This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to Think It Over, *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

A Charged Particle in Perpendicular Electric and Magnetic Fields

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Keywords
Relativity, electric and magnetic fields, cycloid, Lorentz transformation.

Solution

The solution to the question raised is that the coordinate along the electric field is unbounded for the choice of the electric and the magnetic fields made, i.e., the perspective of $\mathcal{S}'$ is correct. The flaw in the analysis of the $\mathcal{S}$ frame presented is that the equation of motion used to obtain the trajectory as cycloid is Newton's law, and not the relativistic equation of motion.

The equation of motion is $\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$, where $\boldsymbol{F} = q\,(\boldsymbol{E} + (\boldsymbol{u} \times \boldsymbol{B}))$ is the Lorentz force, q is the charge, $\boldsymbol{u}$ is the ordinary velocity and $\boldsymbol{p}$ is the momentum of the particle. While Newton's law takes $\boldsymbol{p} = m\boldsymbol{u}$, where m is the rest mass of the particle, the relativistic equation of motion takes $\boldsymbol{p} = \frac{m\boldsymbol{u}}{\sqrt{1-u^2}}$.

The bottom line is that the relativistic equation of motion yields bounded z-coordinate for $E < cB$ and unbounded z-coordinate for $E \geq cB$. In either cases $z = z'$. This means that in relativity, a small magnetic field will be unable to bend the particle back that is accelerated by a large electric field. The well known result stating the trajectory as a cycloid is a non-relativistic one and is applicable when $E \ll cB$.

The relativistic solution to the problem is sketched in only a very

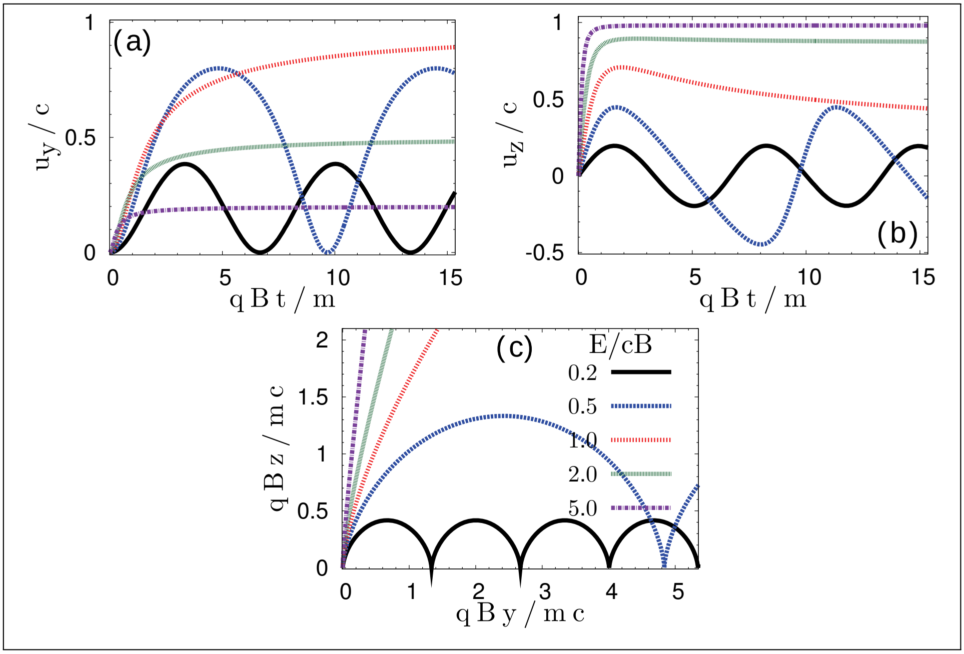

Figure 1. The motion of a particle with perpendicular electric and magnetic fields and the charged particle starting from rest at the origin. **(a)** The y-component of velocity u_y as a function of time, **(b)** the z-component of velocity u_z as a function of time and **(c)** the trajectory in the $y-z$ plane for different values of $E/(cB)$. The legends shown in **(c)** are applicable to all the three. Newtonian dynamics predicts cycloid motion independent of the strengths of $\boldsymbol{E}$ and $\boldsymbol{B}$.

few advanced texts on relativistic electrodynamics (see *Figure* 1 for example) and can be obtained as follows: transform to a new frame (similar to $\mathcal{S}'$ frame) whose velocity is cleverly chosen so as to eliminate one of the two fields (depending on which among E and cB is larger) and the solution obtained in the new frame is then translated back to the language of the original frame. The solution obtained in this fashion (using relativistic equation of motion and suitable numerical techniques) is shown below for selected values of $E/(cB)$.

When $E < cB$, u_y, u_z and z are periodic with frequency $\frac{qB}{2\pi m}\left(1-\frac{E^2}{c^2B^2}\right)^{3/2}$. When,

$$E \geq cB, \text{ as } t \to \infty, u_y \to \frac{c^2 B}{E}, u_z \to c\sqrt{1-\frac{c^2B^2}{E^2}}$$

and the total speed $= \sqrt{{u_y}^2 + {u_z}^2} \to c$.

The solution to this puzzle emphasizes the fact that the relativistic analyses can even *qualitatively* alter some of the popular conceptions based on classical dynamics.

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Suggested Reading

[1] **L D Landau and E M Lifshitz, *Course of Theoretical Physics:*** Vol.2***, The Classical Theory of Fields*, Pergamon Press, pp.62–63, 1994.**