Think It Over

This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

A Charged Particle in Perpendicular Electric and Magnetic Fields

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This question arose during Research Education Advancement Programme conducted by Bangalore Association for Science Education at Jawaharlal Nehru Planetarium, Bengaluru.

Consider an inertial frame S in which there is a uniform and static electric field $\boldsymbol{E} = (0, 0, E)$ acting along the z-axis and a uniform and static magnetic field $\boldsymbol{B} = (B, 0, 0)$ acting along the x-axis as shown in *Figure* 1a. A positively charged particle with charge q and mass m is released from rest at the origin and at time $t = 0$, as viewed from this frame. It is a well-known result (see for example chapter 5 of [1] or chapter 3 of [2]) that under the action of both the fields that are perpendicular to each other, the trajectory will be a cycloid given by:

$$\begin{aligned} x(t) &= 0\ , \\ y(t) &= \frac{E}{\omega B}[\omega t - \sin(\omega t)]\ , \\ z(t) &= \frac{E}{\omega B}[1 - \cos(\omega t)], \end{aligned}$$

where $\omega = qB/m$. The overall motion is along the y-axis as shown in *Figure* 1a. Note here that *the z-coordinate of the particle does not blow up; it is bounded.*

Keywords
Relativity, electric and magnetic fields, cycloid, Lorentz transformation.

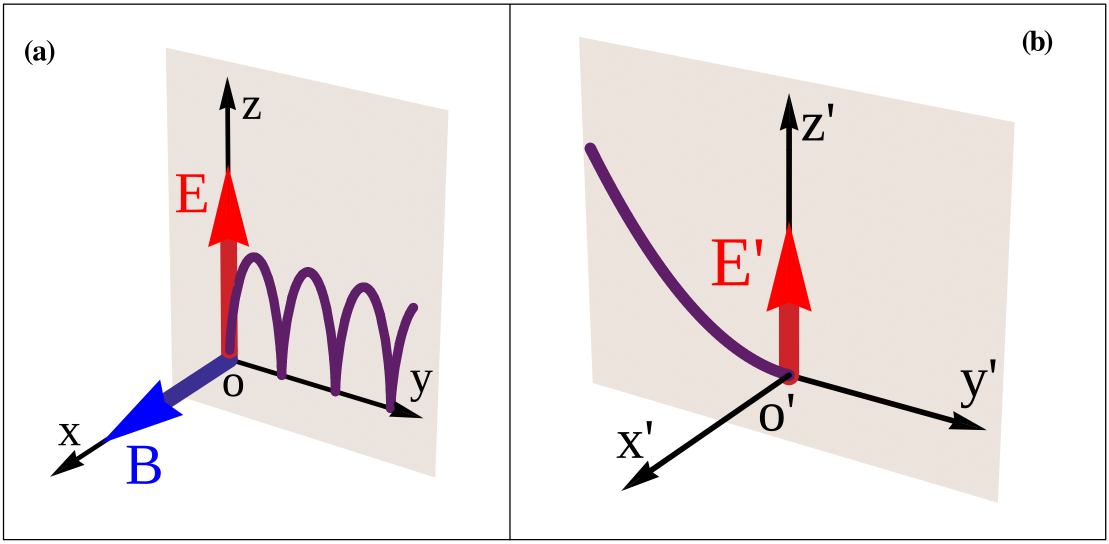

Figure 1. The fields and the trajectory as seen from different frames. (a) The perspective of the S frame: The electric (***E***) and the magnetic (***B***) fields are homogeneous, static, and perpendicular to each other. The trajectory of a charged particle is a cycloid and is *bounded* along the direction of the electric field.
(b) The perspective of the S' frame: The electric (***E'***) field is homogeneous and static with *no* magnetic field. The trajectory is *unbounded* along the direction of the electric field.

Now analyze the same situation from another inertial frame S′ which is moving w.r.t. S along the positive y-axis with velocity $v = Bc^2/E$ (where c is the speed of light), overlapping with S at the time of release of the charged particle. Let us focus on the case $E > cB$ so that $v < c$. This particular speed v is chosen with an intention to help the analysis given below.

If one knows the electromagnetic fields in one inertial frame, the Lorentz transformation gives the fields in any other inertial frame. In this case, the transformation equations are (see chapter 12 of [1]):

$$\begin{aligned}
E'_x &= \gamma(E_x + vB_z) = \gamma(0+0) = 0, \\
E'_y &= E_y = 0, \\
E'_z &= \gamma(E_z - vB_x) = \gamma\left(E - \frac{Bc^2}{E}B\right) = \frac{E}{\gamma},
\end{aligned}$$

$$\begin{aligned}
B'_x &= \gamma\left(B_x - v\frac{E_z}{c^2}\right) = \gamma\left(B - \frac{Bc^2}{E}\frac{E}{c^2}\right) = 0, \\
B'_y &= B_y = 0, \\
B'_z &= \gamma\left(B_z + v\frac{E_x}{c^2}\right) = \gamma(0+0) = 0,
\end{aligned}$$

where $\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$. Therefore, as viewed from S′, there is *no* magnetic field and there is a uniform electric field $\mathbf{E}' = \left(0, 0, \frac{E}{\gamma}\right)$ pointing along the z'-axis.

Note that both $\mathbf{E}\cdot\mathbf{B}$ and $|\mathbf{E}|^2 - \mathbf{c}^2|\mathbf{B}|^2$, which are supposed to be Lorentz scalars, are indeed unaltered. The charged particle appears to be released from the origin with an initial velocity v along the negative y-axis. *The z'-coordinate of the particle in this problem can be easily shown to be unbounded* (prove!) as shown in *Figure* 1b. This is also intuitively evident.

But Lorentz transformation of coordinates says that $z' = z$, i.e., the length perpendicular to the velocity of the frame is unchanged! So, the question is – whether the coordinate along the electric field ($z = z'$) is bounded or unbounded?

Suggested Reading

[1] D J Griffiths, *Introduction to Electrodynamics*, Pearson Education, 1999.

[2] K N Srinivasa Rao, *Classical Mechanics*, Universities Press, 2003.