We discuss some errors in *The Feynman Lectures on Physics* related to the concept of symmetry. We also suggest a possible correction to Fig.1.4 of Vol. I. The discussion may be useful for students of crystallography, solid state physics and solid state chemistry.

1. Introduction:

*The Feynman Lectures on Physics* [1] are justifiably famous as an interesting and inspiring introduction to physics at the undergraduate level. Here I point out some inadvertent mistakes related to symmetry and crystallography that have crept into the text. A discussion of these errors can be illuminating for beginning students of physics, chemistry and materials science.

2. Basic Crystallography

To put the errors in a proper context, we discuss briefly elementary crystallography. A crystal is a 3D periodic arrangement of atoms. A lattice is a 3D periodic arrangement of points. Thus, a crystal can be considered as a lattice with an atom or group of atoms associated with each lattice point. The atom or group of atoms associated with each lattice point is called a motif or basis of a crystal. This relationship between a crystal and the associated lattice and motif is sometimes expressed as

\[
\text{Crystal} = \text{Lattice} + \text{Motif}.
\]

Or, in other words, the lattice tells us *how to repeat* but the motif gives *what to repeat*. A lattice is a geometrical scaffolding to which atoms are attached to get the physical crystal.

The fact that crystals are 3D periodic arrangements of atoms can also be expressed by stating that they have 3D translational symmetry, 3-fold axis, 6-fold axis, hexagonal symmetry, 2-fold axis, mirror plane, bilateral symmetry, lattice, crystal, motif, body-centred cubic lattice, face-centred cubic lattice.
The crystallographic restriction theorem forbids the presence of a five-fold axis in a translationally periodic crystal. But it can be present in quasicrystals, which lack translational periodicity. The 2011 Nobel Prize in Chemistry was awarded for the discovery of quasicrystals.

symmetry. Thus translational symmetry is the defining requirement of a crystal. We can define translational symmetry as follows. If on shifting all points of a crystal by a vector \( \mathbf{r} \), we arrive at points that are indistinguishable from the starting points, then we say that the crystal possesses translational symmetry.

As already stated, all crystals, by definition possess translational symmetry. However, what makes the study of crystals interesting is that they may (and often do) also possess another kind of symmetry called point symmetry. In general, a symmetry operation is one that brings an object into self-coincidence. Translational symmetry discussed above is one example. By definition, a point symmetry is an operation that brings the object into self-coincidence, and also leaves a set of points unmoved or invariant. As translation operation moves all points it is not a point symmetry. Two examples of point symmetry are rotation and reflection.

If a crystal is brought into self-coincidence by a rotation about an axis it is said to possess rotational symmetry. This is obviously a point symmetry operation as points on the rotation axis remain unmoved. If \( \theta \) is the minimum angle of rotation about an axis which brings a crystal into self-coincidence, then in one complete rotation it will come into self-coincidence \( n \) times where \( n = \frac{360^\circ}{\theta} \). The crystal is then said to possess an \( n \)-fold axis. An equilateral triangle has a 3-fold axis passing through its centre, and a square has a 4-fold axis.

An interesting theorem, known as the crystallographic restriction theorem, restricts the values of \( n \) to 1, 2, 3, 4 and 6. This is because other rotational symmetries are incompatible with the translational symmetry of a crystal. In particular, the absence of 5-fold symmetry for periodic crystals should be noted. In 1984, Schetman et al reported a solid phase of metallic alloy having 5-fold symmetry. However, the phase, although ordered, lacked translational symmetry in accordance with the above stated theorem and was called quasicrystal. Schetman was awarded the 2011 Nobel Prize in chemistry for this achievement [2].
Another point symmetry is the reflection symmetry. If a crystal is brought into self-coincidence by reflection in a plane (called mirror plane) then it is said to possess reflection symmetry. This is obviously a point symmetry as points on the mirror plane remain unmoved. This symmetry is also known as bilateral symmetry.

We are now prepared to discuss the errors in *The Feynman Lectures on Physics* (henceforth *FLP*).

3. ERROR 1: Hexagonal vs. Trigonal Symmetry

Following is a quote from Chapter 1, Volume 1 of *FLP* (Fig.1-4 referred to in the text is reproduced as *Figure 1* here.)

“Fig.1-4 is an invented arrangement for ice, and although it contains many of the correct features of the ice, it is not the true arrangement. One of the correct features is that there is a part of the symmetry that is hexagonal. You can see that if we turn the picture around an axis by 120°, the picture returns to itself.”

*Error in text:* Hexagonal symmetry should involve a 6-fold rotation axis with minimum angle of rotation of $360/6 = 60°$ and not 120°.

*Error in figure:* The large dark circles represent oxygen atoms

**Figure 1.** A reproduction of Fig.1-4 of *FLP*, Vol.I.
Figure 2. Fig.1-4 of FLP with the 3-fold and a 6-fold axes marked respectively by triangles and hexagons. The figure is centred on a 3-fold axis which is a wrong choice if one wishes to emphasize the 6-fold symmetry of the structure.

Figure 3. The suggested correction for Fig.1-4 of FLP. The structure shown in Figures 1 and 2 has been extended periodically to include more atoms. The box is centred on a 6-fold axis.

and the smaller white circles hydrogen in Figure 1 (Fig.1-4 of FLP). As claimed by Feynman, this is not the true structure of ice but a 2D representation attempting to show the hexagonal symmetry. But where is the hexagonal symmetry axis in the figure? The centre of the ring of atoms in the middle of the figure is a 3-fold axis and not a 6-fold one. This is shown in Figure 2 with a small triangle (standard symbol for such an axis). This is because the three alternating sides of the hexagonal ring each accommodate a pair of H, but the other three sides do not.

The question now is how to correct this figure without drastically changing the original. If we look carefully, there are three six-fold axes marked by little hexagons in Figure 2. But all of these are on the sides of the figure and not in the centre. Thus the mistake in Fig.1-4 of FLP is that the external box of the figure is incorrectly centred on a 3-fold axis rather than a 6-fold axis. We note that any 2D periodic pattern having 6-fold axes will necessarily have also 3-fold axes. Thus the error in the figure is that although the intention is to show the 6-fold (hexagonal) symmetry, the box is centred on one of the 3-fold axes. With this realization the correction is simple. All that one has to do is to shift the box and centre it on a 6-fold axis. This is shown in Figure 3.
4. ERROR 2: Bilateral Symmetry vs. Two-Fold Axis

The following quote is from FLP, Vol 1, Section 11-1:

“The word “symmetry” is used here with a special meaning, and therefore needs to be defined. When a thing is symmetrical—how can we define it? When we have a picture that is symmetrical, one side is somehow the same as the other side. Professor Hermann Weyl has given this definition of symmetry: a thing is symmetrical if one can subject it to certain operation and it appears exactly the same after the operation. For instance, if we look at a vase that is left-and-right symmetrical, then turn it 180° around the vertical axis, it looks the same.”

In the last sentence of the above quote, reflection or bilateral symmetry (a mirror plane) is casually being mixed up with 180° rotational symmetry (a 2-fold rotation axis). If Feynman’s vase is left-and-right symmetrical, it has a mirror plane; however, it is not guaranteed to look the same after 180° rotation. The only way to establish that the vase has a mirror plane is to carry out a reflection about the imaginary mirror plane in one’s mind, and not a 180° rotation about an axis. Consider a chair as an example (Figure 4). It does have a vertical mirror plane dividing it into symmetric left and right halves. Still it cannot be rotated about any axis by 180° to bring it into self-coincidence.

By the way, the work of Hermann Weyl that Feynman refers to is a collection of four lectures delivered on the concept of symmetry on the eve of his retirement from Institute of Advanced Study, Princeton. This series of lectures, now available as a book entitled *Symmetry* is a highly readable introduction to the concept [3]. Weyl devotes the very first lecture entirely to bilateral symmetry. The second lecture discusses translational and rotational symmetry. He concludes the series of lectures by the final one on symmetry of crystals.
5. ERROR 3: Ignoring the Absence of a Translational Symmetry in a Crystal

The following is a quote from Chapter 30 entitled ‘The Internal Geometry of crystals’ in Vol 2 of FLP (Section 30-4).

“We would like to describe first the simplest lattices, which are characteristic of most of the metals and of the solids formed of the inert gases. They are cubic lattices which can occur in two forms: the body-centered cubic, shown in Fig. 30-4 (a), and the face-centered cubic shown in Fig.30-4 (b). The drawings show, of course, only one cube of the lattice; you are to imagine that the pattern is repeated indefinitely in three dimensions. Also, to make the drawing clearer, only the “centres” of the atoms are shown. In an actual crystal, the atoms are more like spheres in contact with each other. The dark and light spheres in the drawings may, in general, stand for different kinds of atoms or may be the same kind.”

Fig.30-4 of FLP referred to in the above quote is redrawn below as Figure 5.

In Figure 5a the drawing shows a cube with dark spheres at corners and a bright one at the centre of a cube. For the lattice generated by translations of this unit cell to be body-centred cubic (bcc) the atom at the centre of the cube must (and not may as claimed in FLP) be of the same kind as that at the corners. If they are different, the translational symmetry from corner to body centre is lost. This translational symmetry is a required symmetry for the lattice to be bcc. In case of different atoms occupying the centre and corners of the unit cell (say A at the corner and B at the body centre) the crystal can be described as a simple cubic lattice with a two-atom motif. Crystallographers refer to such structures as type B2 or CsCl type.
A more complete description of motif should include the positions of atoms in the motif as fractional coordinates. The position of atoms in a motif is given as their displacement coordinates with respect to the lattice point along the three crystallographic axes as fractions of the respective lattice parameters (or unit cell edge lengths). Thus the motif of CsCl crystal can be more accurately described as Cl at 000 and Cs at $\frac{1}{2} \frac{1}{2} \frac{1}{2}$.

It is interesting here to consider this error in a bit more detail. It can be thought that the error arises from a failure to distinguish between a lattice and a crystal. In a $B2$ structure ($Figure\ 5a$) the atoms at the corners are different from the one at the body centre of the cubic cell. Hence they do not form a lattice. Thus only the corners (or equivalently, only the body centres) can be selected as lattice points. This makes the lattice simple cubic. However, to get the crystal structure we now have to associate two atoms with each lattice point: one at the corner 000 and the other at the body centre $\frac{1}{2} \frac{1}{2} \frac{1}{2}$. This pair of atoms constitutes the motif of the crystal.

A similar argument holds for Fig. 30-4 (b) of $FLP$ ($Figure\ 5b$ of this article) which shows a cubic unit cell with dark atoms at eight corners and light ones at the six face centres. If the cell has to generate a crystal with a face-centred cubic (fcc) lattice, the corner and face centre atoms must be the same and not may be. If they are not the same, we get a crystal structure with a simple cubic lattice and a motif consisting of four atoms. Crystallographers call such structures $L1_2$ or AuCu$_3$.

A crystal having a cubic unit cell with one kind of atom at the corners and another kind at face-centres does not have a face-centred cubic lattice. It is described as a crystal with simple cubic lattice and a motif consisting of four atoms. Crystallographers call such structures $L1_2$ or AuCu$_3$.
Failure to distinguish between a lattice and a crystal is rather widespread in textbooks and on the web. Let us consider another example. There are only three cubic lattices: simple, body-centred and face-centred. Obviously, there is no such thing as a diamond cubic lattice. Instead, one should use the phrase diamond cubic structure. This structure has a face-centred cubic lattice with two-atom motif with fractional coordinates 000 and \( \frac{1}{4} \frac{1}{4} \frac{1}{4} \). However, the incorrect phrase diamond cubic lattice is rather common. A google search for the exact phrase diamond cubic lattice (by putting the phrase in quotes) gave me 6520 results in 0.37 seconds on my computer! Similar is the case with the mistaken phrase hexagonal close-packed (hcp) lattice. There is no such lattice, the only hexagonal lattice being ‘simple hexagonal’. Hexagonal close-packed structure can be described as a simple hexagonal lattice with two-atom motif 000 and \( \frac{1}{3} \frac{1}{3} \frac{1}{2} \).

The present article alerts students to avoid pitfalls described above. Also, it warns them to critically examine statements in their textbooks even if the author is as famous as Feynman.

**Note:** In the digital edition of FLP at the official website (www.feynmanlectures.caltech.edu) Error 1 has now been corrected by redrawing Fig.1-4 as suggested in this article. An attempt has also been made in the web edition to fix Error 2. But this has been achieved by reducing the 3D ‘vase’ of the original FLP to a 2D ‘silhouette of a vase’ in the revised digital edition. In my opinion this is not satisfactory. It would have probably been better to change the phrase ‘turn it 180° around the vertical axis’ to ‘reflect it in a vertical mirror plane’ to preserve the three-dimensionality. Error 3 remains uncorrected even in the latest digital version. It could be corrected simply by dropping the unnecessary last sentence of the paragraph, “The dark and light spheres in the drawings may, in general, stand for different kinds of atoms or may be the same kind ...” and changing the figure to one having all atoms of the same colour.
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