The Conception of Photons – Part II

Bose's Derivation, and the Complete Quantum Description of Light

Urjit A Yajnik

In this second part of the article, we present how S N Bose's 1924 paper provided a systematic derivation of Planck's formula using the conception of photons, filling the major lacuna that was preventing the acceptance of the photon concept by the physics community. This derivation further widened the chasm between classical conceptions and the actual behaviour of the microscopic world as already heralded by the photon proposal. In particular, the very concept of a quantum as an 'independent' entity even when not interacting with other entities was rendered invalid. Classical intuition was subverted in Bose's derivation by a new rule, regarding counting of independent states of the system rather than counting individual quanta. We discuss the implications of the quantum mechanics that eventually emerged, showing that the seeds of some of its uncanny conceptual content were already foreshadowed in Einstein's proposal. While he was instrumental in setting off the revolution, the full implications of the revolution became unpalatable to him. We may expect that as experiments make the quantum world more familiar, the currently projected enigmas will gradually disappear.

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Part 1: Planck, Einstein, and Key Events in the Early History, *Resonance*, pp.1085–1110, 2015.

Keywords
Photons, light quanta, birth of the quantum, Planck spectrum, photoelectric effect, quantum indistinguishability, Bose–Einstein statistics, Glauber–Sudarshan representation, principle of linear superposition.

1. Towards the Birth of the Quantum

1.1 *Seeds of the Dreaded Rules of the Quantum*?

In the first part, we saw how Einstein arrived at the famous formula applicable to photoelectric effect,

$$\mathrm{KE}_{\mathrm{max}} = h\nu - W \,. \tag{1}$$

According to him, once light in this setting was understood to behave like packets of energy, the above formula was simply an energy conservation formula. The proposal that the energy of the light quantum should be proportional to its frequency ran against the grain of Maxwell's electromagnetism, where light could be shown to be a phenomenon similar to waves in any medium.

But the surprise of this proposal is not restricted to this little paradox. The import of the ideas we have now covered – and presumably accepted by you, dear reader, as valid – is truly stupendous. Sometimes, one wonders whether Einstein fully grasped the extent of damage his proposal was doing to some of the well-established classical notions. Let us assume, as clearly articulated by him, that the emitted light was going to proceed without spreading out, as an undivided packet of energy in a specific direction. Considering that the emitting body was a point-like object such as an atom or a molecule, one is immediately faced with the question: 'Which' direction will the emitted quantum proceed in? Even if the emitting body had a size, it could well be very simple, say the hydrogen atom, which can be presumed to be spherically symmetric. Then, simple classical reasoning would suggest that the radiation should emerge as a spherical wave respecting the symmetry of the emitter. But according to the photon hypothesis, the emission process must choose a preferred direction of emission.

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What fundamental principles govern the choice of this direction are not spelt out by the new stipulation. Yet, with a century of experience of quantum mechanics, we know that this is indeed how the emission of a photon occurs and in a sense typifies the nature of all quantum processes. Only under repeated identical observations can we establish the overall isotropy of the emission phenomenon, while in an individual event, the symmetry will not be respected. We may also cite another example of the often studied 'particle in a box'. Consider an electron confined within a box but with no other interactions. From quantum mechanics, we find that its location is not evenly distributed within the box. Depending upon the state it is in, it will be found preferentially at select locations, violating the homogeneity of the container. However, observations of a large number of such boxes will indeed restore the homogeneity of locations. To repeat, the earliest hypothesis made by Einstein already encodes the principle now used by all practitioners of quantum mechanics, viz., isolated quantum processes have to occur with one specific eigenvalue of the concerned observable revealed in a given experiment.

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Thus, in a sense, the seeds of the dreaded quantum mechanics were already sown in Einstein's original proposal when he generalised Planck's law originally proposed for a radiation gas, to individual events of emission and absorption. But an equally drastic phenomenon of Nature had not yet been articulated, and it awaited the correct Boltzmann ensemble of photons as conceived by S N Bose two decades later. And this phenomenon is the intrinsic indistinguishability of quanta which makes us realise that quanta are not at all 'particles' such as billiard balls we are familiar with, but profoundly novel entities.

1.2 *Opportunism of Theorists*

We may view the bold attempts of both Planck and Einstein as an opportunism of sorts, the readiness to jump into the unknown, abandoning the comfortable territory, for the possibility of obtaining a correct answer. As we noted earlier, Planck later came to consider his effort as "an act of desperation".

Einstein on the other hand, faced a stigma. While he

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became famous for his special relativity, the famous relationship between rest mass and energy etc., he was under pressure from senior colleagues to retract his radical ideas about the discrete nature of electromagnetic phenomena. Specifically, it was creating difficulty in getting him elevated as a Fellow of the Prussian Academy. Despite being nominated several times, the committee examining his case seemed to choke at this particular paper. It is reported that in a subsequent nomination his proponents even attempted an apology on his behalf – something to the effect that occasionally in his eagerness to explicate very difficult phenomena he was led to rather radical proposals, but this need not be held against him.

After this went on for several years, he was forced to issue some public clarification about his rather radical and unsavoury paper at the Solvay conference in 1911. But he stood up to the stalwarts, asserting that "I must insist on the validity of the new concept at least within the domain of phenomena for which it furnishes an explanation." His statement made in German was however so carefully worded that it was interpreted as him having reservations about this concept, at least in the English-speaking world. In fact, Robert Millikan who confirmed the photoelectric effect experimentally in his 1916 paper, refers to Einstein's quantum proposal as "bold, not to say reckless", considers it to have been "generally abandoned", and in his conclusions states that the proposal "... is found so untenable that Einstein himself, I believe, no longer holds to it".

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By 1908, Einstein became preoccupied with general theory of relativity and seems to have not returned to the question of light quanta. It is notable that he also proposed an explanation for the behaviour of specific heats of solids based on quantum vibrations in 1907 which again met instant success as a general idea though not quite correct in detail. But his photoelectric effect explanation was shunned by all experts. This caused difficulty in his becoming a member of the Prussian Academy, and since membership of the national academy is a natural step towards the Nobel Prize, also a delay in his getting that coveted Prize. Einstein was quite a celebrity based on his special relativity and was becoming the next genius after Isaac Newton with his formulation in 1915 of the general theory of relativity. But the stupendous intellectual achievement of special relativity did not meet the criteria of new phenomenological content required by the Nobel Committee, while the discovery of phenomena that would conclusively establish general relativity remained far in the future.

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In 1914, Robert Millikan confirmed the formula proposed by Einstein, yet nobody including Millikan seemed to believe the conceptual basis of the formula. Thus it was with much struggle that the well-wishers of Einstein and no doubt well-wishers of the subject of physics managed to convince the Nobel Committee to award the 1921 Prize to Einstein for his discovery of the photoelectric formula. And thus the Prize was awarded to him, taking care to state in the citation that it was "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect". Note that it was not for the correct conceptual basis or theoretical explanation of the effect, it was merely for the correct 'discovery' of the law, the prediction of the equation verified by Millikan.

While talking about opportunism, let us jump ahead a little and refer to Section 2.2 where we discuss the core new concept underlying the contribution of S N Bose. It is common to note in critical assessments that this derivation is technical, brief and while it proves the formula, does not sufficiently explicate the new assumptions involved. One has to note, however, that while Einstein was bold enough to move ahead to make a new prediction, he made no attempt to explain the additional

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$-e^{-h\nu/kT}$ term in the denominator (refer to equation (2) on p.57). While Planck gleaned the formula, and Einstein could grasp the quantum nature of the phenomenon, it was Bose who for the first time clearly derived the whole expression from Boltzmann's ensembles, also incorporating a revolutionary counting for photons.

1.3 *A Curious Case of Inadequate Diffusion of Scientific Knowledge*

It is important at this point to note a few ironical quirks of history and personalities. Einstein, in this paper of 1905, is somewhat circumspect of the methods used by Boltzmann. He does use the microscopic picture of entropy as proposed by Boltzmann. But he carefully avoids using the method of ensembles. For one, he seems to think that listing all the members of an ensemble – all the possible states a system can possibly attain, consistent with energy conservation – is still no guarantee that one has enumerated all possible dynamical effects which occur when a system is undergoing time evolution. This scepticism is similar to what has come to be called the question of ergodicity, but the way Einstein states his objection it seems to be even stronger than the question of ergodicity. Secondly, we are told by the editor John Stachel of the collection of Einstein's papers during that "miraculous year", that there were more reasons for which Boltzmann's contemporaries did not agree with him in detail though many agreed in principle. And this was because Boltzmann had a verbose writing style and the definitions of the concepts he would propose were not sharply defined and would seem to change even within the course of the same long essay. As we now know, there were also a few errors of normalisation in his formulae.

All the issues associated with statistical mechanics had been adequately addressed by Josiah Willard Gibbs in USA by the turn of the 1900's. Had Einstein accessed that treatise, his doubts would probably have been addressed and he would have proceeded to give a detailed derivation of the Planck formula starting from his fundamental conception of photons, using the techniques of Boltzmann, the same way the latter had provided a microscopic explanation of classical thermodynamics. But this was not to be. Probably because Einstein did not read English back then and also perhaps because the centre of gravity of science and intellectual discourse was Europe and Gibbs's treatise was slow in gaining acceptance there. We may then summarise the impasse in the progress towards full understanding of the Planck formula on two ironical circumstances: Albert Einstein firmly believed in photons but would not produce a proof using statistical mechanics, while the rest of the world refused to believe in photons but certainly had many experts who knew the latest reliable methods in statistical mechanics but who probably did not bother to apply them to a gas of photons.

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1.4 *Confirmation from Far Away, Far Later*

It thus fell upon Satyendra Nath Bose, a professor in Dhaka (or Dakka) University in 1924 to produce the required proof. Bose as a younger man venerated Albert Einstein, and being far away from the European crucible of science was perhaps immune to the prejudice prevailing against the notion of photons. Further, as a brilliant scholar, he had no doubt mastered the methods of statistical mechanics, again without too much prejudice because being from colonial India he had ease of access to the English source material, probably including Gibb's treatise. Thus, it was that he set about ascribing a discretised character to the phase space of radiation. In mechanics, where both positions and momenta of particles need to be considered as independent variables, 'phase space' refers to the abstract space labelled by this combined set of coordinates. He made the assumption that in line with quantum principles,

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the phase space needs to be divided into discrete cells of size h^3, a quantity whose unit dimensions match those of a volume element in phase space. To this author's knowledge, this was also the first calculation of density of states for quantised bosons. The previous calculations had introduced frequency-dependent volume factors in phase space within the wave picture. Bose's partitioning of the phase space is what we now call 'box normalisation', and it is clear from his paper that this was conceptually very important to Bose as the full package of the quantum hypothesis.

Bose then proceeded to list the possible states of the ensemble of photons and inadvertently distributed the photons in available phase space boxes without any discrimination among them. He then applied Boltzmann's method to identify the equilibrium distribution which would dominate. It yielded exactly the formula due to Planck.

It is not possible to go into the details of S N Bose's all too brief but paradigm-setting paper. But he had the full answer. There was some imagination and then there was precise logic and a computation. Neither desperate nor opportunistic, this derivation had the entire formula of Planck proved from first principles of statistical mechanics and the conception of radiation as photons. It is said that he sent his paper in English first to the *Philosophical Magazine* in 1923. But it was rejected. He then sent his paper to Einstein addressing him as "Respected Master". Einstein immediately grasped the significance of this paper. This was the calculation he had sought for during the previous two decades. He translated it and communicated it to *Zeitschrift für Physik*. He then proceeded as a follow-up to work out the consequences of the new method of calculation applied to massive particles. The combined general formulation is called the Bose–Einstein statistics to distinguish it from the classical Maxwell–Boltzmann statistics.

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Bose's one-off contribution has evoked puzzlement and its far-reaching implications also perhaps jealousy. True, he did not have a consistent output of scientific contributions within that subject area like a European scientist. Being sensitive to the condition of his country, he shifted his attention to practical problems of semiconductor devices. But nobody denies the brilliance and scholarship of Bose. It appears that he himself did not grasp the novel assumption he had inadvertently made in his derivation. While the discrete partitioning of the phase space is an important step, the success of the derivation relies on an additional crucial assumption. If we read the wording of how Planck finally convinced himself of his derivation in the year 1900, we see a parallel. Planck was thinking of energy as a generic quantity to be distributed among those oscillators in the walls of the cavity. And he spoke of distributing "energy units" into the available excited states of the oscillators. Of course with hindsight, we know that the oscillators were a completely unnecessary scaffolding. Bose, on the other hand, had to contend with the same energy units, now conceived as photons, themselves the objects of statistical mechanics to be handled by set rules. The scaffolding of cavity oscillators was abandoned once and for all. And he implicitly distributed photons among their own available energy levels according to the same indistinguishability approach as Planck. As we elaborate below, this is the key novel assumption, which naturally produces the denominator of equation 3 of Part I, viz.,

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$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\,, \qquad (2)$$

without any reference to any oscillators. Bose himself missed this particular fact, and there is nothing to indicate that even Einstein understood it at the time of communicating his paper. It was indeed something very very subtle. Thus, to Bose, we may attribute the credit for arriving at the "light gas" distribution formula by

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2. Quantum Mechanics

2.1 *Ideas Whose Time had Come*

Between 1905 and 1924, Einstein returned to the physics of light a few times, but the issue of validation of the photon hypothesis remained unresolved. In 1917, as the general relativity revolution was catching on, he devoted attention to radiation again, and wrote his famous insightful paper on the so-called A and B coefficients, concerning emission and absorption of light in atomic sources. These observations went on to become the underlying framework for developing the laser.

But the proof of the Planck formula still evaded Einstein. In 1921, he got his Nobel Prize. But the really eventful year for the story we are pursuing was 1924. It was in this year that Louis-Victor-Pierre-Raymond, 7th duc de Broglie submitted a thesis to the French Academy for a doctorate degree. In it, he proposed that if as per Einstein, electromagnetic waves have a particle-like character, conversely the electron must have a wave-like character. He proposed the equally preposterous formula associating a wave of wavelength λ with an electron of momentum p:

$$\lambda = \frac{h}{p}\,. \tag{3}$$

This is analogous to the relation $\lambda = c/\nu$ for electromagnetic waves if we recognise the Planck relation $E = h\nu$, and the special relativistic relation for photons, $p = E/c$. It appears that the members of the Academy were flummoxed by this hypothesis, and after some discussion sent it off outside France, to Albert Einstein himself for examination. Even Einstein must have been suitably puzzled. However, he had received the letter from Bose just a few months earlier. He had now been fully convinced of his hypothesis of waves behaving as quanta. Much to the Academy's surprise, Einstein approved de Broglie's thesis proposing that material particles behave like waves. I would now like to refer back to Section 4.1 of Part I. There we considered the possibility that the reason why Einstein could withstand the pressure from the stalwarts to withdraw his light quantum paper was perhaps the fact that the unity of the core concepts for matter and radiation was more important to him than reconciling the two alternative descriptions of radiation. The reason why he would readily accept the de Broglie hypothesis can be ascribed to this line of thinking. Until specific writings or records can be uncovered to support this, it is a matter of conjecture whether Einstein's ready acceptance of de Broglie's thesis had anything to do with him having seen a closure to his 1905 hypothesis in the paper of Bose.

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de Broglie had an illustrious career as a philosopher scientist. To begin with he was a duke by inheritance. He quickly became a member of the French Academy. His thesis of 1924 proposed that electrons have waves associated with them, which he called 'pilot waves'. The waves were supposed to escort the particle, like pilot vehicles in front of the car of a dignitary. de Broglie had also conjectured that "the electron has an internal clock that constitutes part of the mechanism by which a pilot wave guides a particle"[1]. With the development of quantum mechanics, and detailed consideration of its implications, it has been recognised that there are no waves that pilot the particle. The particle description and the wave description are complementary to each other, and mutually exclusive. They are not valid simultaneously. This contradicted the original metaphysical motivations of the wave hypothesis. As such, de Broglie remained vehemently opposed to the subsequent development of his wave ideas into wave mechanics. Unlike Bose, about whose contribution questions continued to be asked,

[1] Wikipedia page on Louis de Broglie.

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To the credit of the de Broglie hypothesis is the fact that it bloomed into the landmark new mathematical formulation of quantum mechanics in the hands of Erwin Schrödinger in 1926. Equally importantly, in 1927, the results of the Davisson and Germer experiments at Bell Labs, scattering of slow electrons from crystalline nickel target, matched de Broglie's wavelength formula remarkably. As for the development of quantum mechanics, Heisenberg was the first in making the radical proposal that one must abandon the notion of a trajectory in quantum mechanics, and went on to propose the principles of matrix mechanics. In 1925, this version of the theory was difficult to digest by many, as matrices were foreign to physicists, and the palpable picture that waves offered, and in terms of which Schrödinger's 1926 theory was formulated, rapidly gained acceptance. Although both formulations are equivalent, the wave formulation holds sway in most of non-relativistic problems of quantum mechanics. This is somewhat unfortunate as there are no 'waves' in the ordinary sense of waves in water or strings, but only a method to implement the principle of linear superposition as we explain later.

2.2 *States and Quanta: The Essence of Quantum Physics*

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We finally explain the core new conceptualisation of Nature that the Bose–Einstein statistics offers to us. The novel counting that enabled Bose to arrive correctly at the Planck–Einstein formula was that in his counting, the states containing several quanta received equal weightage regardless of how they were assembled from states of single quanta. To understand this, we consider the example of two coins. Suppose we have two identical coins. And we toss them both independently. Now we try to anticipate the number of times the various possible configurations will show up. There are only

	Classical	$\text{Quantum}_{\text{B–E}}$	$\text{Quantum}_{\text{F–D}}$
H H	1/4	1/3	0
HT or TH	1/2	1/3	1
TT	1/4	1/3	0

Table 1. The weightage factors associated with possible states of two indistinguishable coins in classical, Bose–Einstein and Fermi–Dirac statistics.

three possibilities: both heads, both tails, and a third possibility, one head and one tail. We may list these as HH, TT and HT. Since the coins are indistinguishable, TH is the same as HT and they count as the same configuration. However, we know very well from classical experience that out of the total number of possible configurations, the HT (or TH) occurs in two different ways, and hence twice as often compared to each of the HH and TT configurations. The weightage we associate in Boltzmann statistics with the states of such a system are indicated in the second column of *Table* 1 under the heading 'Classical'.

However, the Bose–Einstein case is different in a very subtle way. In the quantum Bose–Einstein counting, the coins are so completely identical that we are not able to assign twice as much weightage to the HT situation. It has exactly the same weightage as the HH and TT situations. This is indicated in the table in the column with the heading $Q_{\text{B}-\text{E}}$. This kind of counting applies to particles of spin values which are integer multiples of $\hbar$, i.e., 0, $\hbar$, $2\hbar$, etc. For completeness, we have included the last column which corresponds to Fermi–Dirac statistics obeyed by all particles of half-integral spin, i.e., values $\hbar/2$, $3\hbar/2$, etc., examples of which are the electron, the proton and the neutron. If such a species has no quantum numbers other than the one distinguishing H from T, then the Pauli exclusion principle forbids HH as well TT. There is only one state admissible as per quantum principles – HT, with weightage unity!

This state of affairs is called quantum 'indistinguishability' versus classical indistinguishability. But as we

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shall argue, the label of 'indistinguishability' is predicated on a classical prejudice. And this has resulted in enduring confusion and also false hopes of somehow circumventing the unpalatable non-classical content of quantum mechanics! The suggestion in the adjective 'indistinguishable' is that there are two distinct entities to begin with. The quantum logic is however taciturn, and less revealing of its secrets. Let us assign a value $+1$ to H and -1 to T in some units. Now the quantum logic allows an observable called the 'number', and the value of this number is 2 in this example, as we have considered two quanta. However, this system has only one unique state corresponding to total value 0 of the H/T quantum number. The availability of the observable 'number' whose value is 2, should not be confused with there being 'two particles' in the classical sense. So the question of 'distinguishing' between them does not arise. There is only *one* quantum state containing two particles, the H/T quantum number of the state being zero, and the weightage of this state is exactly the same as that of the other states which have H/T quantum number $+2$ or -2.

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At the heart of quantum mechanics is the principle of linear superposition. What we consider classically to be distinct configurations can be 'added' in a precise mathematical sense in quantum mechanics. And the weightages of the superposed states have to be such that the sum of their squares must add up to unity. As we descend from the macroscopic level to the microscopic, there are two ways that the quantum rules set in. One is as the systems become simple, such as small molecules and atoms, systems which are described by a small number of observables. The other important way quantum principles manifest themselves is through the Bose–Einstein or Fermi–Dirac enumeration of states. But the transition to the fully quantum domain most often is not sharply defined. For example, at standard room temperature and pressure, the molecules of hydrogen or carbon dioxide gas obey quantum rules of emission and absorption of radiation, but as a collective system, they obey Maxwell–Boltzmann or classical statistics. What this means is that they behave as independent quantum systems. For each molecule, its internal states would be a superposition of its various standard states (in technical language, eigenstates). However, the collective state is not found to be a superposition of some standard states, but rather just like the states of a small macroscopic particle. This happens because the gas is very dilute, viz., the average separation between the molecules is about 50 times larger than their intrinsic size. Typically, the collective states of a system display quantum mechanical superposition only when the system is densely populated with quanta of a given species.

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In the domain where the quantum rules apply, the counting of the possible states becomes different and defies classical common sense. For a variety of systems, even when dense, an approximate picture which allows thinking in terms of the original isolated quanta works, especially when the quanta have weak mutual interaction. In such situations, one constructs the general states of the system as products of single particle states. This is purely a mathematical convenience. Unfortunately, this leaves behind the feeling that two distinct quanta have been put together, even though the symmetrisation or anti-symmetrisation are applied correctly. Quite a few paradoxes arise simply from this naive thinking. The 'indistinguishability' of quantum systems, more correctly, the appropriate statistics has to be treated as an integral part of the quantum principles and not as an added rule. When this is done consistently, no paradoxes remain, though the rules may continue to intrigue us.

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In the treatment given by Sudarshan, it was emphasised that the quantum mechanical formula given there accounts for all the possible states of light, which subsume the classical states.

3. Conclusion and Outlook

3.1 *Final Story of Light*

The novel description of light that began with Planck's formula and properly recognised as quantum behaviour of light by Einstein, reached maturity with the development of Bose–Einstein statistics. While much of the attention got diverted to condensed matter and nuclear physics, developments in optics continued separately. One of these was the inelastic scattering of light due to the internal structure of molecules and crystals. This effect, discovered by C V Raman, earned him the 1930 Nobel Prize. While quantum electrodynamics, as a dynamical theory of photons and electrons led to profound developments, the physics of photons by themselves had entered the quantum era in the 1950's when the maser was developed, soon leading to the invention of the laser.

In the late 1950's, in the course of using photomultiplier tubes for the study of stars, Hanbury Brown and Twiss developed intensity interferometry, whose quantum principles at first seemed to be unclear. By 1963, Glauber and independently E C G Sudarshan explicated the formalism that applied to the quantised Maxwell field in all possible settings. Glauber received the Nobel Prize for this development in 2005. In the treatment given by Sudarshan, it was emphasised that the quantum mechanical formula given there accounts for all the possible states of light, which subsume the classical states. Put another way, what we usually think of as classical Maxwell waves is actually a state of the quantised Maxwell field, a special state of the photons. Here, the classical description applies exactly and no modifications are needed when quantum mechanics is taken into account. This formulation of Sudarshan can be considered to be the final closure on the theories of light originating with Newton and Huygens, evolved through the historical path of Planck, Einstein and Bose.

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It is intriguing to note that photons have two very special properties, one is zero rest mass and the second is zero charge (or the absence of mutual interactions). These properties have provided us entries, respectively, into the realms of special relativity and quantum mechanics. The zero mass property means that they are always moving at the largest limiting speed permissible in Nature, 'the speed of light', and this property has thus provided us a key to special relativity. On the other hand, zero mass and zero charge properties have facilitated the observation of the peculiar properties of a quantum gas that we are discussing here. Masslessness means that there is no intrinsic 'size' to a photon such as the Compton wavelength for massive particles. Thus, there is no limiting dilution in which this gas begins to obey Maxwell–Boltzmann statistics, unlike in the case of molecular gases as we noted in Section 2.2. And the absence of interactions ensures that it remains a gas of free quanta to which Bose–Einstein statistics can be applied in all laboratory situations. Massive particles such as atoms also display quantum superposition and enter a collective state called a Bose–Einstein condensate. But to see this, we need to prepare their collections with extreme care. Providing sufficient density may trigger interactions; instead, extremely low temperatures are used. For photons, the quantum characteristics are readily visible because they constitute a non-interacting quantum gas, which enabled the revolution in the hands of Planck.

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The theorem of Sudarshan shows that photons provide one more access to the quantum world. Dirac has emphasised in his textbook *Principles of Quantum Mechanics* that the new content of quantum mechanics is the principle of linear superposition. In classical mechanics, there is no meaning to a plus sign between two possible trajectories of a particle. It cannot be following both. In quantum mechanics, it is valid to superpose

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via a plus sign two states of the system which yields a new possible state of the system. The above theorem of Sudarshan then has another intriguing implication. Recall that the classical states of radiation appear without modification in the complete quantum description. As such, the linear superposition principle of electromagnetic fields that is taught at the undergraduate level is actually nothing but the linear superposition principle of quantum mechanics!

3.2 *The Enigmatic Quantum*

Very soon, after the basic rules of the new quantum mechanics were understood, it became apparent that the outcomes of experiments could be predicted only statistically or on the average. Heisenberg had proposed his matrix mechanics with a clear call that the notion of trajectories must be abandoned. He then backed up his abstract formalism operationally through a thought experiment, by showing that attempts to measure one property of the trajectory, say the position, would necessarily mess up the complementary property, the momentum. This consequence was natural because the measuring technique itself had to rely on sending one quantum system, a photon, to 'view' another, the electron. It was impossible to improvise any apparatus that was capable of yielding information of the quantum domain without, at the same time, obeying quantum principles. Ergo, it was impossible to beat the uncertainty in measuring the attributes of a trajectory below a limit set by the new constant of nature, h, or in modern usage, the quantity $\hbar = h/2\pi$.

There was the puzzling property that a quantum state was intrinsically non-local; the wave function was always spread over a space-like domain. This seemed to intuitively contradict special theory of relativity.

Such a probabilistic outcome given by a fundamental framework was anathema to Einstein and to many others of that generation. Needless to say, the debates continue to rage and are also current. Further, there was the puzzling property that a quantum state was intrinsically non-local; the wave function was always spread over a space-like domain. This seemed to intuitively contradict special theory of relativity. The enigma of this situation was formulated by Einstein and his collaborators Podolsky and Rosen with characteristic clarity and has come to be called the EPR paradox. From a pragmatic point of view, paradoxical the situation is, but inconsistent it is not; and no attempts at arriving at an inconsistency with the basic tenets of special relativity, even in thought experiment, have succeeded, nor has a clever experiment been designed that would force an extension of quantum mechanics.

In any one specific measurement that manages to capture only one electron, the answer will be precisely either $+\hbar/2$ or $-\hbar/2$.

The other puzzling aspect of quantum observation is that only specific eigenvalues are returned as the outcome of measurement. For instance, the average spin of an electron in a beam may be $0.35\hbar$, but that only means that if you made measurements on many electrons in that beam, that would be the average outcome. In any one specific measurement that manages to capture only one electron, the answer will be precisely either $+\hbar/2$ or $-\hbar/2$.

This fact has been well verified. But it leads to the following paradoxical situation called Wigner's Friend or Schrödinger Cat, depending on how amicable or macabre your inclination is[2]. Once measured, the system will go on being in that eigenstate, say spin $+\hbar/2$. But now, if you sit quietly after that measurement, your friend who walks in has no way to decide whether it is already in an eigenstate or not without actually making the measurement herself. The dilemma at hand can be seen to result from the rule of quantum mechanics, that once an attribute is measured, the net effect of the measurement process is to leave the system in one of the eigenstates of the particular observable. But this rule creates an unequal situation for different categories of observers – those who first carried out such an observation on a generically prepared system, and those that come later, without knowing whether the measurement has already

[2] Actually Wigner's Friend is a paradox which is a step beyond the more direct paradox presented by Schrödinger's Cat. Due to the brevity of the presentation here, and presuming that many readers are already familiar with these paradoxes, I have taken the liberty to speak of them together.

The dilemma at hand can be seen to result from the rule of quantum mechanics.

been made. Thus, the well-established notion of objectivity even in the classical world of observers seems to be endangered.

Nobody wants to kill a cat ever, let alone twice, so such a paradox jumps out to challenge common sense. But the resolution is very simple. If the first observer has already made the measurement, then the system can be considered to have been prepared in that specific eigenstate for the next observer. No contradictions arise but the challenge to common sense persists. Also, one hopes that a refinement of the formalism would make measurement a more intrinsic and organic part of the formalism than a drastic 'collapse' into specific eigenstates. Constructing such a formalism is an active area of research. But we would expect that such a formalism will only be an extension, without modifying any of the core tenets of quantum mechanics.

Since the quantum is maligned so much in common discourse, it is worth emphasising that there is a lot that is counter-intuitive in physics. As we know, understanding the phenomenology of classical motion was itself a great intellectual enterprise, culminating with the discourses published by Galileo. Its final refined version which we accept with equanimity, is due to Newton. Yet, there is much that is conceptually unsatisfactory about the Newtonian framework which we have come to take for granted. The foremost among them is the notion of limits as needed for infinitesimal calculus. Through the formal concept of instantaneous velocity, we are convinced by Newton that a particle can be at a point and also moving while still being at that point! In fact, it is supposed to possess all orders of time derivatives while still just being at its original point. In the bygone era of theology, this would have remained an active area of debate but not so in modern engineering. While the high level of refinements in real analysis ensure that there is no logical contradiction, the point remains that this is a mind game. Operationally, it is impossible to make your stopwatch measure vanishingly small durations. Indeed, now we know that quantum mechanics will kick in and will show that the Newtonian process of a limit is a figment of our imagination.

It is time we accepted that our intuition is based on the cognitive faculties tuned to the classical experience. And that physical science requires a kind of sophistication that may yield counter-intuitive theorems. And some of the puzzles will fade from common discourse, much as the theology of yesteryears, as highschool students begin to interact with quantum systems and the quantum framework earlier in their physics syllabus.

Suggested Reading

[1] **Wikipedia articles for various specifics of dates, personalities and experiments.**

[2] ***Einstein's Miraculous Year: Five Papers that Changed the Face of Physics*, edited and introduced by John J Stachel, Princeton University Press, 1998.**

[3] **Abraham Pais, *Subtle is the Lord: The Science and the Life of Albert Einstein*, Oxford University Press, 1982.**

[4] **Abraham Pais, *Inward Bound: Of Matter and Forces in the Physical World*, Oxford University Press, 1988.**

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