The Rosetta–Philae Comet Mission as Physics Appreciation

A R P Rau

This essay will show that with no more than simple school algebra and a willingness to wrestle with symbols and simple relations (equations) among them, anyone can appreciate further patterns beyond the purely visual. Far from any diminishing or 'taking away the poetry in Nature', this can only but add to our understanding and delight in the patterns that our senses have found striking.

A Ravi P Rau earned his degrees in physics from Delhi University and the University of Chicago, and has been a professor in the Department of Physics & Astronomy at Louisiana State University since 1974. He is a frequent visitor with adjunct appointments at the Raman Research Institute, Bengaluru, and the Tata Institute of Fundamental Research, Mumbai. His research interests are broadly in theoretical physics in the areas of atomic physics and quantum information.

Keywords
Patterns in physics, dimensional analysis, cometary mission, comet landing, gravity, escape velocity.

The latest spectacular development in space exploration is the European Rosetta–Philae spacecraft that caught up with the comet 67P/Churyumov–Gerasimenko and on November 12, 2014 dropped a lander, Philae, upon it. Two decades of planning, a voyage of ten years and hundreds of millions of miles, and accomplishing such a landing are already things to marvel at. Even more, knowing some of the numbers involved can, with a little further thought of the physics behind them, heighten the appreciation. The analogy is to art appreciation. Many laypeople get pleasure from viewing great paintings in art museums simply for their visual and compositional aesthetics. But, when accompanied by an art expert, we can see deeper into them and learn to cultivate that appreciation.

With the background of dimensional analysis given in *Box* 1, we can take a relook at the comet mission. Among some of the astonishing figures we hear in the TV coverage are its small size of a mere 4 kms (~2 miles) across, with surface gravity ten-thousandth (10^{-4}) or less of that on Earth, and an 'escape velocity' of a mere 2 m/s. This made for a tricky landing of Philae. Let us make further sense of these numbers and concepts. We are familiar with Earth's gravity, and that all objects near the Earth's surface fall with a constant acceleration designated, *g*, of value 9.8 m/s^2 or 32 ft/s^2. This, of course, arises from the gravitational

Box 1. Dimensional Thinking and Analysis in Physics

Quantities characterizing the world we live in and motions in it carry an intrinsic dimensional aspect. First, from the ordinary usage of the word, distances are lengths, areas are formed out of two lengths in perpendicular directions, and volumes from three independent lengths in our three-dimensional world. Using scientific notation of powers to represent successive multiplications, we designate distances by $[L]$, areas by $[L]^2$, and volumes by $[L]^3$. We may use different units of measurement, whether inches, feet, miles, meters (m), and kilometers (km), but all distances are always $[L]$ in their intrinsic dimension. So too, any area, whether measured in acres, square miles, or hectares, is always $[L]^2$.

Proceeding to motion, a new independent element, that of time, has to be invoked with its 'dimension' $[T]$ that is different from and not reducible to any combination of lengths. Time is measured in units of seconds (s), minutes, hours, centuries or billions of years, all sharing the dimensional aspect of being $[T]$. Finally, although the mass of an object will often not be relevant (an example given below), while talking of forces or the energy involved in motion, mass as another independent dimension, $[M]$, whether measured in grams (g) or kilograms (kg), will be needed. All of mechanics, including the most sophisticated space missions such as Rosetta's, can be encompassed with just the three intrinsic dimensions of $[L]$, $[T]$, and $[M]$, and of various products and powers of them.

Thus, a speed or velocity, when a distance changes over a time interval, is a length divided by a time, $[L]/[T]$. A speed is m/s or miles per hour (mph). An acceleration, which is the rate of change of velocity, has one more $[T]$ in the denominator, thus $[L]/[T]^2$ dimensionally. Newton's famous law of motion, that a force (F) cause acceleration (a), and his equation, $F = ma$ which shows how an object's mass (m) indexes the response of a body to changes in its velocity caused by external forces, give the dimension of a force as $[M][L]/[T]^2$.

force exerted by the Earth's mass that for our purposes can be assumed to be at the Earth's geometrical centre – the 'centre of gravity'. The term *near-surface* objects can be applied to all those objects whose distance from the Earth's surface is much less than the Earth's radius, R (6,400 km or 4000 miles). So, an apple hanging from a tree or even a satellite, such as a Space Station, floating 200–300 miles above the surface are very near, in comparison.

Without any further knowledge of gravity, the velocity v_o of a near-surface satellite in orbit, being dimensionally $[L]/[T]$, can be deduced from the only two relevant quantities we have of radius R, a $[L]$, and Earth's g, a $[L]/[T]^2$, in only one unique way:

$$v_o = (gR)^{1/2}.$$

The product, gR, has dimensions $[L]^2/[T]^2$, so that a square root indicated by the ½ exponent is needed to get $[L]/[T]$. Additional dimensionless factors could possibly be involved in more detailed physics but simple dimensional arguments often suffice to get to the essentials and, as in this case, even the exact expressions. This same result can also be viewed alternatively as $v_o^2/R = g$. Here, the left-hand side is called the centripetal acceleration that any circular motion entails, in this case with an orbit of radius R and speed v_o. Gravity being the agent causing this orbital motion, it is g that must be the centripetal acceleration. For the Earth, with $g \approx 10$ m/s^2 and $R = 6.4 \times 10^6$ m, we get $v_o = 8$ km/s or 5 miles/s. This is why near-surface satellites cover the 25,000-mile circumference in approximately 90 minutes.

For the related concept of escape velocity from a heavenly body, it helps to think in terms of energy, the 'kinetic' energy that any mass m carries by virtue of its motion and given by $mv^2/2$, and the gravitational 'potential' energy. As seen from the above expression for kinetic energy, dimensionally, energy is $[M][L]^2/[T]^2$. When an object of mass m is held by gravitation, it is bound and it takes energy to separate it, a 'binding' energy that involves m, g, and a length. The natural and only candidate for that length being R, mgR is the gravitational binding energy; this combination is also $[M][L]^2/[T]^2$. To liberate the mass m from near R to infinite separation requires a kinetic energy supplied by an 'escape velocity' v_e, that is, $mv_e^2/2 = mgR$. As in the previous paragraph, the same dimensional combination of $(gR)^{1/2}$ occurs, not surprisingly, but now with an additional factor of 2 so that $v_e = (2gR)^{1/2}$. Earth's escape velocity is, approximately, $8\sqrt{2} \approx 11$ km/s for any mass m, mass cancelling out in the above equality of two energies.

With a radius smaller by a factor of 3200 and g by a factor of 10,000, the orbital and escape velocities are smaller by a factor of 5,500 so that the escape velocity from the comet is a tiny 2 m/s.

It is now immediate to make the leap from Earth to the comet. With a radius smaller by a factor of 3200 and g by a factor of 10,000, the orbital and escape velocities are smaller by a factor of 5,500 so that the escape velocity from the comet is a tiny 2 m/s. This is barely walking speed! It is what made the landing of Philae so tricky because it had to be dropped from the parent ship Rosetta gently so as to touch down with less than the above number, the design

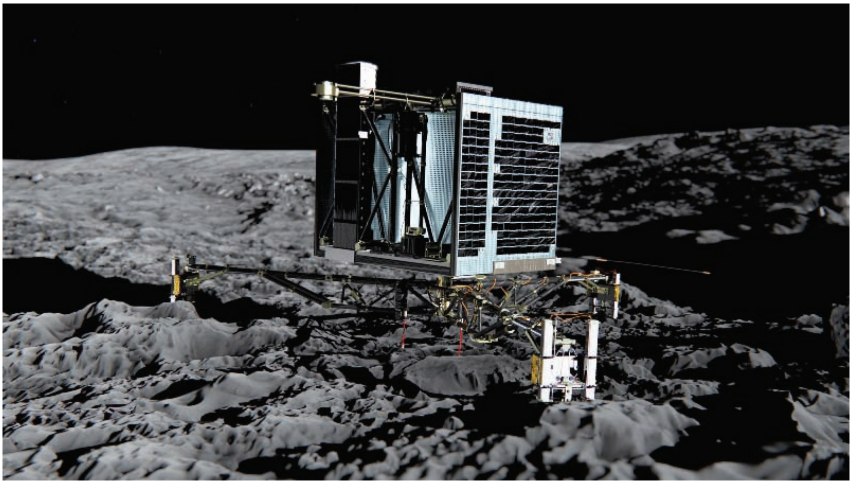

Figure 1. Artist's view of Philae. Courtesy: http://www.esa.int/var/esa/storage/images/esa_multimedia/images/2013/12/philae_on_the_comet_back_view/13467069-1-eng-GB/Philae_on_the_comet_back_view.jpg

speed having been 1 m/s (see *Figure* 1). It did do so but, unfortunately, two mechanisms – a small rocket burn to press it onto the surface and harpoons to anchor it – seem to have failed. And, indeed, the little lander bounced. That, it flew up several hundred meters and touched down again a kilometer away, is also understood with the same terms and expression above: this time that any vertical distance an object reaches or horizontal range covered, being a length, can only be v^2/g for a launch speed of v. With speeds of about 1 m/s and g of 1 mm/s^2, the subsequent rise and landing a kilometer away falls into place, as also that this took about, again dimensionally, v/g or 1000 s (with a factor of 2 for going up and down and allowing for variation in g over this rise, it was about an hour, as reported).

Among the astonishing features of this tiny comet is that it would be unsafe to jump on it! Tread on it carefully. Any one of us would, as in the case of the 200 kg Philae, weigh on that comet only as much as a small slice out of a single butter stick on Earth!

Many details down to quantitative accuracy can be understood on the basis of just two numbers, the size and value of g, of this comet, plus dimensional thinking.

Many details down to quantitative accuracy can be understood on the basis of just two numbers, the size and value of g, of this comet, plus dimensional thinking. Even further, those numbers can themselves be made sense of through simple physics. That g is smaller for a small comet as compared to a large planet, is plausible to

Were g to be smaller, which makes weights lower, mountains could be taller.

anyone but we can go further with the recognition that it stems from gravity. A fundamental constant called Newton's gravitational constant G (always denoted by upper case to distinguish from the lower case used for the acceleration due to gravity) characterizes gravity. Again, details are not necessary, just a recognition of its dimensions, i.e., $[L]^3/[M][T]^2$. Since, $[M]/[L]^3$, mass divided by volume, is density ρ, this means that $(\rho G)^{1/2}$ is an inverse time or frequency.

An acceleration, g, being $[L]/[T]^2$, should then be expected to be the radius R multiplied by ρG. With both R smaller and of lower density, the comet's smaller value of g follows. A feature of the Earth's geology enters here, that while surface rocks, whether basalt or granite, have densities of about 2–3 g/cm^3, the average density of Earth is a much higher 5.5 because of its iron–nickel core with densities of about 7. The small comet is made up of mostly light stuff and its average density can be taken as several times smaller than Earth's (one figure quoted is 0.4 g/cm^3), and this combined with the 3,200 ratio in radius, accounts for g being one ten-thousandth or less.

Finally, the comet is also not a sphere like Earth, more with two lobes and a narrower neck between them (*Figure* 2). It has been called duck-like in appearance. Only when gravitation becomes the determinant, is a large aggregate of matter guaranteed to be spherical. If electromagnetic forces dominate, then objects can have other shapes as indeed true of humans, other animals, or of man-made objects. This is true not just at our own scales but even for the two moons of Mars. A simple dimensional argument gives the transition size. On Earth, the highest mountain is about 6 miles. There is a limiting height because the weight of the piled-up matter would exceed breaking strength (a property arising from electromagnetic forces [1]) of the material at the bottom, causing melting and flow. Were g to

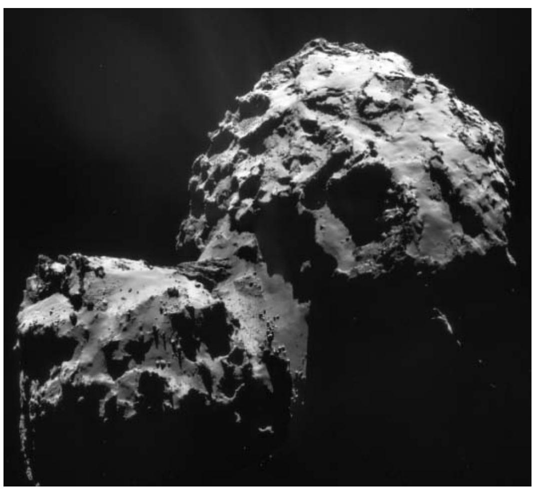

Figure 2. 67P/Churyumov–Gerasimenko photographed on January 10, 2015.
Courtesy: http://sci.esa.int/rosetta/55275-comet-67p-on-10-january-2015-navcam-mosaic/

be smaller, which makes weights lower, mountains could be taller. Thus, the height of protuberances that can be supported is inversely related to g and, as per the previous paragraph, inversely to R. As R decreases, mountains can be taller in inverse proportion. If protuberances become comparable to R, the object can no longer be considered spherical or near-spherical. The transition size, as a length, can therefore be expected to be the geometric mean of the two lengths, R and the highest mountain height. Using 6,400 km and Mt Everest's 10 km, this gives approximately 250 km. Our own Moon and even Pluto are comfortably over this limit and spherical but not Mars's moons or the comet 67P.

Because of its small and odd size and shape (*Figure* 2), the comet's gravity could only be determined by Rosetta itself as it made loops around it, thus mapping out its gravity, a needed input for dropping the lander onto it. This was done from a height of about 25 km, or 12 times the radius of the comet. The time it took to land is also readily understood. Being time, it must involve the combination $(R/g)^{1/2}$ dimensionally. In addition, because of the larger distances involved, the variation in g has to be considered. This introduces a further ingredient, known as Kepler's third law relating distances and time for natural and artificial satellites through a 3/2 power. Thereby, such a power of 12 for the height of the drop in terms of the comet radius enters as an additional factor. In all, these account for the 7 hours the landing took, as reported.

Mark Twain was lampooning science when he said that it gives "wholesale returns out of trifling investments". But it is true that much science appreciation can come out of a little disciplined thinking [2].

Suggested Reading

[1] **V F Weisskopf, Of atoms, mountains, and stars – a study in qualitative physics, *Science*, Vol.187, pp.605–612, 1975.**

[2] **A R P Rau, *The Beauty of Physics: Patterns, Principles, and Perspectives*, Oxford University Press, Oxford, 2014.**

Address for Correspondence
A Ravi P Rau
Department of Physics & Astronomy
Louisiana State University
Baton Rouge 70803, USA.
Email: arau@phys.lsu.edu