How Do Wings Generate Lift?

1. Popular Myths, What They Mean and Why They Work

M D Deshpande and M Sivapragasam

How lift is generated by a moving wing is understood satisfactorily. But the rigorous explanation requires elaborate theoretical background, and hence there are many simpler explanations in the popular literature. In the first part of this article, some such popular models are discussed after briefly dealing with some required preliminary ideas. The concluding part of the article will deal with some simple quantitative models and then tie up many of these simple ideas and propositions together to the rigorous momentum conservation principle placing these popular models in perspective.

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So how do you go about teaching them something new? By mixing what they know with what they don't know. Then, when they see in their fog something they recognise they think, "Ah I know that!" And then it is just one more step to "Ah, I know the whole thing." And their mind thrusts forward into the unknown and they begin to recognise what they didn't know before and they increase their powers of understanding.

Picasso

1. Introduction

A wing that is moving forward in a fluid generates lift. The wing can be a flat plate at incidence or a cambered plate (see *Box* 1). Even after knowing much about the theory of flight, it is difficult not to be impressed when looking at a heavy transport aircraft and wondering that it really does fly. The theory of flight has caught the imagination of the general public, and hence needs simple explanation. It is inevitable that there are many such explanations leading to discussions and objections. This article is meant to dis-

Keywords
Airfoil, Bernoulli's principle, viscosity, streamline curvature, vorticity, irrotational flow.

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cuss such models which are approximations of a rigorous theory, and to see how they are connected and can be made acceptable.

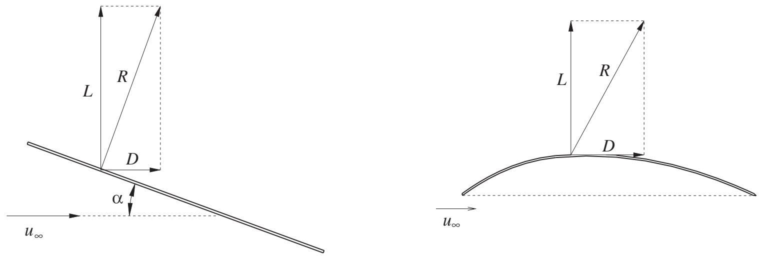
It is a good idea to agree first on what is an acceptable model. Since we are in the realm of classical mechanics we can start with Newton's second law of motion. Hence if a wing can generate lift equal to its weight (total weight of the vehicle) it can balance the gravitational pull and can maintain level flight. The equations for fluid flow that are equivalent to the second law are the well-known Navier–Stokes (N–S) equations [1]. These equations have been known for nearly two centuries. But it is not easy to infer from these equations that a wing develops lift since these are a set of coupled non-linear partial differential equations. They were, in fact, solved only for some simple geometries till the second half of the 20th century.

The advent of digital computers made it possible to compute flow around arbitrarily shaped bodies. But there is a difficulty in such an approach. We can solve the N–S equations using a computer for a given geometry but drawing a general inference like, *a cambered plate in a flow generates lift*, or, *an increase in camber leads to increased lift*, has to be done only through a parametric study (see *Box* 1). This parametric study involves several parameters like wing geometry involving camber and size, angle of attack, flow velocity, etc. This is neither a convenient method nor of interest to a general reader who wants to know how lift is generated, and this is where simple theories giving an explanation have a role to play.

The difficulties mentioned in the previous paragraph may be appreciated further by a quotation from 1895, "Heavier-than-air flying machines are impossible", which is attributed to an accomplished scientist like Lord Kelvin, then the President of the Royal Society. A keen observer like Otto Lilienthal, a contemporary of the Wright brothers, who built gliders, and was trying to build heavier than air flying machines wrote a remarkable book based on his observations on bird flight and experiments [2]. However, his concept of lift (to be interpreted here as upward force) depended solely on the drag of a plate moving in a direction

Box 1. Lift and Drag Forces

A flat plate inclined at an angle α to the flow as shown in the left frame, or a cambered plate as shown in the right frame are subjected to a force $\boldsymbol{R}$. The component of the force perpendicular to free stream velocity $\boldsymbol{u}_\infty$ is defined as lift $\boldsymbol{L}$ and the component of force along $\boldsymbol{u}_\infty$ is defined as drag $\boldsymbol{D}$. Note that a plate stationary in a uniform flow, or a plate moving uniformly in a stationary fluid are equivalent.



(a) **(b)**

Figure A. Schematic illustration of lift, $\boldsymbol{L}$ and drag, $\boldsymbol{D}$ forces on **(a)** An inclined flat plate and **(b)** A cambered plate. $\boldsymbol{R}$ is the resultant force.

perpendicular to its plane (see *Box* 1). Hence a horizontal plate moving down vertically develops lift (this is actually drag but acting upwards and hence helps to lift the body). Obviously, Lilienthal was inspired by the flight of birds. The modern idea of lift, on the other hand, is based on an inclined plate, which may be flat or cambered, moving forward. Note that Lilienthal was working almost a hundred years after Sir George Cayley.[1] He drew in 1799, the lift–drag vector diagram and flat plate wing at an angle of attack to the relative wind (*Figure* 1). In the right frame, a thin plate inclined to the direction of the flow is shown along with the oncoming velocity vector. Apart from this modern representation of forces, hidden here is the concept that flapping is not required to generate lift. Further, the resultant force is shown correctly being perpendicular to the plate, and decomposed into two components – lift and drag. The lift force is perpendicular to the flow and probably given more prominence with an arrowhead. The left frame shows an airship with a boat like fuselage and curved fixed wing. At the rear of this machine we see a long rudder and two

[1] See https:en.wikipedia.org /wiki/George_Cayley for his interesting biography.

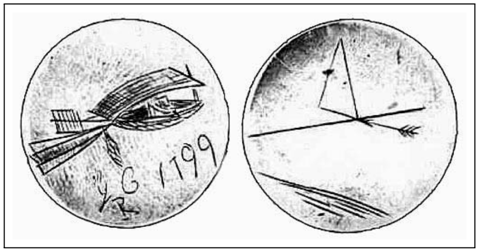


Figure 1. Silver disc showing lift and drag inscribed by Sir George Cayley. (Source: http:/www.ctie.monash.edu.au/hargrave/cayley.html).

pilot-operated propulsive flappers. Thus, the propulsive system is uncoupled from lift generation. There is no sign of flapping of the wing.

2. Fluid Mechanics Preliminaries

2.1 *Bernoulli's Principle*

The explanations in this article involve the use of Bernoulli's principle, and hence it is briefly summarised here. See [3, 4] for details. We have mentioned that a wing can be stationary in a flow and still generate lift. The wing kept in the fluid with uniform flow disturbs the fluid motion leading to streamline distortion. Since Bernoulli's principle relates velocity to pressure at any point on a given streamline (see *Box* 2) it is quite handy. It is sufficient here to restrict to incompressible, steady flow, since it makes analysis much simpler in the present context. The principle is obtained by integrating the momentum equation along a streamline for an inviscid flow. Then we have,

$$p + \frac{1}{2}\rho u^2 + \rho g z = constant. \qquad (1)$$

Here p is the pressure, u is the velocity magnitude, ρ is the fluid density, g is the acceleration due to gravity and z is the height of the point on the streamline above a reference level. The constant of integration on the RHS of (1) is assigned to a streamline and hence can vary from one streamline to the other. However, it can be proved that in an irrotational flow (see *Box* 3) it has the same value in the whole field. This observation will be useful later.

Box 2. Streamlines

A streamline is a curve in the flow such that the tangent to it at any point is the direction of the velocity at that point. If the flow is steady, the streamline is also the particle path. In a steady, two-dimensional, incompressible flow, if the streamlines are getting closer, as shown in the left frame, the velocity has to increase in the flow direction. In a flow, if the streamlines are all parallel straight lines they represent a constant velocity on each streamline.

(a) **(b)**

Figure A. **(a)** Curved streamlines and **(b)** Parallel streamlines.

2.2 *Role of Viscosity*

Viscosity plays an important and interesting role in the motion of fluids. Difficulty in modelling this role slowed the progress in arriving at the N–S equations even after the field equations governing the inviscid flow (Euler equations) were known. Viscous

Box 3. Vorticity and Irrotational Flow

Vorticity is defined as the curl of velocity $\boldsymbol{V}$ and hence it is a vector.

$$\text{Vorticity } \Omega = \nabla \times \mathbf{V}$$

In a two-dimensional flow in the x-y plane, only the z-component of vorticity Ω_z is non-zero and is given by the expression $\Omega_z = (\partial v/\partial x) - (\partial u/\partial y)$, where (u, v) are the (x, y) velocity components. A flow is irrotational in a region if vorticity in that region is zero. In ideal or inviscid flows, vorticity cannot be created and hence an irrotational flow remains irrotational. In the presence of a body, because of the effect of viscosity, vorticity is generated near the wall and transported away, especially in the wake.

Box 4. The Euler and the Navier–Stokes Equations

Newton's second law of motion applied to a fluid element leads to the Euler equations if the stresses due to viscosity are neglected [1]. These form a system of three (one for each direction) first order non-linear partial differential equations. If the stresses due to viscosity are also included, we get the Navier–Stokes (N–S) equations. The dependent variables in these equations are the velocity components (u, v, w), fluid density ρ and pressure p. Since there are five dependent variables (u, v, w, ρ, p) in the Euler (or N–S) equations, we need in addition to these three equations, two more equations in the form of mass and energy conservation. In incompressible flow, since ρ is a known constant, we need only the mass conservation equation.

The complex nature of the N–S equations did not permit this drag to be calculated even for the simplest of bodies – a flat plate of zero thickness at zero angle of attack.

stresses depend on spatial velocity derivatives, and addition of the viscous terms to the Euler equations increases the order of the partial differential equations from one to two (see *Box* 4). This needs an extra boundary condition which is provided by the no-slip or tangential velocity being zero boundary condition on the wall (wing surface) in addition to the no-penetration or normal velocity being zero condition.

An important parameter in the study of fluid motion where the effect of viscosity is important is the Reynolds number *Re* defined as:

$$Re = \frac{\rho V D}{\mu}, \tag{2}$$

where V is the characteristic velocity, e.g., free stream velocity u_∞ for flow around a body (*Figure* 2), D is the characteristic length like chord of the airfoil, ρ is the density, and μ is the viscosity coefficient of the fluid. We can interpret *Re* to be the ratio of the inertia force to the viscous force in a gross way. For a typical

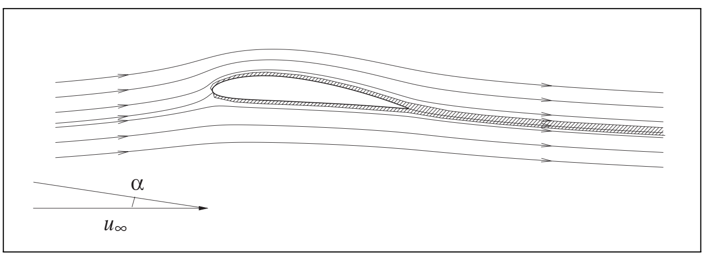


Figure 2. Airfoil with boundary layer. The thickness of the boundary layer and wake region is exaggerated as shown by the hatched region. The line making an angle α to the velocity vector is a line parallel to the airfoil chord line.

aircraft wing, *Re* is of the order of several millions. Hence, if *Re* is large, it indicates that the inertia force is more dominant compared to the viscous force; flow behaves as though it is inviscid. However, application of no-slip boundary condition warrants that a large change in velocity takes place in a thin layer called the boundary layer as indicated in *Figure* 2. We cannot neglect the effect of viscosity in this layer no matter how large the *Re* [5].

Application of the no-slip boundary condition results in a large velocity gradient in a rather thin region, and in a direction normal to the wall, and consequent viscous shear stress on the wall. Integration of pressure and viscous stresses around the airfoil surface gives the net force acting on the airfoil (see *Box* 5).

Inviscid theories admit velocity slip on the wall and viscous stress

Box 5. Forces on a Body Immersed in a Fluid

A body immersed in a fluid is subjected to forces acting on its surface. If the fluid is stationary, the only stress acting on the surface is the pressure acting normal to the surface and acting inwards. This hydrostatic case leads to the Archimedes buoyancy force.

If, on the other hand, there is relative motion of the fluid around the body, the pressure on the surface gets altered and also there is tangential viscous stress τ_w as shown. Integration of the stresses around the whole body gives the net force due to pressure and viscous stress. If the body was immersed in a uniform flow with velocity far away being $\boldsymbol{u}_\infty$, the force acting on the body along the flow is drag *D* and perpendicular to the flow is lift *L*.

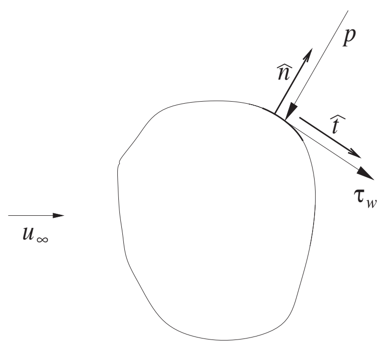


Figure A. Pressure, *p* and tangential viscous stress, τ_w on the wall of a body immersed in a flow.

Thanks to digital computers, we can calculate the lift force on an airfoil using viscous or inviscid governing equations, and smugly state that the results are almost the same. But now we are in a position to ask should not they be?

to be zero. For a streamlined body like an airfoil, as shown in *Figure* 2, this stress is nearly along the flow direction on both the sides and hence adds up to viscous drag. The complex nature of the N–S equations did not permit this drag to be calculated even for the simplest of bodies – a flat plate of zero thickness at zero angle of attack. In a boundary layer, the gradients along the flow direction (x-direction along the wall) are small, compared to the gradients in the y-direction normal to the wall. Further, the boundary layer approximation reveals that the pressure gradient in the direction normal to the wall $(\partial p/\partial y) \approx 0$, and hence one can use the inviscid Euler equations to calculate pressure distribution on the airfoil [5].

If the boundary layer remains thin and attached to the body, we can use the inviscid equations to get pressure distribution on the body. This gives for inviscid, steady, two-dimensional flows zero drag. This celebrated paradox is named after d'Alembert. Despite such a strange result, one can use this pressure distribution and calculate the lift generated.

Thanks to digital computers, we can calculate the lift force on an airfoil using viscous or inviscid governing equations, and smugly state that the results are almost the same. But now we are in a position to ask should not they be? But this does not help us in the present task since the inviscid equations are also quite difficult and have the same difficulty that was stated in a previous section. Then we can solve the equations for a specific case but cannot draw general conclusions.

2.3 *Circulation and Lift* *

[* *Though this section requires a background in vector analysis, it is recommended that readers not familiar with this area may still read on this section to get the general ideas, and not lose the continuity.*]

The concept of circulation will be useful in the present discussion. We restrict the discussion to steady, two-dimensional, incompressible flow. Circulation $\mathbf{\Gamma}$ along a closed curve in a velocity field is given by the line integral along this curve (see *Box* 6),

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} \,. \tag{3}$$

Evaluation of Γ along a rectangular curve ABCDA in the two velocity fields shown in *Figure* 3 is instructive. In *Figure* 3(a), with uniform velocity field, the two sides AB and CD do not contribute to the integral since velocity is perpendicular to these sides, and contributions along BC and DA cancel out leading to $\Gamma = 0$. In the sheared flow in *Figure* 3(b), on the other hand, values along BC and DA do not fully cancel, leading to clockwise value of Γ, which may be assigned a negative sign. If we treat Γ as a vector, the clockwise or negative Γ is along the negative z-direction, or

Box 6. Line Integral

Line integral of a vector $\mathbf{F}$ along a curve C from a_1 to b_1 is defined as:

$$W = \int \mathbf{F}.d\mathbf{s} = \int |\mathbf{F}| \cos\theta \, |d\mathbf{s}| = \int F_s \, ds \,.$$

If $\boldsymbol{F}$ is the force, not necessarily constant, and acting at some angle θ to the curve C as shown in *Figure* A, then W represents the work done by this force from a_1 to b_1. A small circle through the integral sign indicates that integration is done through a closed path C. A similar integral of velocity along a closed path C is called circulation Γ,

$$\Gamma = \oint_c \mathbf{V} \cdot d\mathbf{s} \,.$$

If vorticity $\boldsymbol{\Omega}$ in the region enclosed by C is zero then Γ along C is also zero. Readers may refer to Green's (Stokes') theorem for these interesting results [1].

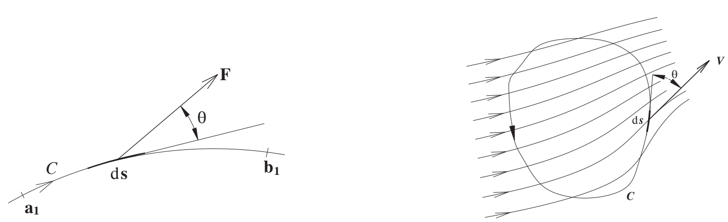


Figure A. Line integral of $\mathbf{F}$ along a curve C from a_1 to b_1 and of $\mathbf{V}$ along a closed curve C.

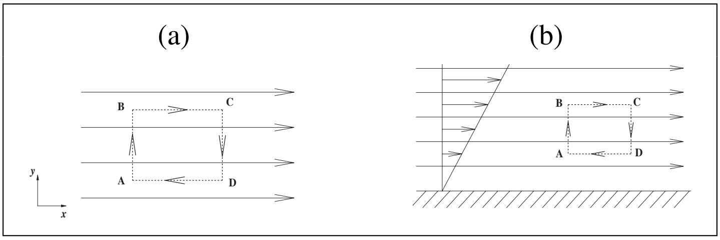


Figure 3. **(a)** Uniform flow with a circulation loop to show $\Gamma = 0$ and **(b)** Shear flow with a circulation loop to show non-zero Γ.

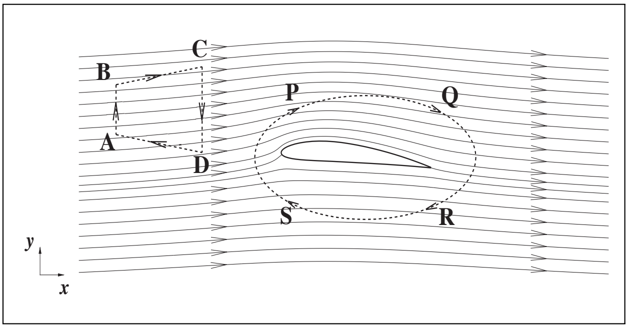


Figure 4. Streamlines around an airfoil kept in a uniform flow. Two loops in the flow to show zero (loop ABCDA excluding airfoil) and non-zero Γ (loop PQRSP around airfoil).

it is perpendicular to the plane of the paper and heading inside as given by the right hand rule. In *Figure* 3(a) Γ is zero no matter how big the rectangle is, and, in fact, it is zero irrespective of the shape of the loop. This is because the flow is uniform in this figure and it can be generalised further: If the flow is irrotational, i.e., vorticity $\mathbf{\Omega} = \nabla \times \mathbf{V}$ is zero inside the loop everywhere, then $\Gamma = 0$.

Now consider flow around an airfoil at a low angle of attack as shown in *Figure* 4. This may be a cambered airfoil or even a symmetrical airfoil at incidence. If the flow is at a sufficiently high Reynolds number, effect of viscosity is restricted to the thin boundary layer and the thin wake, and outside this region, the flow may be considered to be irrotational. Then Γ is zero around the loop ABCDA shown. In fact, it is zero around any loop that does not enclose the airfoil and the boundary layer including the wake. The loop PQRSP, on the other hand, encloses the rotational flow and hence Γ is non-zero. By looking at the spacing between the streamlines, it can be noted that velocity is higher than u_∞ at the upper region, and lower at the bottom region. An unsymmetrical cambered airfoil has led to this type of streamlines. Now comparing this with *Figure* 3(b) it appears obvious that Γ on the loop PQRSP shown is non-zero and clockwise. Thus Γ is a global measure of streamline curvature.

An important result in fluid mechanics follows from the momentum equation and is for incompressible, inviscid flow. It is called the Kutta–Joukowski theorem.

There is an important result in fluid mechanics for two-dimensional flows relating lift L on a cylinder of any cross-section to the circulation Γ around it. Again this follows from the momentum equation, and is for incompressible, inviscid flow. It is called the Kutta–Joukowski theorem [1]:

$$\mathbf{L} = \rho \mathbf{u}_\infty \times \mathbf{\Gamma} . \tag{4}$$

Thus lift is in the plane we are considering and is perpendicular to free stream velocity u_∞, direction being decided by the sign of $\mathbf{\Gamma}$ (i.e., clockwise or counterclockwise and lift being upwards for clockwise Γ). It should look obvious now that to calculate Γ the loop selected can be of any size and shape but it should include the cylinder only once.

This interesting theorem relates two kinematic quantities, $\boldsymbol{u}_\infty$ and $\mathbf{\Gamma}$ to lift. But the catch here is determining $\mathbf{\Gamma}$ is as involved as determining $\mathbf{L}$. If we have to answer why or how lift is generated, the same question applies to Γ as well, which also requires additional information.

A cylinder with circulation in a free stream with uniform velocity u_∞ as shown in *Figure* 5(a) is a typical example. It is now intuitive that the flow field has a clockwise circulation if the loop includes the cylinder. But circulation is zero around a loop if it does not include the cylinder.

However, there is a hidden catch here. The flow is irrotational since the effect of viscosity is neglected, but still circulation Γ is imparted to the flow, say by the rotation of the cylinder. It is equivalent to assuming that the cylinder imparts the velocity of the rotating wall to the fluid adjacent to it to make relative velocity zero to enforce the no-slip boundary condition (see *Box*

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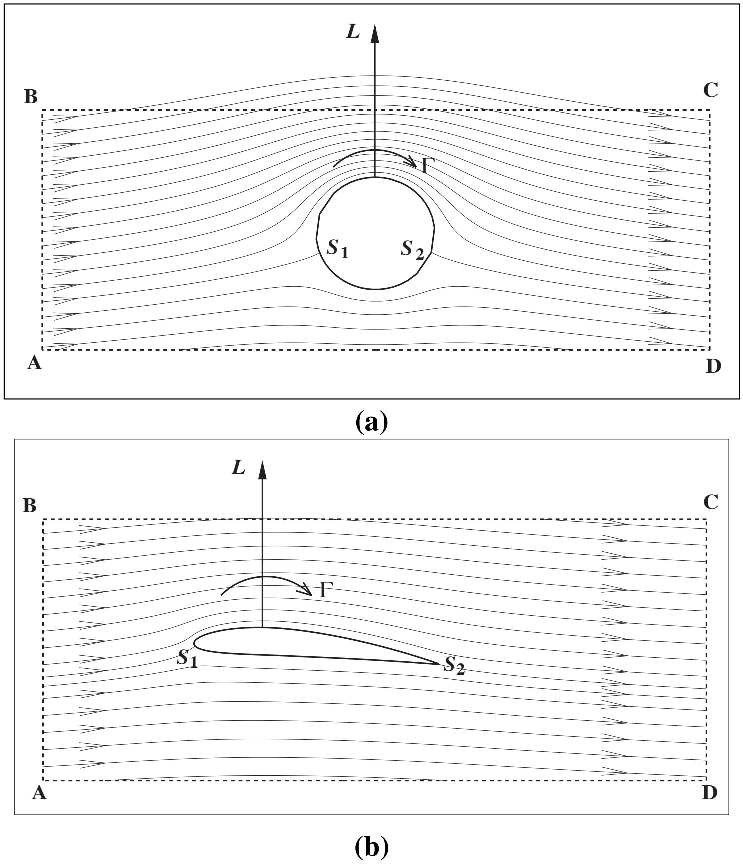
Figure 5. **(a)** Uniform inviscid flow around a cylinder with circulation. **(b)** Uniform inviscid flow around a cambered airfoil to show non-zero Γ and lift.



7). Thus the effect of viscosity is tacitly assumed but its influence is restricted to a thin boundary layer. Outside this boundary layer, flow is nearly irrotational as observed experimentally, and Γ is zero if the loop does not enclose the cylinder with the boundary layer. However, Γ is non-zero for a loop around the cylinder and has the same value irrespective of size and shape of the loop. It is not simple to relate the value of Γ to the rotational speed of the cylinder unless one solves the viscous equations or measures it experimentally.

It is interesting to note that Newton examined if the planetary motion around the Sun corresponds to vortex motion established by the Sun rotating about its axis. He considered both axisymmetric and spherically symmetrical models [6, 7].

Box 7. Streamlines Around a Rotating Cylinder

A rotating cylinder immersed in a flow from left to right imparts momentum to the adjacent fluid and sets it in motion. Role of viscosity and no-slip boundary condition are tacitly assumed here. The streamlines appear different from those in *Figure* 5(a) for various reasons, but can be made to resemble them approximately by adjusting the rotational speed of the cylinder. A more vigorous rotation of the cylinder is seen by the nature of the streamlines in the frame to the right.

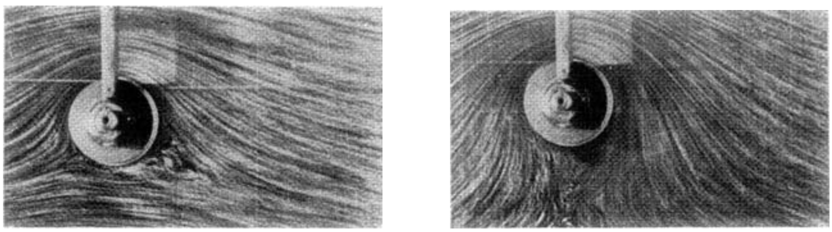


Figure A. Streamlines around a cylinder with circulation observed experimentally. Rotational speed of the cylinder, and hence Γ, is higher in the right frame (from [9]).

Now look at the streamline pattern around a cambered surface in *Figure* 5(b). The streamline shape is assumed and plotted. Comparing this pattern with that for the cylinder with circulation in *Figure* 5(a), we can argue that there is circulation around a loop enclosing the surface and hence there is lift in a direction perpendicular to velocity u_∞ as given by the Kutta–Joukowski theorem. The tacit assumption, however, is the shape of the streamlines. If we accept that, then a rigorous mathematical theory tells us that there is lift.

3. Simple Models

We are now in a position to consider simpler models and see how wings generate lift, and also clearly see what are the underlying assumptions. Before considering the simpler models, let us summarise what we have learned till now. It is possible to simulate the flow around a cambered plate by solving the N–S equations using a digital computer and calculate the lift and drag generated. The experiments can be done in a wind tunnel, but it will not quite

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tell us how lift is generated in terms of the simpler laws we know. The inviscid flow is the next lower level of simplification. But this method leads to a strange but expected conclusion that the drag force is zero. Even if we are ready to accept the Kutta–Joukowski theorem, this opens up a question – *Why should a model that gives an absurd answer like zero drag be correct in case of lift?* or, *why should such an inviscid model be assumed to give correct results for lift in a real fluid*?

It so turns out that these legitimate questions have a pleasant answer. The inviscid model indeed gives the correct answer for lift, even quantitatively, provided we know the range of its applicability. This will be discussed in detail in a section on the thin airfoil theory in Part 2.

What should we expect from simpler models? To generate lift we need a plate that can generate higher pressure at the lower surface compared to that at the upper surface. A cambered plate or a flat plate at an angle of attack can do that. The popular simple theories should explain this.

3.1 *Idea of Longer Particle Path*

This intuitive model is very popular, and it again depends on the assumption of shape of the streamlines as indicated in *Figure* 5. Because of the camber of the airfoil in *Figure* 5(b) or the circulation around the cylinder in *Figure* 5(a) the streamlines on the upper part take a longer path compared to the lower ones. Hence, the fluid elements along the longer paths should hurry up to join the other elements that were co-travellers earlier but chose the shorter lower path.

But here is a point that keener observers may feel uncomfortable with, and also some critics of such models train their guns on. Just because a fluid element takes a longer path why should it hurry up?

But here is a point that keener observers may feel uncomfortable with and also some critics of such models train their guns on. Just because a fluid element takes a longer path why should it hurry up? Where is the need to finish the race at the same time? In fact, in the inviscid model there is no need for this to happen.

But because of the effect of viscosity, the fluid does not have so much of freedom and it has to pull together. The sharp trailing edge plays a crucial role here in the form of Kutta condition which will be discussed later.

If we solve the inviscid equations of motion, luckily we get sufficient warning. Then the solution is not unique. The flow elements from the upper and lower streams are not compelled to finish the race together. Then the velocity on the upper side need not have to be higher or pressure need not have to be lower. There is no lift. In terms of the earlier terminology we have learned, the circulation Γ around the airfoil is not unique. But this is not what happens. We will see how it is forced to be unique later; viscosity comes to our help here. In the case of a circular cylinder, uniqueness can be achieved by specifying the circulation.

3.2 *Idea of Streamline Curvature*

This intuitive model is intimately connected to the previous section since we assume here too, the pattern of streamlines as shown in *Figure* 5. From Newton's first law of motion it follows that a fluid element has to move along a straight-line with uniform velocity if there is no net force acting on it. In a fluid velocity field, it amounts to uniform pressure with parallel streamlines and constant velocity on each streamline. If the streamlines are curved there has to be a centripetal acceleration (u^2/R) pointing inwards towards the centre of curvature, and an associated pressure field with higher pressure on the convex side as shown in *Figure* 6. It is this extra pressure that causes centripetal acceleration of the fluid elements associated with curved streamlines. Then the radial pressure gradient is given by,

$$\frac{\partial p}{\partial r} = \frac{\rho u^2}{R}, \tag{5}$$

where R is the radius of curvature of the streamline at this location.

Now this handy formula can be used along with assumed streamline patterns as shown in *Figure* 7 to convince ourselves that there

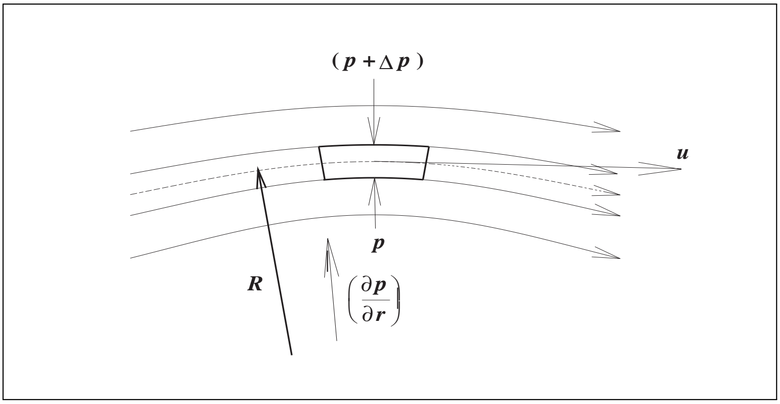

Figure 6. Curved streamline pattern and consequent pressure increase at the outer side.

is a lift force. We start with the assumption that pressure p far away from the airfoil is the constant free stream pressure p_∞. The presence of the cambered airfoil has changed the pressure field locally, and the disturbances caused in the velocity and pressure fields go to zero as we move far away. If we select a point B on the upper surface and roughly at mid-chord of the airfoil and reach it by moving vertically downwards from a far off point A, then the pressure should decrease to a value lower than p_∞ as shown in *Figure* 7. It is because of the curvature of the streamlines. Conversely, we select a point D on the lower surface and reach it by moving vertically upwards from a far off point C. Due to the curvature of the streamlines, the pressure p at point D should be higher than p_∞. Comparing pressures at points B and D, it is clear that there is a pressure jump across the airfoil surface and consequent upward lift force. Associated with the pressure jump there should be a velocity jump with higher velocity on the upper side as required by Bernoulli's principle and as seen in the previous section. This velocity jump, of course, ignores the existence of the boundary layers and zero velocity on the wall due to no-slip condition on either sides.

If the vertical line ABDC in *Figure* 7 were in front of the airfoil or behind it, there cannot be any pressure or velocity jumps or discontinuities. This leads us to the inevitable conclusion that there must be something drastic at the leading and trailing edges.

We have described two simple models which only give qualitative

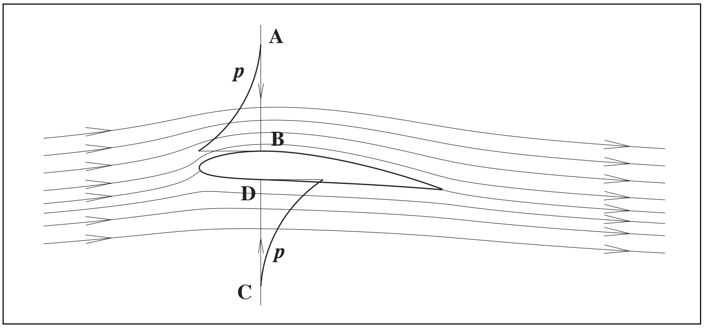

Figure 7. Curved streamline pattern and consequent pressure difference on the airfoil.

explanations. An acceptable model should be quantitative, based on rigorous principles. In the next part, we will discuss the thin airfoil theory leading to some wonderful quantitative results. The momentum conservation principle will help us to put all these models on a logical footing.

Suggested Reading

[1] A M Kuethe and C–Y Chow, *Foundations of Aerodynamics: Bases of Aerodynamic Design*, Fifth edition, Wiley, New Delhi, 2010.

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