Classroom

In this section of *Resonance,* we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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An Experiment on Projectile Motion

A simple and inexpensive experimental setup for studying projectile motion using a low-cost projectile launcher and a microcontroller-based photogate timer is described. Using this setup, all three kinds of projectile motion (horizontal, oblique – ground to ground, and oblique – from a height) are studied.

Introduction

The motion of projectiles known to mankind since the times of Archimedes is an example of two-dimensional motion. This motion occurs in a vertical plane defined by the direction of launch. In the simplest case (when air resistance is neglected and motion occurs close to the surface of earth), the projected body experiences a uniform acceleration along the vertical direction and a uniform velocity along the horizontal direction. A study on projectile motion helps in a thorough understanding of the basic concepts in kinematics like accelerated motion, uniform motion, equations of motion and so on. Though most students are exposed to an extensive theoretical treatment, hardly any experiments are done. In recent times, some experimental kits on projectile motion have become available but they are too expensive to be affordable to most schools and colleges. In this article, a simple experimental

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Projectile motion, Arduino microcontroller, photogate timers, kinematics experiment.

setup is suggested for performing a comprehensive study of projectile motion.

The setup consists of a projectile launcher made from a de-soldering pump fixed to a semicircular plate graduated with angular positions. The estimation of the velocity of the projectile and time of flight requires an accurate measurement of time. A photogate timer is built with the help of microcontroller-based open source hardware called Arduino, which is a flexible and easy-to-use hardware and software platform. This is a good example of a new technology available in the market that can be utilized to perform some basic physics experiments. The entire setup can be built in an undergraduate lab at a cost of INR 3,000. The procedure to build the photogate timer using a microcontroller is comprehensively discussed. Using this setup, all three types of projectile motion are studied in detail[1].

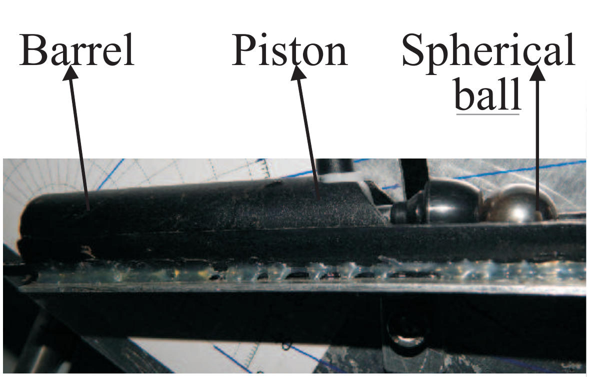
The projectile launcher is made from a de-soldering pump.

[1] All experiments were repeated at least 5 times. In cases where the spread was high, more readings were taken till the experimenter was satisfied. Typically 7 to 10 readings were taken.

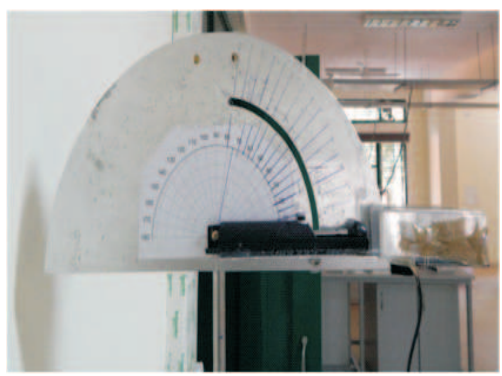
2. Experimental Setup

2.1 *Projectile Launcher*

The projectile launcher is made from a de-soldering pump that is used for removing the unwanted solder in an electronic circuit (*Figure* 1a). The O-ring attached to the piston of the pump is removed so that it moves freely in the barrel when released. The pump is attached to a semicircular board as shown in *Figure* 1b. A small spherical ball can be placed on the piston and when



(a)



(b)

Figure 1a. De-soldering pump used as a projectile launcher.

Figure 1b. Semicircular board with angular graduations to which the launcher is fixed.

released, it can launch the ball with speed. The pump acts as a projectile launcher. One end of the launcher is fixed to the center of the board and the other end is supported by an L-clamp which is free to move along the groove in the board. The launcher can be placed at different angles by rotating it and the L-clamp helps in securing it. The speed of the launch will depend on the spring constant and the mass of the ball. By choosing balls of different masses, the speed of launch can be changed. The semicircular board can be attached to a stand and its height can be adjusted. Thus, the height, angle and speed of the launch can be varied. This helps in studying the dependence of range and time of flight of the projectile on all these parameters.

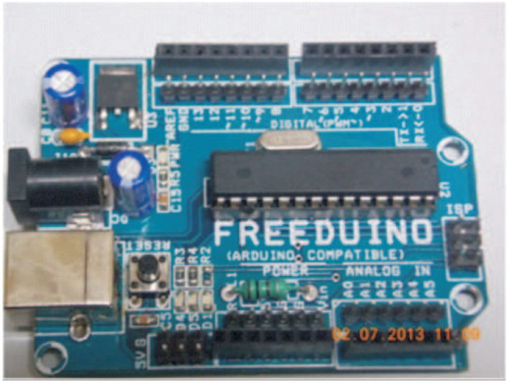
2.2 *Microcontroller (Arduino)-based Photogate Timer*

Arduino is a single-board microcontroller designed to make interfacing with hardware and software easier. Arduino is based on a free hardware and software concept, i.e., the hardware and software details are freely available and any modification to either is not only legally allowed but encouraged. The Arduino programming software is freely available for various operating systems. The Arduino board (*Figure* 2a) comes preloaded with a boot-loader which allows the board to interface directly with a computer. The board connects to the computer directly with a USB cable. When connected, the computer can be used to program the microcontroller as well as provide power to the board.

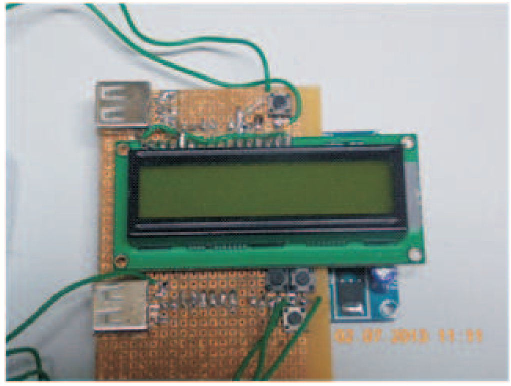
Arduino can be used to run low power devices such as LEDs and LCD-panels (*Figure* 2b) directly. The board used in this project has an Atmel 8-bit ARM microcontroller. It has 13 digital input/output (IO) pins and 6 analog pins. The analog pins act as input pins reading a voltage of up to 5V with a 10 bit resolution.

Arduino is based on a free hardware and software concept.

Because of the microcontroller used, the Arduino also provides an internal clock which counts the microseconds since the last reset. This clock has an accuracy of +0.05% and a resolution of 4 μs. This makes it ideal for use as a timer for externally triggered events.



(a)



(b)

Figure 2a. Arduino compatible single board microcontroller.

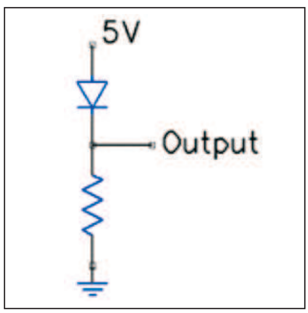
Figure 2b. LCD panel attached to Arduino.

2.3 *Photogate*

For measuring the speed of a projectile, a photogate is used. A photogate is a device which produces an output (generally a change in voltage) when an object cuts the path of a light beam in the device. It is ideal for detecting moving objects without interacting with them or changing their momentum. A photogate consists of an infrared (IR) LED-detector pair (IR is used since ambient IR noise is generally very low and it is cost effective). The LED and the photodetector are set up directly opposite to each other. When the projectile passes between the pair, the light from the LED is cut off. The beam diameter is close to 1mm, so any projectile larger than 1mm will register, when properly centered.

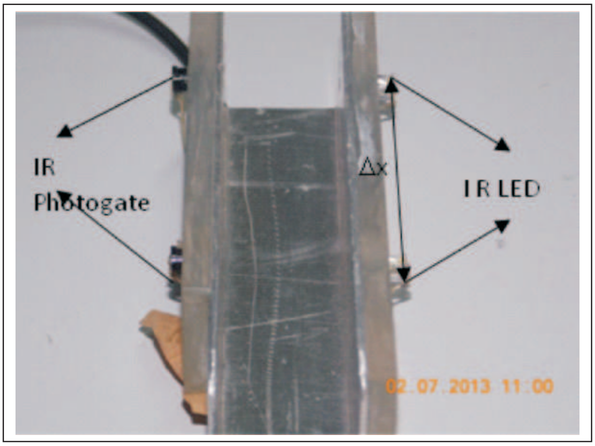
The photodetector itself consists of a photodiode in series with a large resistance (56KΩ) connected across a 5V source (*Figure* 3). The photodiode acts as a photoresistor with a large resistance when the IR light intensity is low and vice versa. In this case, the output voltage is close to 3.5V when the beam is blocked and close to 5V, otherwise. This voltage is fed to one of the analog inputs of the Arduino.

Figure 3. Photodetector.



A set of two photogates are mounted on the launcher with a separation of Δx as shown in *Figure* 4. When a projectile passes through the two photogates, the time taken to cross this distance

Figure 4. Photogate sensor attached to the launcher.



can be measured by noting the time duration (Δt) between the two triggering. The speed of the projectile is $\Delta x/\Delta t$. The entire unit is fixed in the front portion of the projectile launcher such that the projectile passes through the photogate and hence its speed could be recorded for every launch.

2.4 *Contact Sensor*

To measure time of flight of the projectile, two time sensors are required; one at the point of projection and the other when it hits the floor. The photogate, which is placed at the launcher, will itself act as the first sensor. A contact sensor pad is used for the second sensor. The contact sensor pad is basically a large plate supported on a switch as shown in *Figure* 4. When an object falls on the plate, the switch gets pressed due to the impact. The switch itself is connected between 5V and a digital input pin of the Arduino. Thus, the impact of an object changes the voltage at the input pin which can be easily detected. This method is preferred over other techniques such as piezoelectric sensors because it is sensitive enough to be triggered just by the weight of the projectile.

The contact sensor pad is basically a large plate supported on a switch.

2.5 *Switches and Ports*

The timer can also be used for other applications apart from projectile motion. To select the required application, switches are provided (*Figure* 5). By pressing the switch S_1, the timer can be used for the projectile motion experiment. In this case, the

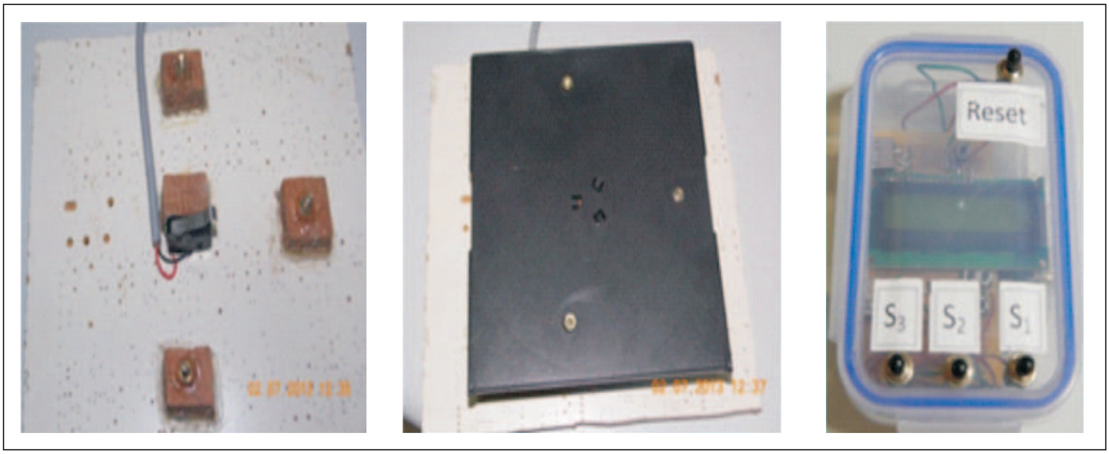


Figure 5. Contact switch mounted on a base on which a plastic pad is anchored and the Arduino setup is placed in the box with switches and ports.

velocity of launch and time of flight can be determined. Once reading is taken, the reset switch is pressed for taking the next reading. The photogates (attached to launcher and the contact sensor pad) are connected to the microcontroller through USB ports. The timer can also be used in simple pendulum and free fall experiments. In this article, only the projectile motion experiment is discussed.

2.6 *Software*

The software for this kit is written using the Arduino programming environment. The software already provides a clock function which gives the number of microseconds since the start of the microcontroller with 4 μs resolution. To time the duration of an event, the software needs to note the clock time at the start of the event and at the end of the event. The duration is simply the difference of the two values. So, to measure the speed of the projectile, the time when the first photogate is triggered is noted and then the output of the second photogate is checked in a loop (continuously, until the photogate is triggered). The time is noted again and their difference gives the duration. The same concept applies for finding the time of flight. Although the clock resolution is 4 μs, the limiting step is reading the analog voltage output of the photogates, which can only be done at 10KHz. Thus, the effective time resolution is 0.1ms, which is comparable to the commercially available timers.

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3. The Experiment

In a projectile motion experiment, the horizontal range (R) and time of flight (T) are some of the parameters of interest. In the following experiment we explore the dependence of these parameters on the initial conditions (speed, angle and height of launch). Using the appropriate relations the value of g can also be determined.

All three kinds of projectile motion are studied using the projectile launcher and photogate timer. The experiment on the horizontal projectile is exploratory type. In the case of the ground-to-ground projectile and oblique projectile from a height, experimental results are compared with those predicted by theory.

3.1 *Horizontal Projectile Motion*

The projectile launched from a height in the horizontal direction, as shown in *Figure* 6, is called a horizontal projectile. The range of the projectile depends on the initial velocity u, the height h from which the projectile is launched and g. The expression for R can be written as

$$R = Ku^m h^n \,, \qquad (1)$$

The experiment on the horizontal projectile is exploratory type.

where K, m and n are constants to be determined in the experiment. The value of g is included in K.

The experiment is conducted in two parts.

In the first part, the height of the launch is fixed at 0.406m to study the variation of range with respect to speed of launch. Different speeds are obtained by taking spherical balls of different sizes and material.

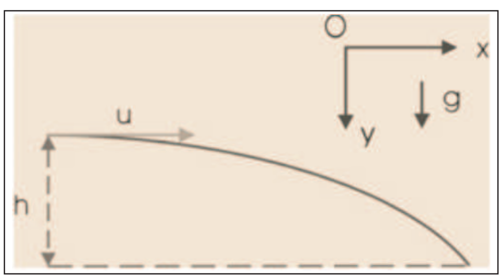


Figure 6. Schematic representation of horizontal projectile.

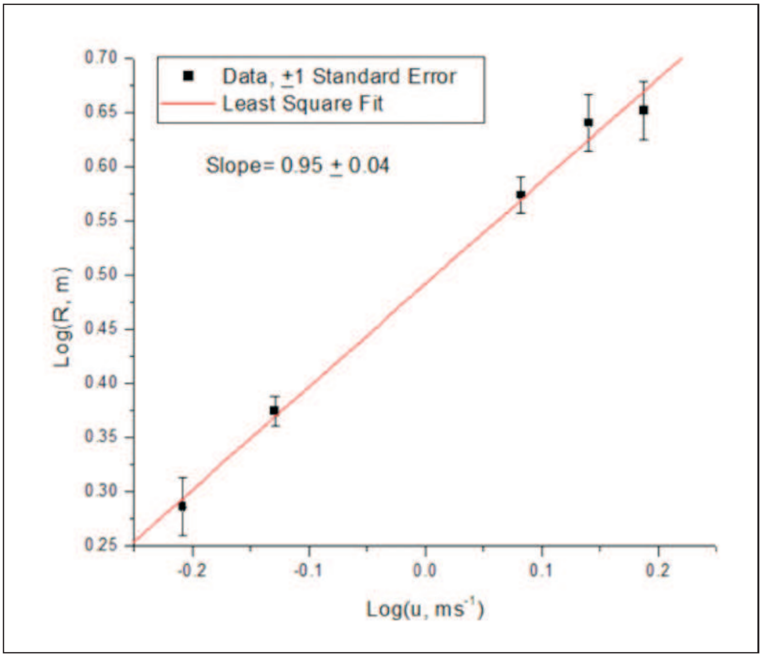


Figure 7. Graph of log R vs. log u.

The range is measured by placing a carbon sheet on the table where the projectile hits. The horizontal distance between the point of projection and the carbon mark is measured with a scale of least count 1mm. Multiple trials in each case show the uncertainty in measurement of range to be less than 3% and uncertainty in measurement of velocity to be less than 2%. The time of flight was found to be constant with 1% deviation.

The value of m can be determined by plotting the log of range versus the log of velocity. The slope gives the value of m. The graph is shown in *Figure* 7. The value of 0.95 ± 0.04 indicates that m is very close to unity which suggests that range is linearly dependent on the velocity of the projectile.

In the second part, the variation of range with respect to height is studied keeping the velocity of the projectile constant. The launcher is placed at different heights and a spherical ball is launched. The range is measured using the carbon sheet method. The experiment is repeated with the same ball so that the speed of launch remains the same. In order to explore the relation between R and h, a graph of log R vs. log h is plotted. This is shown in *Figure* 8 along with

The range is measured using the carbon sheet method.

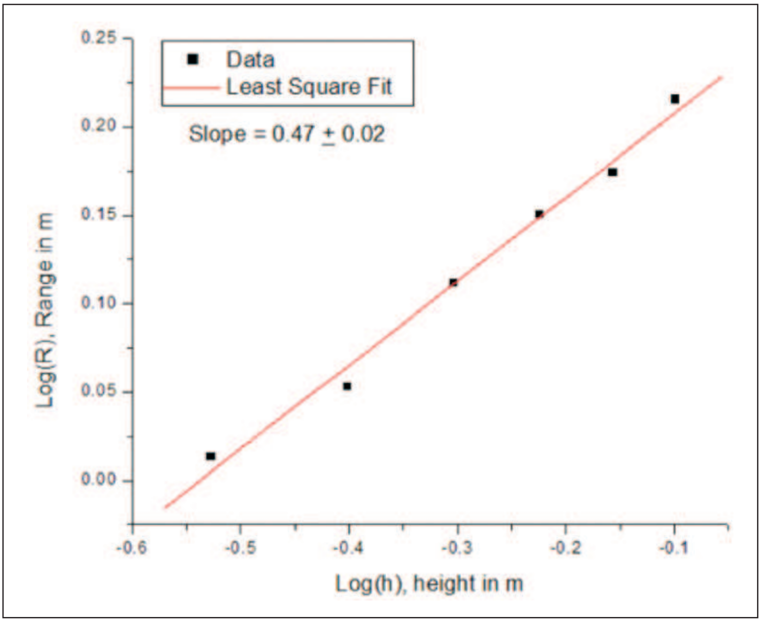


Figure 8. Graph of log R vs. log h is shown along with the value of n.

the value of the slope equal to 0.47 ± 0.02. This suggests that the value of n is 0.5 and hence, $R \propto \sqrt{h}$.

The results of the above two parts can be combined to obtain the value of K. Range (R) is plotted as a function of $u\sqrt{h}$. The slope of the straight line graph gives the value of K which is seen to be 0.46 ± 0.02 in SI units (*Figure* 9). Thus, the empirical relation for range can be written as

$$R = 0.46\, u\sqrt{h}\,. \qquad (2)$$

The theoretically obtained expression for range is given by

$$R = u\sqrt{\frac{2h}{g}}\,.$$

Here, g is the acceleration due to gravity. Comparing the two expressions, the constant K can be expressed as

$$K = \sqrt{\frac{2}{g}}\,.$$

The value of g obtained using this expression is (9.3 ± 0.7) ms^{-2}. Good agreement in the value of g implies that the empirical

Good agreement in the value of g implies that the empirical expression for range is indeed correct.

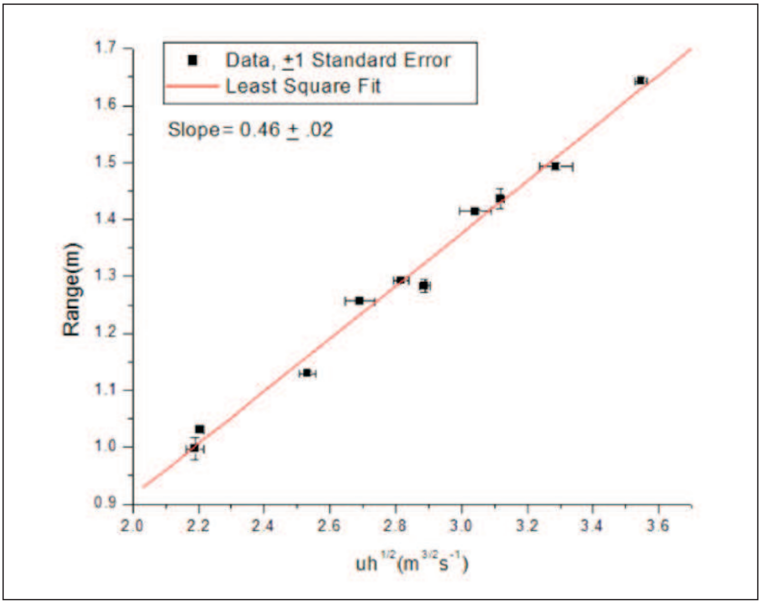


Figure 9. Graph of R vs. $u\sqrt{h}$ is shown along with the value of slope.

expression for range given in equation (2) is indeed correct.

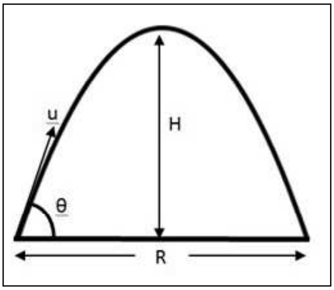
3.2 *Oblique Ground to Ground Projectile*

When a projectile launched from the ground reaches the same horizontal level as its initial position, after its flight in air (*Figure* 10), it is called a ground to ground projectile.

The range, R, and the time of flight, T, attained by the projectile depend on the angle of projection, θ, and the velocity of launch, u, given by

$$R = \frac{2u^2 \sin\theta \cos\theta}{g} \text{ and } T = \frac{2u \sin\theta}{g}. \qquad (3)$$

Figure 10. Schematic representation of ground-to-ground projectile.



In the experiment, the variation of the horizontal range of the projectile as a function of its angle of projection is studied. The launcher is oriented at angles ranging from 10° to 80° in steps of 5°. As in the previous case, multiple trials were conducted for each angle. The range was measured using the carbon sheet method. When the launcher was oriented at an angle, care was taken to ensure that the point of projection was at the same level as that of the table. The data obtained was compared with the

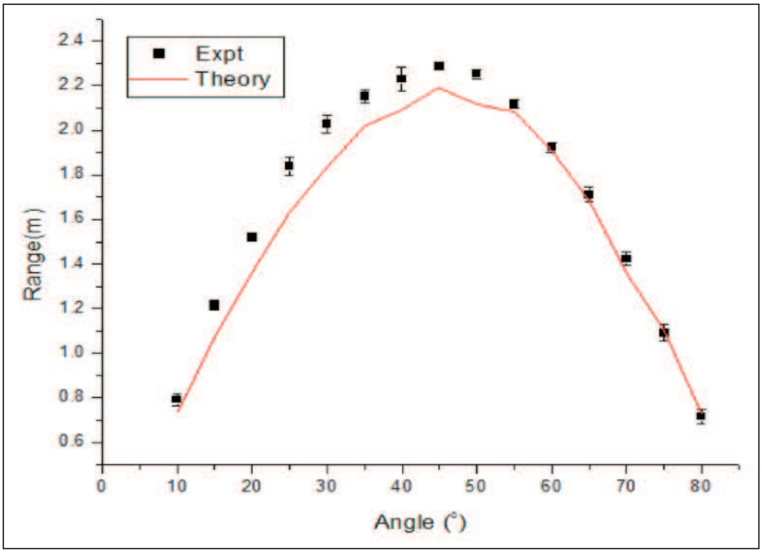


Figure 11. Comparison of experimental and theoretical values of R vs. θ; ground to ground projectile.

The comparison between values obtained from theory and the experiment is good

theoretical estimation. The u measured in the experiment was used for the theoretical calculations. The graph of range as a function of angle is shown in *Figure* 11. The comparison between values obtained from theory and the experiment is good in the given range of angles. It can be observed that the maximum range occurs at 45°. The graph of time of flight as a function of angle is shown in *Figure* 12. An excellent agreement with theory is observed.

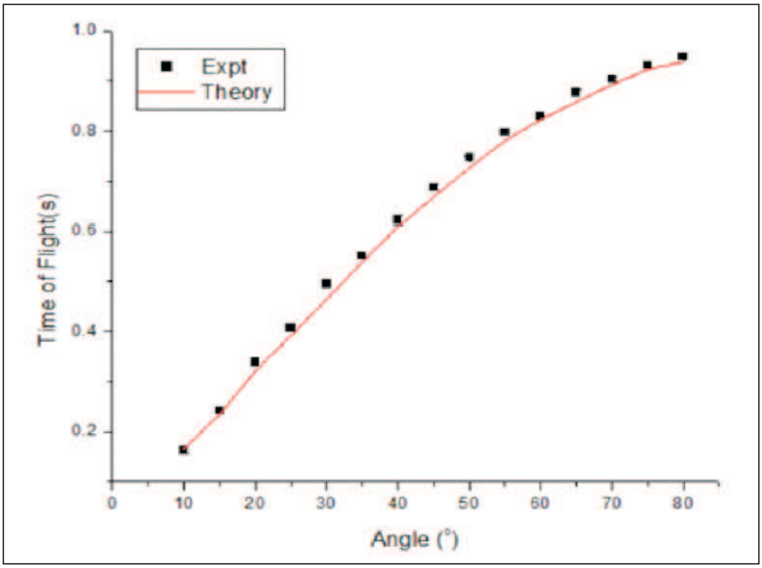


Figure 12. Comparison of experimental and theoretical values of T vs. θ; ground to ground projectile.

3.3 *Oblique Projectile from a Height*

The projectile motion as shown in *Figure* 13 can be analyzed by considering vertical and horizontal motion independently. The motion along the vertical direction can be described using the equation:

$$-h(t) = u\sin\theta t - \frac{1}{2}gt^2. \quad (4)$$

Here, t is the time of flight of the projectile. The motion along the horizontal direction can be described using the equation,

$$R = (u\cos\theta)t. \quad (5)$$

Theoretically, the time of flight can be obtained by solving the quadratic equation (4). Substituting for t, the range of the projectile can also be estimated.

In the experiment, the launcher is placed at a height of 30cm. The angle of launcher is changed from 10° to 80°. Each time, care is taken to fix the point of projection at a height of 30 cm from the ground. Multiple trials were conducted for each angle. The variation of range with angle of projection is shown in *Figure* 14. This is compared with the theoretical calculation of range described the previous paragraph. A good agreement is observed. *Figure* 15

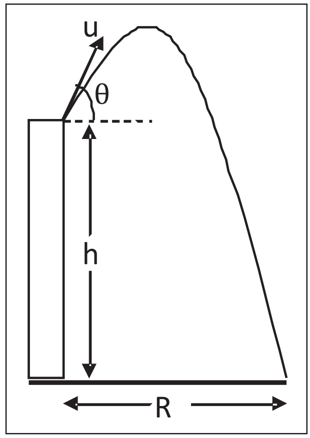


Figure 13. Schematic showing oblique projectile from a height.

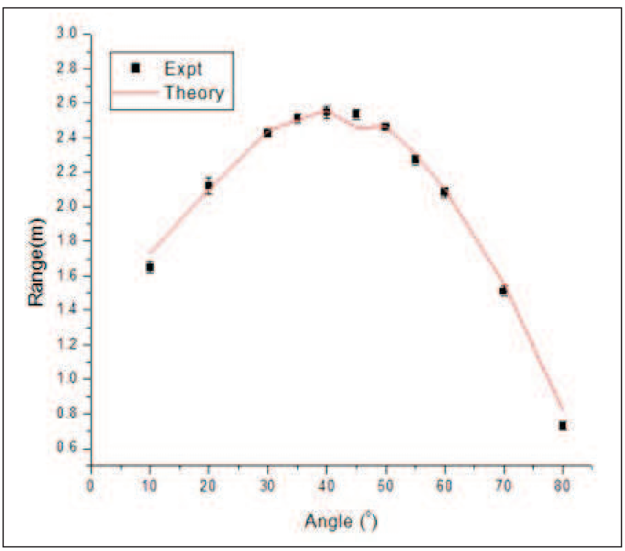


Figure 14. Comparison of experimental and theoretical values of R vs. θ; oblique projectile from a height.

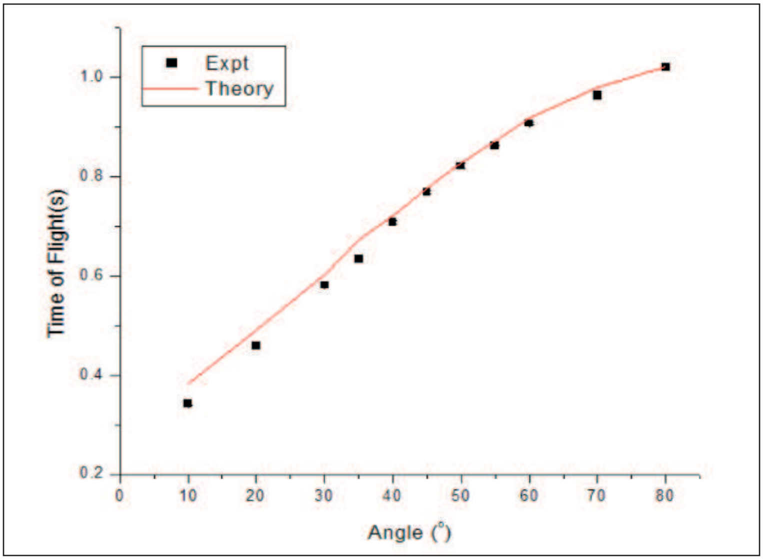


Figure 15. Comparison of experimental and theoretical values of T vs. θ; oblique projectile from a height.

variation of time of flight with angle of projection. Again, a good agreement is observed.

4. Results and Conclusion

1) The projectile launcher fabricated using a de-soldering pump seems to provide a consistent initial velocity to the spherical ball. The variation in the velocity of the steel ball used in most of the experiments was found to be about 2%.

2) The photogate sensor along with the timer unit obtained from Arduino is found to be precise and accurate. The time resolution is around 0.1ms. The velocity has been determined to an accuracy of 0.01m/s.

The photogate sensor along with the timer unit obtained from Arduino is found to be precise and accurate.

3) The empirical formula obtained in the case of horizontal projectile motion agrees well with the theoretical expression. Students should be made familiar with the exploratory type of experiments. Without knowing the mathematical relation or the theoretical basis, the dependence of a physical quantity on different parameters can be determined by performing experiments. In the development of physics, this method has led to the discovery of many important phenomena.

4) In the case of oblique projectile motion (ground-to-ground and from a height), good agreement between experimentally determined horizontal range and time of flight as functions of angle of projection with theory gives confidence in the treatment of 2D motion as two independent 1D motions. Further, the assumptions of constancy of g and neglecting air resistance appear to be correct.

This method has led to the discovery of many important phenomena.

5) This can be one of the experiments on kinematics of a body that can be introduced in the undergraduate laboratory curriculum. With the help of a photogate timer, other experiments like simple pendulum, free fall, etc., can be studied more accurately.

6) More physics experiments using the microcontroller-based open hardware platform are being explored.