Identical Distinguishable Gas Particles in the Real World

Carl E Mungan

Carl Mungan is an Associate Professor of Physics at the United States Naval Academy, educating approximately 4000 future Navy and Marines officers. His research interests are in optical spectroscopy of condensed matter, with emphasis on laser cooling of solids, stimulated Brillouin scattering in optical fibers, and photothermal effects.

In undergraduate courses in statistical or quantum mechanics, students are exposed to quantum and classical particles that are described as being identical or indistinguishable. It is difficult to appreciate this terminology unless real-world examples are given to illustrate the different possibilities. In particular, while it is easy to think of systems of identical indistinguishable particles or of nonidentical distinguishable particles, it requires more care to identify a gas of identical distinguishable particles.

In undergraduate physics courses treating either quantum mechanics or classical statistical mechanics, students are introduced to the concept of a gas of particles which may be identical and may be indistinguishable [1]. In this short contribution, the definitions of these terms are considered, and examples of real gases which fit the various possibilities are presented.

Define particles to be *identical* if they have the same intrinsic properties (such as mass, charge, shape, and spin). Adopt an operational definition in which two gas particles are said to be *distinguishable*, if and only if one can keep track of which particle is which, following any and every possible collision between them [2], regardless of whether such collisions between the two particles have occurred. This definition presupposes that the particles can actually collide; particles that are kept spatially isolated from each other are always distinguishable. (Further, the definition restricts attention to *two* particles, to sidestep the impracticality of tracking a thermodynamically large number of gas particles at a time [3].) Consequently, nonidentical particles are necessarily distinguishable. Is the inverse true: Are identical gas particles necessarily indistinguishable? That is an interesting

question to ask of students. To keep the issue concrete, examples of actual gases are to be discussed, and not merely hypothetical or computer-simulated systems.

Start by considering a cold dilute gas of spin-polarized ^{1}H, where the average distance between particles is much larger than their thermal de Broglie length, which in turn is larger than their mean atomic radius [4]. Such a system is described by Boltzmann statistics and is therefore a 'classical gas' even though the atoms themselves are 'fuzzy quantum particles.' In this case, the overlap of their wavefunctions during collisional mixing makes it impossible to distinguish individual particles. Any gas of identical *quantum* particles will necessarily be indistinguishable, even if it is described by classical statistics.

What if one, that considers *classical* particles? To ensure they are in the classical limit, one might first think of relatively large particles such as in a colloid, e.g., fat globules in milk [5]. The problem with large particles is that, like the proverbial snowflakes, they are all somewhat different in shape, not to mention in atomic arrangements and impurities. To avoid that problem, one might next think of isotopically pure molecules, such as buckyballs [6] which might appear to be a physical realization of the prototypical system of 'identical impenetrable spheres'. However, molecules have internal modes such as vibrations and rotations that can be both thermally and collisionally excited, so that these particles are again not identical.

One is left with a hot monoatomic gas as the only real system of colliding particles that can be simultaneously identical and distinguishable. Even then, the atoms must have identical spin orientations, which cannot be maintained at high temperatures. So one must restrict attention to spin-zero atoms such as ^{40}Ar. At a temperature that is, say, ten times the boiling point, the thermal de Broglie length of argon is twenty times smaller than its van der Waals radius and it can be treated as a gas of identical classical particles.

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identical are, on closer inspection, actually nonidentical. That includes molecular gases, colloids, and collections of ball bearings or plastic beads. Even gas atoms of a single element are not identical unless they have the same isotope, spin, ionization, and excitation [7]. An actual example of a gas of identical distinguishable particles is a spin-zero monatomic isotopically pure gas that is hot enough to be in the classical limit [8]. Instructors of statistical mechanics or quantum mechanics discussing effects such as Gibbs paradox, Bose–Einstein condensation, or atomic beam scattering [9] should specifically focus on such systems to help students appreciate the real-world distinction between 'identical' and 'indistinguishable', instead of vaguely referring to unspecified 'classical identical particles.'

Acknowledgements

Suggested Reading

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Address for Correspondence
Carl E Mungan
Physics Department
US Naval Academy
Annapolis, MD 21402-1363
USA
Email: mungan@usna.edu