The Creative Genius: John Nash

"Nash's theory of noncooperative games should now be recognized as one of the outstanding intellectual advances of the twentieth century. The formulation of Nash equilibrium has had a fundamental and pervasive impact in economics and the social sciences which is comparable to that of the discovery of the DNA double helix in the biological sciences", says Roger Myerson, who won the Nobel Prize in Economic Sciences in 2007 and whose textbook on game theory is a classic. The celebrated journalist Erica Klarreich says of John Nash's work: "No one cites Nash's papers any longer because *Nash equilibrium* is standard vocabulary; every mathematician knows what it means. While he published only a small handful of papers, John Nash will be remembered as one of the most original and influential mathematicians of the 20th century, whose work continues to inspire new results and research directions." Barry Mazur, Professor of Mathematics at Harvard University says: "Jane Austen wrote six novels. I think Nash's pure mathematical contributions are on that level. Very, very few papers he wrote on different subjects, but the ones that had impact had incredible impact." John Nash is surely one of the most brilliant minds of the twentieth century.

Early Years

John Forbes Nash Jr. was born on 13 June 1928, in a small city called Bluefield in the Appalachian Mountains in West Virginia, USA. He was named after his father John Forbes Nash, who after his BS in electrical engineering from Texas A&M University, served in France during World War I and subsequently moved to Bluefield and worked for the Appalachian Electric Power Company as an Electrical Engineer. His mother, Margaret Virginia Martin, studied at West Virginia University and was a school teacher before her marriage. Nash (that is, John F Nash Jr.) had a sister, Martha Nash, who was born on 16 November 1930 [1, 2]. He did his schooling, from kindergarten to high school, in Bluefield where, during his final year, his parents arranged for him to take supplementary mathematics courses at the Bluefield College.

At Carnegie Institute of Technology

After completing high school, Nash went to Carnegie Institute of Technology (now, Carnegie Mellon University) in Pittsburg, on a George Westinghouse Scholarship, and started his BS in chemical engineering. He soon switched to chemistry and subsequently, to mathematics major. Because of his progress, he was given an MS degree in mathematics along with BS degree, in 1948. Even while he was an undergraduate student, Nash published an article on number theory and an article on topology in the university journal *Carnegie Technical*, a magazine published by students of the College of Engineering and Science.

When he graduated from Carnegie Institute of Technology, he had offers from Harvard University as well as Princeton University, to pursue his PhD and he accepted the offer from Princeton. Richard J Duffin, Nash's advisor at the Department of Mathematics, Carnegie Institute of Technology, in his letter recommending Nash to the graduate college at Princeton, wrote, "He is a mathematical genius." John L Synge, Head, Department of Mathematics, Carnegie Institute of Technology, in his recommendation letter to Princeton, wrote [3, 4], "Mr. Nash is unique in my experience of students. I would rank him among the best I have had, and possibly he is the very best. At first impression, he might seem inferior, since he does not write out his work in polished form, nor does he lecture impressively. However, this external clumsiness is more than compensated by quickness of understanding, originality, and capacity for seeing the inner meaning of an argument, all unrivalled in my experience."

His publication titled 'The Bargaining Problem' was the result of an idea that stuck him when he did an elective course in 'international economics' at Carnegie Institute of Technology. This work was published in *Econometrica* in 1950, and is currently known as the 'Nash bargaining solution'.

At Princeton University

At Princeton, Nash was J.S.K. Fellow in Mathematics during 1948–49 and Atomic Energy Commission Pre-Doctoral Fellow during 1949–50. He submitted his 27-page typescript PhD dissertation titled 'Non-cooperative Games', under the supervision of Albert W Tucker and was awarded his PhD degree in 1950, which according to Milnor was his most widely influential work [5]. Further, according to Milnor, Nash was "an amazing person, highly original, and always attempting to make a name for himself by attacking the most difficult and significant mathematical problems." Nash conceptualized non-cooperative games and proved that any *n*-person non-cooperative game with finite action spaces has an equilibrium point, that is, a tuple of strategies of the players from which no player has an incentive to unilaterally deviate. His seminal publications on the topic include 'Equilibrium Points in *n*-person Games' which appeared in the *Proceedings of the National Academy of Sciences* in 1950, and 'Non-cooperative Games' which appeared in the *Annals of Mathematics* in 1951. Over the years, his ideas have become central in diverse fields such as economic theory and evolutionary biology.

At Massachusetts Institute of Technology

In 1951, Massachusetts Institute of Technology (MIT) offered Nash a position in their Mathematics faculty as 'C.L.E. Moore Instructor'. During his early years at MIT, he solved the open problem in differential geometry on isometric embeddability of abstract Riemannian manifolds in Euclidean spaces. His seminal paper titled 'Real Algebraic Manifolds' was published in the *Annals of Mathematics* in 1952. Nash's proof of the isometric embedding problem came as a complete surprise to much of the mathematical community. His methods were revolutionary. The great mathematician Mikhail Gromov said that Nash's work on the embedding problem struck him to be "as convincing as lifting oneself by the hair". But after great effort, Gromov finally understood Nash's proof: at the end of Nash's lengthy argument, Gromov said, Nash "miraculously, did lift you in the air by the hair!" Mikhail Gromov further stated [5] that in his opinion, Nash's contribution to geometry was incomparably greater than his contribution to economics by many orders of magnitude. To this remark, Nash mentioned that it was a matter of taste. Nash met Alicia Larde, who was a student at MIT, majoring in physics. They married in 1957.

At Institute for Advanced Study, Princeton

Around the same time, when Nash was on a sabbatical at the Institute for Advanced Study, Princeton, he started working on David Hilbert's 19th problem, 'Are the solutions of regular problems in the calculus of variations always necessarily analytic?' Ennio de Giorgi from Italy was also working on the problem at the same time, and both of them independently came up with their results. It is widely believed that if only one of them had derived this result, he would have been immediately awarded the Fields Medal in Mathematics.

Later Years

In 1958, Nash became a tenured Professor at MIT. In 1959, around the same time when Alicia was pregnant with their son, Nash started showing signs of paranoid schizophrenia. Subsequently, he resigned from MIT and was admitted to a hospital for treatment. His final discharge from hospital was in 1970, after which he slowly attained stability. Princeton allowed him to audit courses and he slowly got back to teaching and research. In 1978, he won the John von Neumann Theory Prize of the Institute for Operations Research and Management Sciences (INFORMS).

Nobel Prize in Economic Sciences

For his work on introducing "the distinction between cooperative games, in which binding agreements can be made, and non-cooperative games, where binding agreements are not feasible" and developing "an equilibrium concept for non-cooperative games that now is called Nash equilibrium", Nash was awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel in 1994 (Nobel Prize), along with John C Harsanyi and Reinhard Selten, "for their pioneering analysis of equilibria in the theory of non-cooperative games". In 1995, Nash became Senior Research Mathematician at Princeton. He also won the American Mathematical Society's Steele Prize for his seminal contribution to research, in 1999.

Abel Prize and the Tragic End

In 2015, the Norwegian Academy of Sciences and Letters awarded the Abel Prize to Nash and Louis Nirenberg "for striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis." "Their breakthroughs have developed into versatile and robust techniques that have become essential tools for the study of nonlinear partial differential equations. Their impact can be felt in all branches of the theory" [6]. They received the Abel Prize from His Majesty King Harald V at the award ceremony in Oslo on 19 May 2015. Nash and Alicia died in an unfortunate taxi accident in New Jersey on 23 May 2015, when they were returning from Norway.

Epilogue

The National Security Agency (NSA) declassified in 2014, a sensational letter that John Nash had sent to the agency in 1955. This is reported in detail in the Blog 'Turing's Invisible Hand' [7]. Since 1950, Nash was developing an encryption machine and tried to explain his ideas to some US security agencies. Not much is known about whether Nash's ideas were implemented by these agencies. At the start of this letter, Nash says: "In this letter I make some remarks on a general principle relevant to enciphering in general and to my machine in particular. I hope my handwriting, etc. do not give the impression I am just a crank or circle-squarer. My position here is Assist. Prof. of Math. My best known work is in game theory (reprint sent separately)." In this letter, he remarkably talks about many ideas underlying computational complexity theory as well as modern cryptography. He goes beyond Shannon's information-theoretic formalization of cryptography and proposes that security of encryption be based on computational hardness, an idea that cryptographers embraced almost two decades later. He also talks about the distinction between polynomial time and exponential time computation, which forms the very basis of computational complexity theory that was developed almost a decade later. According to Ron Rivest and Adi Shamir, Turing awardees and cryptography czars, "Nash anticipated the birth of complexity theory a decade later (1960s), and the birth of modern cryptography two decades later (1970s)." This sums up the intellectual genius of one of the most brilliant minds of the twentieth century.

Suggested Reading

[1] www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1994/nash-facts.html

[2] www.britannica.com/biography/John-Nash

[3] www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1994/nash-bio.html

[4] webspace.princeton.edu/users/mudd/Digitization/AC105/AC105_Nash_John_Forbes_1950.pdf

[5] Interview with Abel Laureate John F Nash Jr by Martin Raussen and Christian Skau, *Notices of the AMS*, Vol.63, No.5, pp.486–506, 2015.

[6] www.abelprize.no/nyheter/vis.html?tid=63589

[7] https://agtb.wordpress.com/ – Turing's Invisible Hand : Economics, Computation, and Game Theory.

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