Hermann Weyl (1885–1955)

Mathematician, Physicist, Philosopher and Poet

It is well known that the heart of mammals is an asymmetric screw. If nature were all lawfulness then every phenomenon would share the full symmetry of the universal laws of nature as formulated by the theory of relativity. The mere fact that this is not so proves that 'contingency' is an essential feature of the world.

– Hermann Weyl

Hermann Weyl was born on November 9, 1885 in Elmshorn, Germany. He received his doctorate from the University of Göttingen under the direction of David Hilbert. Following a short teaching stint at Göttingen, he took up a chair of mathematics at the Swiss Federal Institute of Technology in Zürich where he had Einstein for a colleague, working on the finishing touches to his general theory of relativity. In 1913, Hermann Weyl married Helene Joseph who was a philosopher and an expert in Spanish language. Both Weyl and his wife were interested in philosophy, and each had a flair for languages. Weyl's writing in English was almost poetic. As a young man in Zürich, Weyl was a romantic figure; his close friend Schrödinger's wife fell in love with him. In 1930, Weyl left Zürich, and after a three-year position in Göttingen, occupying the chair vacated by Hilbert, moved to the Institute for Advanced Study in Princeton in 1933. He remained there for the last 22 years of his life.

Weyl had a life-long interest in physics, and more specifically, in the theory of relativity. In fact, his fundamental contributions to Lie group representations were directly motivated by his desire to find what he called 'the formal apparatus' behind the theory of relativity. He was the first to attempt interpreting electromagnetism via a so-called connection on a bundle. In 1918, in his book *Space, Time, Matter*, Weyl initiated the study of gauge theory. While looking at one aspect of general relativity, viz., the non-preservation of direction in space with curvature, he looked at the possible situation that length was also not preserved. He proposed another metric related to length which resulted in the appearance of Maxwell's equations. Magical though this idea seemed, it was pointed out by Einstein that it was flawed as it implied that the size of a particle would depend on its past history; something not supported by physics. However, it turned out to be not a complete failure because this led to the modern gauge theory, and more importantly, got other scientists believing in the possibility of unifying electromagnetism and gravitation. Weyl's popular book *Symmetry* written in 1952 bears witness to his artistic temperament. Many of his contributions to physics involve symmetry; for example, his book

The Theory of Groups and Quantum Mechanics written in 1931 revealed the importance of symmetry in quantum theory. In contrast to symmetry, Weyl also recognized that asymmetry plays a fundamental role in the laws of physics. In 1929, being interested in the discrete symmetries P, C, T, he introduced the massless wave equation (the so-called Weyl equation) as referred to by physicists. Amusingly, mathematicians call this the Dirac equation.

Weyl was a universal mathematician – unifying and explaining disparate themes. His pioneering contributions cover a very broad spectrum. They encompass eigenvalue distribution of differential operators, representations of compact Lie groups and invariant theory, the spectrum of self-adjoint operators under compact perturbation, and the uniform distribution of numbers modulo 1; to name a few. These works have proved to be seminal in the development of non-commutative topology and geometry, and in the physics of gauge theories. His papers on representations of Lie groups during 1923–28 are some of the most influential works in mathematics.

Weyl had a poetic disposition and was kind and generous, even extravagant in his praise. During the International Congress of Mathematicians in 1954, Weyl said of Serre (the youngest Fields medallist at 28), "Never before have I seen such a brilliant ascension of a star in the mathematical sky as yours." He introduced the subject of the other Fields medallist – Kodaira's work, with a wonderful description of sheaf theory and went on to say, "Your work has more than one connection with what I tried to do in my younger years; but you have reached heights of which I never dreamt. Since you came to Princeton in 1949 it has been one of the greatest joys of my life to watch your mathematical development. If I omitted essential parts of misrepresented others, I ask your pardon, Dr Serre and Dr Kodaira; it is not easy for an older man to follow your striding paces. The mathematical community is proud of the work you both have done. It shows that the old gnarled tree of mathematics is still full of sap and life. Carry on as you began!" He was self-effacing as the above quote as well as the following quote shows. When he was 65, he referred to his book *The Concept of a Riemann Surface* written when he was 26, and told Raoul Bott, "I can't bear to look at it now; how pompous one can be at 26!"

Despite beginning his famous book on classical groups with the words, "The gods have imposed upon my writing the yoke of a foreign language that was not sung at my cradle", Weyl has shown a gift for the mot juste in English as we mentioned above. He coined the terms 'symplectic', 'Lie algebra', and 'maximal torus' which has become standard nomenclature ever since. A glimpse to his personality is afforded by his oft-mentioned quote claiming that even though he usually attempted to unify truth and beauty, when forced to choose one over the other, he always chose beauty!

I want to describe just one problem to which Weyl contributed, and which goes under the name "Can one hear the shape of a drum?" (See S Kesavan, Listening to the Shape of a drum – The Mathematics of Vibrabting Drums, *Resonance*, Vol.3, No.9, pp.26–34, 1998.) This ponders whether someone hearing a drum, and identifying all the normal modes of vibration can discern its shape (its geometry). If a 2-dimensional drum is beaten, its displacement v at time t is described by the equation,

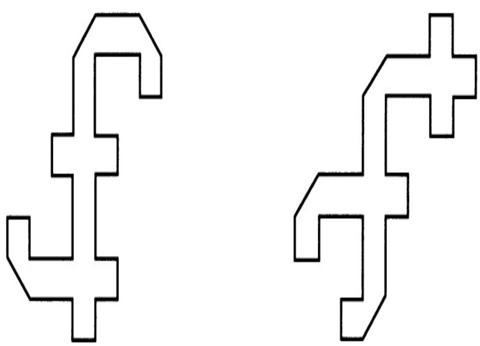
$$\frac{\partial^2 v}{\partial t^2} = c^2\Big(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\Big).$$

One looks for solutions in separate variables as $v = F(t)u(x, y)$. Then, u satisfies the stationary equation (taking $c = 1$ for simplicity) $\Delta u + \lambda u = 0$, and the boundary condition that $u = 0$ on the boundary – as the drum is attached tautly at the ends. The set of eigenvalues λ form a discrete infinite unbounded set,

$$0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \lambda_4 \cdots$$

The domains of vibrations of two such drums are said to be isospectral if they have the same set of eigenvalues, and they are isometric if they are congruent in the usual sense of Euclidean geometry. The amusing phrase 'hearing the shape' refers to the possibility of the spectrum determining the geometry. Sometimes, one also looks at Neumann conditions where, instead of the function u, its normal derivative is constrained to vanish on the boundary. More generally, one considers domains Ω in $\mathbf{R}^n$ for any $n \geq 2$, and Weyl showed that the r-th eigenvalue λ_r is asymptotically (under either of the Dirichlet and Neumann boundary conditions) equal to $4\pi^2(r/Vol(B_n)Vol(\Omega))^{2/n}$ as $r \to \infty$, where B_n is the ball of unit radius in n-space. This remarkable result of Weyl shows that the volume of the domain is determined by its spectrum!

In 1992, Carolyn Gordon, David Webb, and Scott Wolpert came up (see *Bulletin of the American Mathematical Society*, Vol.27, pp.134–138, 1992) with the two very simple domains in the figure here, which have the same Dirichlet and Neumann spectra but are not isometric. This subject is one of the very active areas of research today.



Hermann Weyl passed away quite unexpectedly on December 9, 1955, less than a month after his 70th birthday. He was returning home after posting a letter thanking a friend who had wished him on his birthday, when he suddenly collapsed and passed away.

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