower than that at the lower surface only on an average.

The thin airfoil theory, on the other hand, quantifies the lift generated and also predicts the existence of the aerodynamic centre. This is also an inviscid model ignoring the presence of the boundary layer. But the presence of viscosity is utilised in the form of Kutta condition. Quantification fails if the boundary layer separates or at very low Reynolds numbers like $Re = 1{,}000$. It is interesting to see the contrast. The presence of viscosity which is a requirement to generate lift itself has led to the breakdown of the model at high incidence. But the model is still correct qualitatively since lift remains positive.

The central idea in the models is to explain how circulation is generated which is directly connected to lift. To generate circulation viscosity is required. A wing is seen as a body that locally disturbs the otherwise uniform flow. The disturbances in velocity and pressure caused are such that they aid to generate lift but damp down to zero far away from the wing. The momentum theorem connects these ideas and explains how the reaction force to the lift on the wing is finally transferred to the ground.

Vether it's worth goin' through so much, to learn so little, as the charity-boy said ven he got to the end of the alphabet, is a matter o' taste.

Charles Dickens, *Pickwick Papers*

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Suggested Reading

[1] A M Kuethe and C -Y Chow, *Foundations of Aerodynamics: Bases of Aerodynamic Design*, Fifth Edition, Wiley, New Delhi, 2010.

[2] J D Anderson, *Fundamentals of Aerodynamics*, Fifth Edition, McGraw-Hill, New Delhi, 2010.

[3] I H Abbott and A E Von Doenhoff, *Theory of Wing Sections: Including a Summary of Airfoil Data*, Dover, New York, 1959.

[4] M Van Dyke, *An Album of Fluid Motion*, The Parabolic Press, Stanford, 1982.

[5] J D Anderson, *Modern Compressible Flow: With Historical Perspective*, Third Edition, McGraw-Hill, New Delhi, 2012.

[6] L Prandtl and O G Tietjens, *Applied Hydro- and Aeromechanics*, Translated by J P Den Hartog, McGraw-Hill, New York, 1934.