# Ed Lorenz: Father of the 'Butterfly Effect'

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**Ed Lorenz, rightfully acclaimed as the father of the 'Butterfly Effect', was an American mathematician and meteorologist whose early work on weather prediction convinced the world at large about the unpredictability of weather. His seminal work on a simplified model for convections in the atmosphere led to the modern theory of 'Chaos' – the third revolutionary discovery of 20th century, the other two being relativity and quantum physics. The possibility of unpredictability in certain nonlinear systems was vaguely mentioned earlier by J C Maxwell and clearly asserted later by H Poincaré. But it was the work of Lorenz in 1963 that indicated clearly that the sensitive dependence on the initial conditions (also called 'SIC'-ness) of such systems can lead to unpredictable states. This strange and exotic behavior was named the 'Butterfly Effect[1]' by him in a lecture that he delivered in December 1972 in Washington DC.**

[1] See Classics, p.260.

Edward Norton Lorenz was born in West Hartford, Connecticut on May 13, 1917. His father, Edward Henry Lorenz, was a mechanical engineer involved in designing machinery, but interested in all aspects of science, especially mathematics. His mother, née Grace Norton, was a teacher in Chicago. As a small boy, he was interested in numbers, and when taken out in a go-cart, he used to read all the numbers on houses before he was two-years old. When he learned multiplication, he became fond of perfect squares and could recite all of them from 1 to 10,000. He used to spend hours solving mathematical puzzles with his father. He graduated in

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mathematics from Dartmouth College in 1938, followed by a postgraduate degree from Harvard in 1940. During the Second World War, he had to work as a weather predictor for US Air Corps which kindled in him an interest in weather science or meteorology that he pursued later [1,2]. He earned his master's (1943) and doctoral degrees (1948) in meteorology from the Massachusetts Institute of Technology (MIT, Boston). He joined MIT as a research scientist in 1948 and continued there till he became Emeritus Professor. For his substantial contributions in meteorology, Lorenz was elected to the National Academy of Sciences in 1975. He won numerous awards, honours and honorary degrees. Among them is the Kyoto Prize for basic sciences awarded in 1991 in the field of Earth and Planetary Sciences for his 'boldest scientific achievement' in discovering 'deterministic chaos' [3].

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The three deceptively simple but nonlinear equations, which Lorenz derived for modeling convection in the atmosphere, are now known as the Lorenz system [6]:

$$\begin{aligned} \mathrm{d}x/\mathrm{d}t &= a(y-x), \\ \mathrm{d}y/\mathrm{d}t &= -xz+cx-y, \\ \mathrm{d}z/\mathrm{d}t &= xy-bz. \end{aligned} \tag{1}$$

Here, $x$ is proportional to convective intensity, $y$ to the temperature difference between descending and ascending currents and $z$ to the difference in vertical temperature profile from linearity. The system parameters, $a, b, c$ are respectively the Prandtl number, relative Rayleigh

**Box 1.**

130 JOURNAL OF THE ATMOSPHERIC SCIENCES VOLUME 20

## Deterministic Nonperiodic Flow[1]

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ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

### 1. Introduction

Certain hydrodynamical systems exhibit steady-state flow patterns, while others oscillate in a regular periodic fashion. Still others vary in an irregular, seemingly haphazard manner, and, even when observed for long periods of time, do not appear to repeat their previous history.

These modes of behavior may all be observed in the familiar rotating-basin experiments, described by Fultz, *et al.* (1959) and Hide (1958). In these experiments, a cylindrical vessel containing water is rotated about its axis, and is heated near its rim and cooled near its center in a steady symmetrical fashion. Under certain conditions the resulting flow is as symmetric and steady as the heating which gives rise to it. Under different conditions a system of regularly spaced waves develops, and progresses at a uniform speed without changing its shape. Under still different conditions an irregular flow pattern forms, and moves and changes its shape in an irregular nonperiodic manner.

Lack of periodicity is very common in natural systems, and is one of the distinguishing features of turbulent flow. Because instantaneous turbulent flow patterns are so irregular, attention is often confined to the statistics of turbulence, which, in contrast to the details of turbulence, often behave in a regular well-organized manner. The short-range weather forecaster, however, is forced willy-nilly to predict the details of the large-scale turbulent eddies—the cyclones and anticyclones—which continually arrange themselves into new patterns. Thus there are occasions when more than the statistics of irregular flow are of very real concern.

In this study we shall work with systems of deterministic equations which are idealizations of hydrodynamical systems. We shall be interested principally in nonperiodic solutions, i.e., solutions which never repeat their past history exactly, and where all approximate repetitions are of finite duration. Thus we shall be involved with the ultimate behavior of the solutions, as opposed to the transient behavior associated with arbitrary initial conditions.

A closed hydrodynamical system of finite mass may ostensibly be treated mathematically as a finite collection of molecules—usually a very large finite collection—in which case the governing laws are expressible as a finite set of ordinary differential equations. These equations are generally highly intractable, and the set of molecules is usually approximated by a continuous distribution of mass. The governing laws are then expressed as a set of partial differential equations, containing such quantities as velocity, density, and pressure as dependent variables.

It is sometimes possible to obtain particular solutions of these equations analytically, especially when the solutions are periodic or invariant with time, and, indeed, much work has been devoted to obtaining such solutions by one scheme or another. Ordinarily, however, nonperiodic solutions cannot readily be determined except by numerical procedures. Such procedures involve replacing the continuous variables by a new finite set of functions of time, which may perhaps be the values of the continuous variables at a chosen grid of points, or the coefficients in the expansions of these variables in series of orthogonal functions. The governing laws then become a finite set of ordinary differential

[1] The research reported in this work has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, under Contract No. AF 19(604)-4969.

number and aspect ratio (ratio of vertical height to horizontal size) of rolls. The equations primarily arise in a simplified model of Rayleigh–Bénard convection for two-dimensional fluid flow in a thin layer heated uniformly from below [7]. His discovery of the strange properties of this system was quite accidental [8].

One day, he wanted to check in detail the solution he had obtained earlier and so he reentered data from the printout of a weather simulation he had run the previous day into his computer, a Royal McBee LGP-30 and went out for a coffee. He returned an hour later to find that the new solution did not agree with the previous one. Instead of ignoring this as a bug in the algorithm or a computational artifact, he compared the new solution step by step with the old one and found that at first both agreed but soon they started departing, the difference becoming larger and larger until after some time the solutions were unrecognizably different. He remembered that the only difference was that the numbers that he had entered on the second day were rounded off to three places while earlier they had been to six places. Then, he realized to his surprise and to the benefit of all humanity, that "slightly differing initial states can evolve into considerably different states". Considering the inevitable inaccuracy and incompleteness of weather observations, the important realization dawned on him then itself that precise long-range forecasting of weather would be impossible.

Considering the inevitable inaccuracy and incompleteness of weather observations, the important realization dawned on Lorenz that precise long-range forecasting of weather would be impossible.

Since then, his name is also associated with the trajectory of this system in a 2-D $(x,z)$ plane: the '*Lorenz attractor*'. It is called an attractor since the Lorenz system is dissipative and so trajectories starting from a large number of initial conditions will eventually settle on to dynamics on the attractor shown in *Figure* 1. As Lorenz pointed out, this dynamics has sensitive dependence on initial conditions and so nearby starting points will end up at widely separated points on the attractor. This is

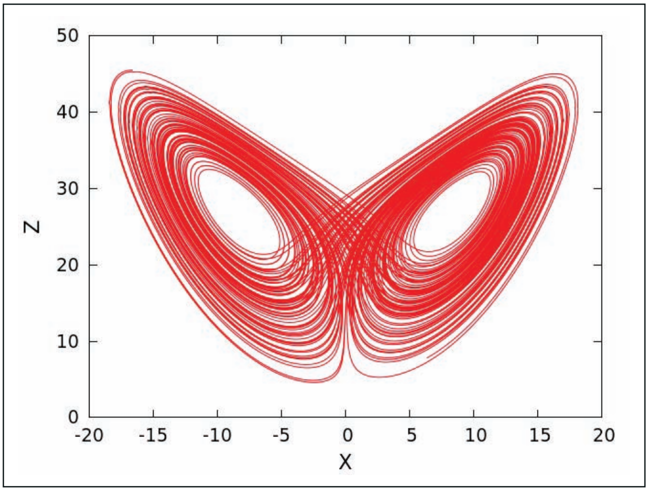


**Figure 1.** Lorenz attractor. Trajectories of the Lorenz system in the (*x,z*) plane with initial values (*x,y,z*) = (2,5,6) and parameter values *a* =10, *b* = 8/3 and *c* = 28.

evident from *Figure* 2, where the values of $x(t)$ obtained by solving the Lorenz equations using two slightly differing initial conditions are plotted. Thus, we can say that if $d_0$ is the distance between two nearby starting points in the phase space $(x, y, z)$, then after time $t$, this distance will evolve into

$$d(t) = d_0 e^{\lambda t}, \tag{2}$$

where $\lambda$ is a positive index called the 'Lyapunov exponent'. Thus, a positive value for $\lambda$ is now identified as the signature of deterministic chaos in any system, and

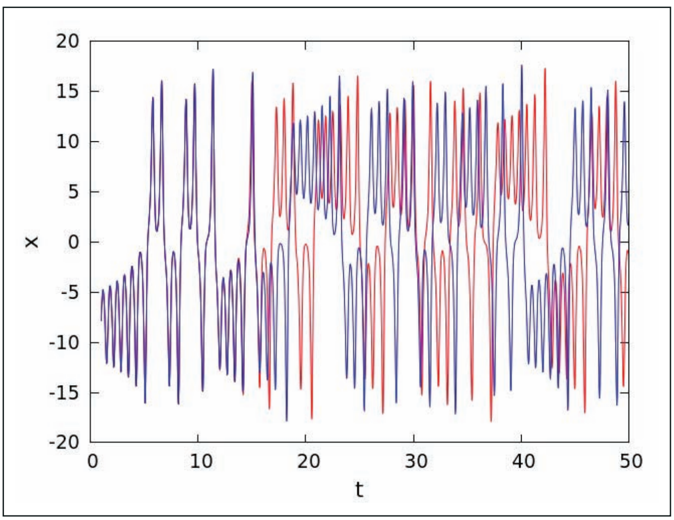


**Figure 2.** Two time series, *x(t)* of the Lorenz attractor starting from two different initial conditions differing by 0.001. Though they move together initially, after some time they start moving apart indicating sensitivity to initial conditions.

the Lorenz system is a minimal nonlinear system with this property.

Later, the Lorenz attractor was identified as belonging to the class of fractals, with a geometry that goes beyond Euclidean. The concept of fractals [9], introduced by B Mandelbrot in the mid 70s, is now found to exist in objects at all levels from the bacterial colony to distribution of stars in the galaxy. The geometry of a fractal can be captured by defining a box-counting dimension, known simply as the fractal dimension [10,11]. It indicates how the number $N(\varepsilon)$ of boxes of size $\varepsilon$, which are required to cover the object, scales, as $\varepsilon$ is reduced, i.e., $N(\varepsilon) \sim \varepsilon^{-D_0}$. Here, $D_0$ is the fractal dimension that can thus be computed as

$$D_0 = \frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)}, \quad \text{in the limit } \varepsilon \to 0. \tag{3}$$

The fractal dimension of the Lorenz attractor is estimated to be 2.02. While the dynamic complexity of a chaotic attractor is measured using the Lyapunov exponent, its geometric complexity can be understood using its fractal dimension. The Lorenz attractor, which soon became an iconic object claiming enviable esteem in the nonlinear community, is indeed a thing of beauty as is evident from the endless variety of its realizations and their variations (http://paulbourke.net/fractals/lorenz/). Several other dynamical properties of this system like its bifurcation scenario, fixed points and their stability, etc., were exhaustively studied in later years [12].

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It is interesting that equations which are structurally similar to Lorenz system arise in very different contexts like lasers, dynamos, thermosyphons, brushless DC motors, electric circuits, chemical reactions and forward osmosis. Also, chaotic behavior was observed experimentally in laser systems, Josephson junctions, plasma systems and nonlinear electronic circuits. A tabletop demo model can be easily fabricated for a chaotic water

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wheel, with dynamics that follows the Lorenz equations.

The irregular, complex and unpredictable behavior of simple nonlinear systems, christened later as deterministic chaos by Jim Yorke, drastically changed our understanding of Nature and opened up a totally new area of intense research activity. This later led to many related concepts like stochastic resonance, synchronization, secure communication, etc., that have a wide variety of useful technological applications. It is indeed a marvel that high-speed long distance communication through optical fibers was realized based on chaos synchronization and using an optical carrier wave generated by a chaotic laser, across a distance of 120 kms in the metropolitan area network of Athens, Greece in 2005 [13]. Nonlinear scientists still use the Lorenz equations as the prototype in their modeling and characterization of measures and as the standard system to illustrate various features of nonlinear systems. Being easy to fabricate in circuits or physical systems, this system is being extensively used in several studies and applications.

Ed Lorenz was a perfect gentleman, incredibly quiet and reported as the most humble and kind among the geniuses of that era. In an article, 'A scientist by choice' [8], he recaptures several turning points of his life and ponders on the essential characteristics that make a scientist: an interesting article appealing to all scientists at large. He was active in his work well into his seventies. An exceptionally fit man, he enjoyed hiking, climbing, and cross-country skiing. He enjoyed a happy home life and was attended by his family during his last days. He died of cancer on April 16, 2008, at home in Cambridge, Massachusetts. The unpredictable but unavoidable end was at the age of 90.

On the fiftieth anniversary of the discovery of chaos (Chaos at Fifty [14, 15], MIT started a new center, The Lorenz Center (http://web.mit.edu/lorenzcenter/)

which is a new climate think tank devoted to curiosity-driven research and learning, fostering creative approaches to increase fundamental understanding of global climate changes, one of the major challenges of the day.

The Butterfly Effect led to a new discipline of study and research.

The scientific community will remember Lorenz for the Butterfly Effect that led to a new discipline – an uncommon mix of determinism and unpredictability built on the framework of fractal geometry. That Nature perfects her creations and their functions not through pure stochasticity or noise alone but through randomness of a different kind, generated by chaos and nonlinearity, is now established and is evident to us from the fractal structure of a leaf to the intricate pattern formations of neurons in the brain. While a complete characterization of their complexity still eludes us, their unpredictability and diversity serve to make us humble. Our quests and consequent realizations in this context were all kindled mainly by the discovery of deterministic chaos by Lorenz fifty-one years ago.

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