Think It Over

This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

An Interesting Problem on Diophantine Triple

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Solution:

We want to find three integers a, b, c with $1 \leq a \leq 12$, $1 \leq b \leq 31, 1 \leq c \leq 99$ such that $ab + 1$, $ac + 1$ and $bc + 1$ are perfect squares. We desire 'c' to have the largest possible value since we want the last day of the century.

It is well known that the general form of a Diophantine triple is $(a, b, a + b \pm 2x)$, where $x^2 = ab + 1$. As we want c to be large, we should choose the '+' sign in the third term of the above triplet. The maximum possible value of ab as per our conditions is $(12)(31) = 372$. Now the largest square less than 373 is 361. Hence we get $a = 12, b = 30$ and the corresponding value of x as 19 which gives $a + b + 2x$ as 80.

Thus the required date is **12/30/80.**

Remark. For each choice of a, b where $ab + 1 = x^2$, there is another choice of c other than $a + b \pm 2x$ which is given by $4x(a + x)(b + x)$. But this choice is not to be considered as we want $c \leq 99$.