Joseph L Doob and Development of Probability Theory

Joseph Leonard Doob was a great mathematician from the USA. He played a leading role in the development of probability theory in the US and the world. This article is an account of his life, career and contributions. For an introduction to probability theory, see the article by K B Athreya on page 292.

Early Life

Joseph L Doob was born in Cincinnati, Ohio, USA on February 27, 1910 and passed away in Urbana, Illinois, USA on June 7, 2004.

After finishing high school in Cincinnati, Ohio, Doob became an undergraduate student (equivalent to BSc in India) in mathematics at the famous Harvard University, Boston, Massachusetts, USA. He was just about sixteen years old. In an interview with J L Snell [1], Doob says, "I knew nothing about college education or colleges when I finished my high school education in 1926". On the advice of his high school principal, Doob applied to Harvard. He says in the interview with Snell [1] that he was accepted by Harvard without any examination or test because he was in the upper seventh (top 14%) of his high school class. Doob graduated with a BS degree from Harvard in 1930 (at the age of twenty).

PhD at Harvard

He started his PhD work in the same department right away in 1930. One W Seidel was a young faculty member at Harvard. Doob and Seidel worked on the behavior of analytic function near the boundary points of the unit disc $D = \{z \in \mathbf{C} : |z| : \leq 1\}$. He pursued it further and got some interesting results. He wrote these up as a paper. He showed it to his formal PhD guide, J L Walsh, who apparently did not read it but asked for Seidel's evaluation of it. Seidel said it was a good piece of work and that was that. Doob got his PhD from Harvard University in mathematics in 1932 at a young age of 22. In the interview with Snell, Doob says that even though he got his PhD at the age of 22, he felt woefully ignorant of almost everything in mathematics that was not connected with his thesis work. There were excellent mathematicians at Harvard at that time including Garret Birkhoff, Kellogg and Morse, all leaders in their fields. But, Doob says that he completely missed contact with any of these three and many other good mathematicians at Harvard.

Over the last sixty years, most top class US universities insist that their PhD students go through two years of course work in a variety of basic topics. In mathematics, this includes analysis, algebra (abstract) and topology. Then, the doctoral program also insists on students passing a written qualifying examination on basic subjects before starting their PhD thesis work. This course work is an excellent preparation for research work. Unfortunately, many universities in India do not insist on this. Many students in India start their PhD thesis work soon after their MSc

and are usually not well rounded and thus miss the chances of doing an outstanding research work for their PhD.

Economic Depression in USA

When Doob finished his PhD at Harvard in 1932, Garrett Birkhoff of Harvard managed to arrange a National Research Council (of USA) Fellowship for two years for him. Doob used this Fellowship to go to Columbia University, New York City, USA to work with J F Ritt. Doob had chosen Columbia University because his wife Elsie was a medical student in New York. Doob and Elsie had met during Doob's Harvard years and got married. After two years at Columbia University, Doob started to look for jobs. The 1930's were a period of serious economic depression in the USA and it was not easy to find a job. In his interview with Snell, Doob says, "The streets of New York were full of unemployed men begging or selling apples to make a few cents and no jobs were opening up either in industry or academic. I humiliated myself trotting after big shots at the annual American Mathematical Society meeting."

Bernard O Koopman of Columbia University advised him to approach Harold Hotelling, the Professor of statistics at Columbia University. Fortunately, Hotelling could arrange a Carnegie Corporation grant for Doob provided he agreed to work with him on topics related to statistics. Doob says that the force of economic circumstances got him into probability theory. It is probability theory that provides the mathematical basis for the subject of statistics. He worked with Hotelling during 1934–35.

Fortunately for Doob, he got a faculty position at the mathematics department, University of Illinois, Urbana, Illinois, in 1935. He liked the small town atmosphere of Urbana and never wanted to leave. He stayed there as a faculty for the next forty-three years until his retirement in 1978. He spent the rest of his life there as well and passed away in 2004.

Random Walk

There was a Saturday walk (called Saturday Hike) program in Urbana–Champaign, (a town adjacent to Urbana), established in 1909 by a Professor of Classics (that of subjects like Greek, Latin, etc.) at the University of Illinois. Each Saturday, the members of the Saturday Hike group would drive to nearby woods along a river. They would have a long walk, play some games and in the evening, build a fire and have dinner and then go home. Doob joined this walk in the 1940s. He was the Commissioner of the Champaign–Urbana Saturday Hike for about twenty five years after World War II (roughly from 1945–1970).

Honours and Recognition

Here are some of the honours and awards conferred on Prof. J L Doob. He was elected a member

of the National Academy of Sciences, USA and Foreign Associate of the Academy of Sciences, France. He was President of the Institute of Mathematical Statistics, USA in 1963 and 1964. Doob was given the National Medal of Science by US President Jimmy Carter in 1979. In 1984, the American Mathematical Society USA awarded him the Career Prize for his "fundamental work in establishing probability theory as a branch of mathematics and for his continuing profound influence on its development."

Many of Doob's PhD students have gone on to become distinguished mathematicians. Among them are Paul R Halmos, Warren Ambrose, David Blackwell, J L Snell and John Walsh. Also, Professor Donald Burkholder was a distinguished colleague of Doob at the University of Illinois, mathematics department.

Books

J L Doob established probability theory as a major discipline of study in the USA and the world [2]. Apart from his ground breaking research in probability theory, he also wrote three books published in 1953, 1984 and in 1994. The first book, *Stochastic Processes,* published in the Wiley series in 1953 [3] was a very influential book in the development of probability theory in the US and the world.

A famous French probabilist, P A Meyer, calls the book, *Stochastic Processes,* "The bible of new probability". In this book, Doob shows that measure theory, appropriately developed, provides the basics and allows one to solve a number of problems in probability theory. In the same book, Doob develops the theory of martingales – both discrete and continuous time – and a detailed study of Ito's stochastic differential equations with respect to Brownian motion.

Doob, in his interview with J L Snell, jokes that, "I had the best possible proof that my book, *Stochastic Processes*, was read carefully by a large number of readers: a blizzard of letters arrived pointing out mistakes". He adds that his second book, *Classical Potential Theory and its Probabilistic Counterpart* (1984, 800 pages) had no such reception [4].

He wrote a third book, *Measure Theory*, which was published when he was eighty four (1994) [5]. For a more detailed account of Doob, see Snell's article [1].

Suggested Reading

[1] J L Snell, A conversation with Joe Doob, *Statistical Science*, Vol.12, No.4, 1997.
[2] K B Athreya, What is Probability Theory?, *Resonance*, Vol.20, No.4, 2015.
[3] J L Doob, *Stochastic Processes*, Wiley, NY, 1953.
[4] J L Doob, *Classical Potential Theory and its Probabilistic Counterpart*, Springer, NY, 1984.
[5] J L Doob, *Measure Theory*, Springer, NY, 1994.

K B Athreya
Email: kbathreya@gmail.com