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## Georg Cantor (1845–1918)

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Georg Cantor is the mathematician who introduced us to the notion of set theory, a concept taught today even at upper primary school level. It was Cantor who dared to bring the notion of infinity into mainstream mathematics and make it a subject of formal study; an action for which, sadly, he suffered greatly in later life. It was Cantor who helped lay the foundations of the theory of fractals – a notion which has now become part of everyday language and which has played a role in (of all things) the design of the landscape in science fiction films like *Jurassic Park*. And it is Cantor, about whose work one can hear views from leading mathematicians that are as diametrically opposed as the following – Henri Poincaré: “Later generations will regard set theory as a disease from which one has recovered”; Leopold Kronecker has said, “I don’t know what predominates in Cantor’s theory - philosophy or theology, but I am sure that there is no mathematics there” while David Hilbert has written: “No one shall expel us from the Paradise that Cantor has created”. Cantor was nothing if not controversial!

Georg Cantor was born in St. Petersburg, Russia, on 3 March 1845 in a rich and cultured family. His father – Georg Woldemar Cantor – was born in Denmark and was a Protestant. His father had a great love for art and culture. He had a flourishing business spread over several countries. Cantor’s mother – Maria Anna Böhm – was a Russian and a Roman Catholic. She was an accomplished musician. Cantor inherited this musical and artistic talent and was an outstanding violinist.

The climate of St. Petersburg did not suit Cantor’s father. In 1856, the Cantor family moved to Wiesbaden in South Germany where Cantor attended Gymnasium (1856–1863). His university education was in Berlin where he attended lectures of illustrious mathematicians like Weierstrass, Kummer and Kronecker. Hermann Schwarz was a fellow student at Berlin. At Berlin, he submitted a dissertation in number theory. In 1866, he spent a term in the University of Göttingen. He joined the University of Halle soon after as a privatdozent, and before long became an ‘Extraordinary Professor’. By 1879, he was full Professor; he was just thirty-four years old at the time. He served out his entire career at this University.

At Halle, Cantor’s research interests changed from number theory to analysis, mainly at the instance of Heinrich Heine, one of his senior colleagues at that University; it was Heine who introduced him to the theory of trigonometric series



and urged him to attack an open problem in this field – a problem which had resisted the efforts of the leading mathematicians of the day. In 1870, Cantor proved the uniqueness of the representation of a function by trigonometric series. Soon after, he developed a friendship with Dedekind. They exchanged several important letters over a period of many years. During this period, Cantor proved several important results. He gave his construction of real number system and introduced the notions of countable and uncountable sets. In a paper that appeared in 1877, he introduced the notion of one-to-one correspondence. He showed that there are uncountably many reals and there are as many transcendental numbers as real numbers, etc. He showed that there is a one-to-one correspondence between  $[0, 1]$  and  $\mathbb{R}^n$  for all  $n \geq 1$ . He was greatly surprised by this result and wrote, “I see it, but I don’t believe it.” Between 1879 and 1894, he developed set theory, introducing such concepts as cardinal and ordinal numbers. During this period, he formulated the continuum hypothesis on which he struggled for the rest of his life, but without success. (Now we know the reason for that!)

Cantor’s interests extended well beyond mathematics. He had a love for literature, with a deep interest in Shakespeare. He also had strong philosophical and religious interests which led him to exchange letters with many theologians (and even the Pope!). He did not consider his interest in these matters as separate from his love for mathematics. To him, the notion of the infinite was not just a concept in mathematics; it had religious significance as well.

Cantor married Vally Guttamann and they had six children. His profession did not bring in much of an income, but thanks to a generous inheritance from his father, he was able to get by.

Cantor had several mental breakdowns during his later life, resulting in hospitalizations. His condition was aggravated by the sudden death of his youngest son in 1899. Severe criticism of his theory of the infinite by mathematicians of the caliber of Kronecker and Poincaré, not to mention similar reactions from philosophers such as Wittgenstein, his repeated failures to prove the continuum hypothesis and well-order the set of all real numbers, and the discovery of numerous paradoxes in set theory – all added to his miseries. Cantor died on 6 January 1918 at Halle.

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