Life and Times of Bourbaki

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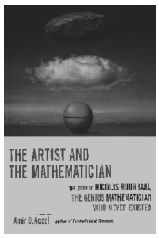

The Artist and the Mathematician – The Story of Nicolas Bourbaki, the Genius Mathematician Who Never Existed,
Amir D Aczel
High Stakes Publishing, London
Price: ₹1475/-, Pages: 272, 2007.

The author of the book under review, Amir Aczel, presents a thesis about the relation between developments in Mathematics and the general Culture surrounding it. He attempts to illustrate this taking the example of Bourbaki's work and the extraordinary developments in Algebraic Geometry led by Alexander Grothendieck on the one hand, and creation of Modern Art by Pablo Picasso / Georges Braque and the inauguration of structuralism by the Anthropologist, Claude Levi–Strauss with its effects on Linguistics, on the other hand. This reviewer, with his very limited knowledge of the fields of Art / Linguistics / Anthropology (and Mathematics), primarily read the book for the story of the Bourbaki phenomenon and the stories about the creators of Bourbaki. Thus, this review serves only to bring this very readable book to the notice of those interested in the history of intellectual endeavours.

Forest fires are quite devastating and for a while it looks as if life had no chance against them. But then, they also signal the beginning of the next generation of huge trees to grow in the wild. A similar thing happens during a long drawn out war, the two World Wars, for example. Thousands of young lives are lost, life is thrown out of gear, school / colleges get disrupted and the future looks bleak. But the indomitable spirit of Life will be striving to re-establish itself and, in the process, creates many a phenomenon. It seems as if a severe jolt is necessary to shake up the established norms and relay the paths as per the requirements of newer generations.

Bourbaki may perhaps be one such phenomenon conceived during the after-years of WW-I and nurtured through WW-II resulting in an extraordinary resurgence of mathematics in France in the years following it. Certainly, there are such examples in other spheres of human endeavour and their value in reestablishing humanity across the borders created by the War is immense.

The geographical setting for the book is, mainly, France. During WW-I, France had lost most of its intellingentsia and academia. About half of the graduating classes of years 1910–16 had perished in the war. This resulted in a dearth of young and enthusiastic teachers for the post WW-I generation of students at all levels. The creators of Bourbaki, Henri Cartan (1904–2008), Claude Chevalley (1909–84), Jean Delsarte (1903–68), Jean Dieudonne (1906–92), Rene de Possel (1905–74) and Andre Weil (1906–98) grew up and had their schooling during the War. They graduated in an atmosphere of uncertain and an absence of a challenging academic environment, and started teaching in the 1930s.

Andre Weil and Henri Cartan both studied in ENS and started teaching mathematics at the University of Strasbourg in 1933. The curriculum needed revamping, books were inadequate and direction / guidance for research was lacking. Constantly subjected to complaints and questions by Henri Cartan as to how to reorganise the curriculum, and himself trying to find ways out of this situation, Andre Weil had an inspiration. In his own words: "One winter day toward the end of 1934, I came upon a great idea that would put an end to these ceaseless interrogations by my colleague. 'We are five or six friends; I told him sometime later, 'who are in charge of the same mathematics curriculum at various universities. Let us all come together and regulate these matters once and for all; and after this, I shall be delivered of these questions.' I was unaware of the fact that Bourbaki was born at that instant."

Weil called for a meeting on December 10, 1934, in a cafe in Paris and Bourbaki as we know today was born on that day, but was christened* so during the group's congress, July 10–17, 1935, held in a small countryside village near the city of Clermont–Ferrand; Eveline de Possel, (then) wife of Rene de Possel (she would later divorce him and marry Weil), was chosen as Bourbaki's Godmother and she gave the first name of Nicolas. The group immediately proceeded to publish a note in the *Compte Rendu* of the French Academy of Sciences in the name of Nicolas Bourbaki, thus bringing his existence to the notice of the mathematical community. The rest is history. The fact that N Bourbaki was not a real person and represented a group of mathematicians was known to very few, for example, Ralph Boas, but the world of mathematics had come to believe in his existence. In an article on mathematics for the *Encyclopaedia Brittannica,* Boas revealed the truth; he was severely reprimanded in a letter to him "From my ashram in the Himalayas", beginning with "You miserable worm, how dare you say that I do not exist?" and signed 'Nicolas Bourbaki'!

Alexander Grothendieck was born in March 1928 and so he started his schooling around the time N Bourbaki came into existence. By the time he finished college, Bourbaki was a

*The name chosen for the group has a history, and an Indian connection. In 1923, a certain General in the army of Napoleon III who played a prominent part in the Franco–Prussian war of 1870–71, General Charles Denis Sauter Bourbaki, came in handy to Raoul Husson, a third-year student at ENS, Paris, when he wanted to play a prank on the first-year students. Husson wore a false beard and proceeded to give a lecture, which was made compulsory for first-year students, under the name of (an imaginary) Prof. Holmgren. The lecture concluded with 'Bourbaki's theorem'! Andre Weil, who was in the audience, appreciated the prank so much that he recounted the story to his friend D D Kosambi during his short stint in Aligarh Muslim University in 1930; he further suggested to Kosambi that he write a mathematical paper extending the (imaginary) results of Bourbaki. Kosambi did just that: in 1931–32 he published his paper '*On a generalisation of the second theorem of Bourbaki*' in the *Bulletin of the Academy of Sciences of the Agra and Audh Allahabad Provinces*! The work discusses the work of an "unknown Russian mathematician, D. Bourbaki", who was poisoned during the Revolution.

well organised group with its own constitution and whose members were among the best French mathematicians. However, the world had not yet recognised the impact of Bourbaki's contributions; it seemed as if the foundational approach to mathematics by Bourbaki was only a different point of view and might not produce much that is new. In a matter of five-six years, Grothendieck changed this perception by taking the Bourbaki approach to great heights in his relaying the foundations of algebraic geometry and, during the course, solving many deep problems in other areas of mathematics, most noticeably, arithmetic and topology. The climax, perhaps, is the proof of Ramanujan's conjecture in 1974 by Grothendieck's student Pierre Deligne.

Srinivasa Ramanujan made his conjecture in 1916 in his paper 'On certain arithmetical functions', *Transactions of the Cambridge Philosophical Society*, XXII, No.9, 1916, 159 – 184. (see paper No. 18 in http://ramanujan.sirinudi.org/html/published_papers html) and when it was made, it belonged to the domain of analytic number theory. The conjecture said that a certain arithmetical function, Ramanujan tau-function denoted by $\tau(n)$, was of the order of $n^{11/2}$. Tantalising part is that a basic course in complex function theory would enable one to show that its order is less than n^6; so analytic number theorists, notably R A Rankin, worked hard on it but could only proceed very slowly and in very small increments. After the work of E Hecke, its relation to elliptic curves became known and Weil put this matter in a general perspective through Weil conjectures. Deligne's proof of the last part of Weil conjectures, using the abstract algebraic tools created by Grothendieck, settled Ramanujan's conjecture and convinced mathematicians about the power of abstraction. Here was a problem belonging to analytic number theory, a very classical and concrete subject, and it was solved by some highly abstract algebraic machinery. This cleared away any suspicions mathematicians had about the need for the high level of abstractions which came out of the Bourbaki school. A quote from a letter of C L Siegel, considered to be one of the giants of mathematics of the 20th century, to L J Mordell in March 1964 will give an idea of the sentiments of the time: "I am afraid that mathematics will perish by the end of this century if the present trend for senseless abstraction – as I call it: theory of the empty set – cannot be blocked up."

The author seems to suggest that Bourbaki's decline coincided with Grothendieck's shift away from mainstream mathematics which happened in 1970's and subsequently 'died'. Though the publications under the name of N Bourbaki have become few and far between (the latest publication is the eighth chapter of their algebra book in 2012), there are some who believe that Bourbaki is still alive.

Another project which keeps the Bourbaki name alive is the *Seminaire Nicolas Bourbaki,* which is a series of seminars, about 12–20 per year, on contemporary mathematics started in 1948. It is considered an honour to be invited to give a seminar in this series; the only Indian

Figure 1. The house in Lasserre in which Grothendieck lived for more than the last 23 years of his life.

to figure in the seminar so far is Harish Chandra, who gave a talk in the 1957–58 series and, apparently, thus lost the chance of winning the Fields medal in 1958! (Siegel was the chairman of the Fields medal committee in the ICM 1958!)

Amir Aczel begins his book with the disappearence of Grothendieck in 1991: "In August 1991, Alexander Grothendieck, widely viewed as the most visionary mathematician of the twentieth century, ..., suddenly burned 25,000 pages of his original mathematical writings. Then, without telling a soul, he left his house and dissappeared into the Pyrenees. .... And for ten years now [as of 2006], no one has reported seeing him. ..... It seems as if AG has simply vanished off the face of earth."

Now we know that Grothendieck lived alone for more than 23 years of the last part of his life in the little village of Lasserre – with about 200 residents – and died in the hospital at Saint-Girons in south-west France on November 13, 2014, at the age of eighty-six.

The author has taken great pains to collect the information and has told the story in an interesting way. The story he has chosen to tell is rather spread out and complex, and the people who figure in the story are strong personalities with enormous contributions to the field of human thought. Naturally, the writing looks thin at many places but the book is a good place to start with for many of the topics treated in it.

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