Ramanujan's Circle

Inspirors, Patrons and Mentors

Utpal Mukhopadhyay

Ramanujan is one of the greatest mathematicians that India has ever produced. Eminent mathematicians of the world have admitted that Ramanujan was a genius. On the path to achieving fame, Ramanujan received generous support from various persons. In this article, an account of the history of the nurturing of Ramanujan by persons around him is given, along with a brief introduction to each of those generous people from India as well as from abroad.

Introduction

The Indian mathematical prodigy Srinivas Ramanujan Iyengar (1887–1920), commonly known as Ramanujan (not Ramanujam as written in his certificate of Primary Examination), achieved his legendary fame largely, but not solely, through his own talent. His great gifts were supported by the concerted effort of a number of personalities (mostly mathematicians) of India and abroad. Ramanujan was born in British India in a lower middle class family without any sophisticated background. So, in spite of his phenomenal intuition, it is doubtful whether it would have been possible for him to acquire so much fame in his very short lifetime without the support offered by his mentors and well-wishers. For instance, a genius like Niels Hendrik Abel (1802–1829) remained unnoticed during his lifetime. In this article, a historical account of the support provided by persons from various quarters has been presented along with brief introductions of those generous persons who lent a helping hand to Ramanujan in his tireless striving for opening up new pathways in mathematical research.

An Unsuspected Stimulant

The first intellectual inspiration that Ramanujan received came from a 'friend'. It was a book, *A Synopsis of Results in Pure and Applied Mathematics*, written by George Shoobridge Carr (see *Box* 1). This book was first published in England in 1880. Ramanujan got its 1886 edition sometime in 1903, a few months before his matriculation examination. In this book, Carr provided more than five thousand mathematical results from various branches of mathematics, viz. algebra, theory of equations, plane trigonometry, spherical trigonometry, elementary geometry, geometrical conics, differential calculus, integral calculus, differential equations

Box 1. George Shoobridge Carr (1837–1914)

George Shoobridge Carr was born in Teignmouth and had his schooling in Jersey. He studied at Gonville and Caius College as well as at Cambridge University for his BA degree in 1880 and MA in 1883. He was a *Senior Optime*, i.e., he secured second class in mathematical Tripos Examination. The first class holders of this examination were called *Wranglers* while those who stood first were known as *Senior Wranglers*. Carr used to coach students preparing for Cambridge Tripos Examination and his book was prepared from his coaching notes, carefully edited by him over fourteen long years. In fact Ramanujan has made Carr immortal in the history of mathematics by making the fullest utilisation of his book.

The speciality of the book was the style of presentation of the results. No complete proof of any result was given in that book; only in some cases hints were supplied. This provoked a genius like Ramanujan to exercise his mental power for proving the results of the book in his own way through heuristic approach. Carr himself commented – *Let them be read once, but recalled often. The difference in the effect upon the mind between reading a mathematical demonstration, and originating one wholly or partly, is very great. It may be compared to the difference between the pleasure experienced, and interest aroused, when in the one case a traveller is passively conducted through the roads of a novel and unexplored country, and in the other case he discovers the roads for himself with the assistance of map.* According to Kanigel [1] – *One can only guess at the effects of a book like Carr's Synopsis on a mediocre or even normally bright student. But in Ramanujan, it had ignited a burst of fiercely single-minded intellectual activity*. After getting this book, Ramanujan's world changed completely and he was so much obsessed with mathematics that he was ready to take the risk of a failure in his FA Examination which ultimately happened. However, this book opened up the eyes of the genius and in this way played a vital role in shaping Ramanujan's future career.

A Quartet of Well Wishers

After Ramanujan's failure in FA Examination in 1905 and 1907, he did his mathematical research in his own way without any interference for five long years. Financial stringency in terms of termination of his scholarship and other mundane difficulties could not divert his concentration from mathematics, his greatest love of life. However, after his marriage with Janaki on 14 July 1909, a secure job became a bare necessity for Ramanujan. In search of a job he first went to V Ramaswamy Iyer (see *Box* 2), the Deputy Collector of Tirukoilur, in early 1910 along with his 'notebooks' in which he used to write down his mathematical results. Ramanujan showed his notebooks to Iyer who, being a mathematician himself, readily recognised the high level of mathematics done by Ramanujan and commented – *I was struck by the extraordinary mathematical results contained in it*. That he had no intention to set up any kind of bondage to hinder Ramanujan's progress in mathematical research is clear from his comments made afterwards – *I had no mind to*

Box 2. V Ramaswamy Iyer (1871–1936)

Ramaswamy Iyer was born at Satyamangalam in the then Coimbatore. After completing his FA course in a local college, he obtained his graduate and postgraduate degrees in mathematics from Presidency College, Madras. He served initially at Central College, Bangalore and afterwards he was appointed to Maharaja's College, Mysore for assisting Principal Weir in teaching of mathematics. Then he joined Madras Provincial Service in 1898 and in 1901 he was appointed as Deputy Collector. His mathematical articles or works were published in *Education Times* and other journals of mathematics. Ramaswamy Iyer founded the Indian Mathematical Society on 4 April 1907 and acted as its Founder Secretary till 1910.

smother his genius by an appointment in the lowest rungs of the revenue department. However he helped Ramanujan by handing him a letter and advised him to see P V Seshu Iyer for fulfilling his need.

Seshu Iyer was Ramanujan's teacher at Government College, Kumbhakonam. At that time Seshu Iyer did not disturb Ramanujan, when in his class the latter was deeply involved in mathematical calculations of his own. Even Iyer inspired Ramanujan to solve mathematical problems published in *London Mathematical Gazette*. Ramanujan met Seshu Iyer, then a Professor at Presidency College, Madras (now Chennai), after a lapse of four years and showed him the letter of recommendation of Ramaswamy Iyer and approached for a job. Seshu Iyer handed over a note of introduction to Ramanujan and advised him to see Dewan Bahadur Ramachandra Rao, the Collector of Nellore.

In December 1910, Ramanujan, accompanied by his friend R Krishna Rao (a nephew of Ramachandra Rao) went to meet Dewan Bahadur who was initially skeptical about Ramanujan's talent. In fact Ramanujan had to meet him four times, the last time against Dewan Bahadur's permission. When Ramanujan showed Dewan Bahadur a letter of Professor Saldhana, an eminent mathematician of Bombay, as well as some of his mathematical results, Rao's doubt was dispelled. Later, Dewan Bahadur Ramachandra Rao wrote about this final meeting in the words – *These transcended existing books and I had no doubt that he was a remarkable man. Then, step by step he led me to elliptic integrals, and hypergeometric series. At last, his theory of divergent series, not yet announced to the world, converted me. Asked him what he wanted.* When Ramanujan replied that he wanted nothing but simple food and sufficient leisure for uninterrupted mathematical research, then just like Ramaswamy Iyer, Dewan

Dewan Bahadur Ramachandra Rao, Collector of Nellore.

Professor P V Seshu Iyer, Maths Professor at Kumbakonam.

Box 3. S Narayan Iyer (1874–1937)

Narayan Iyer was born on 15 December, 1874 at Cumbum near Madurai in Tamilnadu. He came out his examinations with flying colours. After obtaining M A degree in mathematics from St. Joseph's College in Trichinpally, he was appointed as a college teacher. During his tenure in teaching profession, he became acquainted with Sir Francis Spring who was then serving as a railway officer. Spring was very much pleased with Narayan Iyer's mathematical knowledge. After accepting the position of a Chairman of Madras Port Trust, Spring invited Narayan Iyer to Port Trust for working as his Office Manager which Iyer accepted. Afterwards, Iyer was promoted to the Chief Accountant. Being pleased with Narayan Iyer's efficiency, Govt. of India conferred the title 'Rao Bahadur' on him.

Bahadur also thought that Ramanujan should not be bogged down by offering him a job at '*taluk*' office. So he once again sent him to Seshu Iyer with the advice that Ramanujan should stay at Madras and that he would shoulder the expenses for his stay. Since then, Ramanujan received an amount of 25 rupees per month from Dewan Bahadur for about a year. At that time Ramachandra Rao was wise enough to advise Ramanujan to make copies of his notebooks so that they did not get lost.

Then through one of his well-wishers, Ramanujan got a temporary job at the Office of the Accountant General of Madras and worked there for a few weeks. When his job was terminated after a few weeks, Ramachandra Rao sent him to S Narayan Iyer (see *Box* 3), the Chief Accountant of Madras Port Trust. Narayan Iyer was himself a very good mathematician and the then Treasurer of Indian Mathematical Society. So, he could rightly judge that Ramanujan was a genius. Since then Narayan Iyer became one of Ramanujan's mentors. On his advice, on 9 February 1912 Ramanujan submitted an application to Madras Port Trust for a job with a letter of recommendation from E W Middlemast (see *Box* 4), a Professor of Madras Presidency College. This letter of recommendation was a result of Ramanujan's meeting with Middlemast.

Box 4. Edgar William Middlemast

E W Middlemast, son of Edward William Middlemast and Margaret, was born in Northumberland on 9 December, 1864. After his schooling at the Royal Grammar School, he studied in St. John's College, Cambridge during 1882–1886. He was the tenth Wrangler in the Mathematical Tripos, Part I in 1886. From 1888 to 1897 he served as a Professor in Madras Engineering College. Then he was appointed as Principal of Rajmundry College and in 1903 he became the Deputy Director of Public Instruction in Madras Presidency. In 1910, he was appointed as Professor of Mathematics in Madras Presidency College. Middlemast was the President of the Indian Mathematical Society for the year 1915.

Box 5. Sir Francis Joseph Edward Spring (1849–1933)

Sir Francis Spring, son of Rev. Edward Spring and Matilda, was born on 20 January in Magourney in UK of Great Britain and Ireland. After obtaining higher education at Middleton College and Trinity College in Dublin, he joined the Indian Engineering Service in 1870. As a Consulting Engineer to the Govt. of India, he was instrumental in the development of railways in India. Afterwards, Spring was appointed as Chairman of Madras Port Trust. During his Chairmanship, Ramanujan served as a Clerk in Madras Port Trust during 1912 to 1914. Spring was conferred with 'Companion of the Order of the Indian Empire' in 1894 and 'Knight Commander of the Indian Empire' in 1911.

On 1 March, 1912 Ramanujan secured his coveted job when he was appointed as a Class III, Grade IV clerk in the Accounts Section of the Madras Port Trust.

Sir Francis Spring

Ever since Ramanujan's appointment in Madras Port Trust, Narayan Iyer took over the role as one of his mentors. After making arrangements for financial security to Ramanujan, he began to provide intellectual support to him by working with him on mathematics after regular office hours. Moreover, he introduced Ramanujan to Sir Francis Spring (see *Box* 5), the then Chairman of Madras Port Trust and asked for his support. In fact in some of the papers within his file, Spring found some results on elliptic integrals. After enquiring Narayan Iyer about it, he came to know that these results were due to Ramanujan. It will be shown later how Francis Spring also played the role of a patron in the life of Ramanujan. In the meantime, after getting a job in the Madras Port Trust, Ramanujan began to pursue his mathematical research more vigorously and went on discovering many new results. On the other hand, Sir Francis was trying to judge the real depth of Ramanujan's work and sought the advice of A G Bourne, Director of Public Instruction in this regard. Bourne suggested two names; one was W Graham, the Accountant General of Madras who did not give his opinion clearly. In the meantime, at the request of Dewan Bahadur Ramachandra Rao, Professor C L T Griffith of Madras Engineering College sought the help of his teacher Professor M J M Hill of London.

The First Real Judge

Hill wrote his first letter on 12 November, 1912 which Griffith received in the middle of December. In that letter, Hill critically analyzed Ramanujan's mathematical results and pointed out some loopholes in Ramanujan's argument and advised him to go through the book *Theory of Infinite Series* by Bromwich. He also suggested a careful preparation of manuscripts before communicating them for publication. However, decisive comments regarding Ramanujan's originality came from the second letter of Hill (written on 3 December, 1912) in which he wrote

to Griffith – *He has in fact observed certain properties of the earlier Bernoulli numbers and assumed them to be true of them all without proof. For [this and other] reasons, I feel sure that the London Mathematical Society would not have accepted the paper for their Proceedings*. But, afterwards, with respect to certain vital points undermining Ramanujan's work, Hill made the significant comment – *When I was a student in Cambridge, 1876–9, these things were not properly understood and the modern theory has only recently been established on a firm basis. Many illustrious mathematicians of early days stumbled over these difficulties, and so it is not surprising that Mr. Ramanujan, working by himself, has obtained erroneous results. I hope he will not be discouraged*. Thus, Hill for the first time judged Ramanujan's work from the viewpoint of a mathematician without being guided by any kind of emotion. However, these letters failed to satisfy the well-wishers of Ramanujan. Some of them, viz., Singaravelu Mudaliar, Ramanujan's ex-professor at Pachaiyappa's College, Bhavaniswami Rao, one of Ramanujan's professors at Kumbakonam College, Ramanujan's friend Narasimha and mentor Narayan Iyer advised him to send letters to other European mathematicians for their comments.

On the Way to a Historic Collaboration

In late 1912 and the early months of 1913, Ramanujan tried to communicate with reputed European mathematicians. First of all he wrote to H F Baker (1866–1956), a Fellow of Royal Society (FRS) and past President of the London Mathematical Society. But no response came from Baker. Then Ramanujan sent a letter to E W Hobson (1856–1933), another FRS as well as Cambridge's Sadleirian Professor of pure mathematics during 1910–1931. Hobson also remained silent.

When one day Ramanujan showed some of his results on prime numbers, Seshu Iyer advised

Box 6. Godfrey Harold Hardy (1877–1947)

G. H. Hardy was born on 7 February in Cranleigh of Surrey in England. After completing his school education, he entered Trinity College, Cambridge in 1896. He was the fourth Wrangler in Mathematical Tripos Examination. His doctoral supervisors were A E H Love and E T Whittaker. Hardy brought rigour in British mathematics which was earlier a characteristic of French, Swiss and German mathematics. In 1906 he was appointed as 'Cayley Professor' of mathematics. In 1910, he became an FRS. From 1911, he collaborated with J E Littlewood (1885–1977) in mathematical research. He passed away on 1 December, 1947 on the very day in which he was scheduled to receive 'Copley Medal', the highest award of the Royal Society. After his demise, one of his doctoral students Edward Titchmarsh wote in an obituary – *He was an extremely kind-hearted man, who could not bear any of his pupils to fail in their research.*

Box 7. Ramanujan's First Letter to G H Hardy

16 January 1913
Madras

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Madras Port Trust Office at Madras on a salary of only £ 20 per annum. I am now at about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation on divergent series in general and the results I get are termed by the local mathematicians as "startling".

Just as in elementary mathematics you give a meaning to a^n when n is negative and fractional to conform the law which holds when n is a positive integer, similarly the whole of my investigations proceed on giving a meaning to Eulerian Second Integral for all values of n. My friends who have gone through the regular course of University education tell me that $\int_0^\infty x^{n-1}e^{-x}\,dx = \Gamma(n)$ is true only when n is positive. They say that this integral relation is not true when n is negative. Supposing this is true only for positive values of n and also supposing the definition $n\Gamma(n) = \Gamma(n+1)$ to be universally true, I have given meanings to these integrals and under the conditions I state the integral is true for all values of n negative and fractional. My whole investigations are based upon this and I have been developing this to a remarkable extent so much that the local mathematicians are not able to understand me in my higher flights.

Very recently I came across a tract published by you styled **Orders of Infinity** in page 36 of which I find a statement that no definite expression has been as yet found for the number of primes less than any given number. I have found an expression which very nearly approximates to the real result, the error being negligible. I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

I remain, Dear Sir, Yours truly
S. Ramanujan

P. S. My address is S. Ramanujan, Clerk, Accounts Department, Port Trust, Madras, India.

Ramanujan to go through the paper 'Order of Infinity' by the famous Cambridge mathematician Godfrey Harold Hardy (see *Box* 6) published in *Cambridge Tracts in Mathematics*. In page 36 of that paper Ramanujan found that "The exact order of $\rho(x)$ [defined by the equation: $\rho(x) = \pi(x) - \int_2^x \frac{dt}{\log t}\int_2^x \frac{dt}{\log t}$ where $\pi(x)$ denotes the numbers of primes less than x] has not yet been determined" and told Seshu Iyer that he had discovered a formula which would indicate the 'order

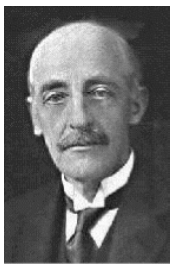
of $\rho(x)$'. Then Seshu Iyer advised Ramanujan to write a letter to Hardy. This letter (*Box* 7), written on January 16, 1913 became a historic one as it was instrumental in the beginning of a significant collaboration between Ramanujan and Hardy.

In his first letter, Ramanujan sent a large number of mathematical results derived by him. Hardy called on Littlewood at night and the two began to evaluate the real merit of the results. According to Hardy, although some results were familiar some others "seemed scarcely impossible to believe". In his lecture to an audience at Harvard, Hardy recalled the experience of going through that letter by the words – *I should like you to begin, by trying to reconstruct the immediate reactions of an ordinary professional mathematician who receives a letter like this from an unknown Hindu clerk*. Observation of the outstanding mathematical results sent by Ramanujan prompted Hardy to comment – *I was the first really competent person who had the chance to see some of his works and can still remember with satisfaction that I would recognize at once what treasure I had found*. Hardy replied to this letter on 8 February, 1913 in which he wrote that he was very much impressed by the theorems sent by him. But, we shall come to that collaboration after reviewing what was going on in another front.

Walker–Spring Venture

Ever since Francis Spring was introduced to Ramanujan, he was trying his best to secure a scholarship for Ramanujan. Sir Gilbert Walker (see *Box* 8), an FRS and a Senior Wrangler, was the Director General of the Indian Observatories in the Indian Meteorological Service. On 25 February, 1913 Walker went to visit the tide related laboratory of Madras Port Trust. There, Sir Spring showed Walker the first letter (dated 8 February, 1913) of Hardy. Reading that letter, Walker was amazed with Ramanujan's mathematical talent and was sure that Ramanujan's work could be compared with that of any of the Fellows of the Cambridge. Being impressed

Box 8. Sir Gilbert Thomas Walker (1868–1958)



Sir Gilbert Walker, son of Thomas Walker and Charlotte Haslehurst, was born in Rochdale of Lancashire. After studying at St. Paul's School, he obtained a degree in metallurgy from Imperial College, London. Walker became a Senior Wrangler of Trinity College in 1889. An applied mathematician Gilbert became Director General of the Observatories in India in 1904. He studied Indian monsoon extensively. In 1923 he published his findings on the alternate change in surface air pressure at Darwin, Australia and the Tahiti Island in the South Pacific known as Southern Oscillation. The 'Walker Circulation' (a particular model of the air flow in the tropics in the Troposphere) is named after him. Sir Walker described it for the first time. Walker was the President of the Royal Meteorological Society during 1926–27. He died at Coulsdon of Surrey.

greatly, Gilbert immediately sent a letter (dated 26 February, 1913) to Dewsbury, the Registrar of the University of Madras recommending Ramanujan's name for a scholarship so that he could continue his mathematical research without any interruption.

On March 19, 1913 the Board of Studies of the University of Madras met and Narayan Iyer was invited (by Hanumanth Rao of Mathematics Department, Engineering College) to present Ramanujan's case to the Board. This resulted in the Board recommending Ramanujan's name to the Syndicate for a research scholarship. On the consent of Pentland, the Governor of Madras, the Board of Studies of the Madras University accepted Walker's recommendation. It was decided that Ramanujan would get a scholarship of Rs. 75/- per month for two years on the condition that Ramanujan would have to submit a quarterly report of his work. On April 17, 1913, after discussion on Walker's recommendation, the Registrar of Madras University penned down the resolution – *The regulations of the University do not at present provide for such a scholarship. But the Syndicate assumes that Section XV of the Act of Incorporation and Section 3 of the Indian Universities Act, 1904 allow of the grant of such a scholarship, subject to the express consent of the Governor of Fort St. George in the Council*. No doubt it was a historic moment when the existing rules bowed down to a genius and it could happen as a joint effort of Francis Spring and Gilbert Walker. Ramanujan received the first research scholarship of the University of Madras from May, 1913 (on leave, on loss of pay, from Madras Port Trust).

In September, 1913 Narayan Iyer communicated some theorems of Ramanujan to the *Journal of the Indian Mathematical Society* with the comments – *The following theorem is due to Mr. S. Ramanujan, the Mathematics Research Student of the Madras University*. Afterwards, on October 26, 1913, by the initiative of Narayan Iyer, Ramanujan met Richard Littlehailes, Professor of Mathematics of Presidency College, Madras. Perhaps, this meeting was arranged to nullify the opposition against Ramanujan regarding granting of scholarship to him.

Neville–Ramanujan Connection

Even before writing his first letter to Ramanujan, Hardy made contact with the India Office. As a result, on 3 February, 1913 one Mallet had sent a letter to Arthur Davies, Secretary to the Advisory Committee for Indian students in Madras. After some days, Davies met Ramanujan and subsequently, due to Francis Spring's initiative, Narayan Iyer informed Ramanujan that Hardy wanted Ramanujan to come to England for continuing his mathematical research. Initially Ramanujan declined to go to England mainly due to religious constraints. To find a way out, Hardy approached Neville (see *Box* 9), who was scheduled to travel to India for delivering a series of 21 lectures on differential geometry, for convincing Ramanujan so that he might go

Box 9. Eric Harold Neville (1988 –1961)

Eric Harold Neville was born on 1 January in London. While studying at William Ellis School, he was encouraged, for his proficiency in mathematics, by his mathematics teacher T P Nunn. He entered Trinity College in 1907 where he came in contact with G H Hardy and Bertrand Russell (1872–1970). He specialised in geometry, particularly differential geometry. Afterwards, he was appointed to the Chair of Mathematics at the University College, Reading. Neville is known for the algorithm invented by him and named after him. Neville's algorithm is used for polynomial interpolation. Neville breathed his last on 22 August at Reading.

to England. After delivering one of his lectures, Neville met Ramanujan in early January, 1914 in the Senate House of Madras University and the latter showed him his notebooks. After some more such meetings, Neville became very pleased as well as astonished with the works of Ramanujan. In the meantime, Ramanujan had changed his mind and agreed to go to England. So, in his letter dated 22 January, 1914 he wrote to Hardy in which he thanked Hardy and Littlewood *to be good enough to take the trouble of getting me to England.* On the other hand Richard Littlehailes took an active role in introducing Neville to high-ranking officials of Madras University and public administration. Neville himself wrote a letter on 28 January, 1914 to Dewsbury for granting a scholarship to Ramanujan so that he could continue his research work in England. On the very next day, Littlehailes also sent a letter to Dewsbury seeking a scholarship for Ramanujan.

Francis Spring also met Lord Pentland, the Governor of Madras and in a letter to his Secretary C B Cottrell on 5 February, 1914 Spring commented – *I am anxious to interest him in a matter which I presume will come before him within the next few days – a matter which under the circumstances is, I believe, very urgent. It relates to the affairs of a clerk of my office named S Ramanujan, who, as I think His Excellency has already heard from me, is pronounced by very high mathematical authorities to be a Mathematician of a new and high, if not transcendental, order of genius.* As a result of these combined efforts of the high ranking persons, the Government of India sanctioned a scholarship for Ramanujan (for two years) on 12 February, 1914 which included free passage to England and a certain amount of money as means of subsistence. On February 26, 1914 Ramanujan received a second class ticket from Binny & Co. for his voyage. Since he was a strict vegetarian, on March 11, 1914 Francis Spring instructed the steamer agents to make sure that Ramanujan could get his vegetarian food during his journey. After everything was settled, Ramanujan started his historic voyage on 17 March, 1914 on board the ship S S Nevasa.

A Historic Collaboration

Ramanujan's first letter to Hardy played a pivotal role in establishing an important collaboration, probably the most significant one, not only for Ramanujan but also in the history of mathematics. During his stay in India, Ramanujan had many well-wishers and mentors. However, his attachment to Hardy was of another dimension and at a much higher level. Seshu Iyer, Narayan Iyer, Ramaswamy Iyer, Ramachandra Rao and Francis Spring, were all persons in the world of mathematics who helped Ramanujan a lot in one way or the other. All of them could recognize Ramanujan's exceptional merit. But, Hardy's place in the life of Ramanujan was of a different nature. It was Hardy who actually guided Ramanujan in his mathematical research, removed some of his misconceptions and taught him the style of writing a research paper. Moreover, through Hardy's effort Ramanujan received due recognition in his short lifetime.

Before going to England, six papers of Ramanujan were published. In England, Ramanujan was trying to re-assess and extend the results (written in his *Notebooks*) he obtained at Madras. When Hardy saw those results he commented – *He (Ramanujan) combined a power of generalization of a feeling far from a capacity of rapid modification of his hypothesis that were really startling and made him in his own peculiar field, without a rival in his day*. Around June, 1914 Ramanujan and Hardy started working on two papers of which one was nearly ready. Hardy used to attend the meetings of the London Mathematical Society on the second Thursday of every month by catching the train at 2:15 pm. In one of those meetings in June, 1914 Hardy presented a paper of Ramanujan in the presence of Hobson, Bromwich, Love and Littlewood. However, Ramanujan abstained from attending that particular meeting.

After going abroad, Ramanujan could publish only one paper in the year 1914. This particular paper, entitled 'Modular Equations and Approximations to Pi', was published in the *Quarterly Journal of Mathematics*. However, as many as nine papers were published in 1915. Of those nine papers, 'Highly Composite Numbers', published in the *Proceedings of the London Mathematical Society* (Vol.2, No.14, pp.347–409), was extraordinary. For this particular paper, Ramanujan was awarded a B A degree (*honouris causa*) from London University. Three papers of Ramanujan were published in the year 1916 – two in *Messenger of Mathematics* and one in *Transactions of Cambridge Philosophical Society*.

It may be mentioned here that according to the condition set up by Madras University, Ramanujan (or his research guide) had to submit a report regarding Ramanujan's progress in research work. In the report on behalf of Ramanujan in June, 1916 Hardy wrote to Dewsbury – *India has produced many talented mathematicians who attained high academical distinction.*

They will be the first to recognize that Ramanujan's work is of different category. Twenty-one papers of Ramanujan were published during his stay in England of which five were joint papers with Hardy. Besides those papers, five 'Short Notes' and six more articles were published in the 'Records' of the *Proceedings of the London Mathematical Society*. In his very short life period, 37 research papers of Ramanujan were published, of which seven were coauthored by Hardy. The topics of the papers published during Ramanujan's stay in England covered definite integral, modular equation, Riemann zeta function, infinite series, summation of series, analytic number theory, asymptotic formulae, modular function, theory of partition, combinatorial analysis. Out of the seven joint papers of Ramanujan and Hardy, two particular papers, viz., 'Proof that almost all numbers *n* are composed of about log log *n* prime factors (published in the *Proceedings of London Mathematical Society*, Vol.2, No.16, 1917) and 'Asymptotic Formulae in Combinatorial Analysis' (published in the *Proceedings of London Mathematical Society*, Vol.2, No.17, 1918) are considered as most significant.

Hardy not only shaped Ramanujan's research career, but also exercised all possible influence to bring the desired recognition for Ramanujan. Hardy strongly believed that selection as a Fellow of the Royal Society was a necessity for Ramanujan to boost his spirit. In the application form for FRS of Ramanujan, Hardy proposed his name and was seconded by P A McMohan (1854–1929). Eleven other signatories in that form were J H Grace (1873–1958), Joseph Larmour (1857–1942), T J Bromwich (1875–1929), E W Hobson (1856–1933), H F. Baker (1866–1956), J E Littlewood (1885–1977), J W Nicholson (1881–1955), W H Young (1863–1942), E T Whittaker (1873–1956), A R Forsyth (1858–1942) and A N Whitehead (1861–1947). Among the signatories, all except McMohan were Wranglers of Cambridge Mathematical Tripos (including six Senior Wranglers). Ramanujan's application form was submitted to the Royal Society on 18 December, 1917. Ramanujan's deteriorating health condition was a source of anxiety for Hardy. So, he wasted no time in convincing the high-ranking persons of the Royal Society regarding Ramanujan's mathematical talent. He communicated detailed information about Ramanujan to the then President of the Royal Society and Nobel Prize winning physicist J J Thompson (1856–1940). Hardy also mentioned in his letter the poor health of Ramanujan and warned that if the Royal Society lingered on Ramanujan's selection then *The Society would have to live forever with its failure to honour him*.

All these efforts of Hardy culminated in the selection of Ramanujan as FRS in the meeting of the Royal Society on 28 February, 1918. After 2 May, 1918 Ramanujan was entitled to write FRS after his name, being the second Indian to achieve this honour. It may be mentioned here that prior to his selection as an FRS, Ramanujan was selected Member of the London Mathematical Society on 6 December, 1917 and Member of the Cambridge Philosophical Society on 18 February, 1918. On 10 October, 1918 Ramanujan was selected as a Fellow of

Trinity College, being the first Indian to achieve this honour. In Ramanujan's selection as a Fellow of Trinity College, Littlewood played a leading role in nullifying the racial issues raised against Ramanujan. When questions arose whether Ramanujan was mentally fit, Littlewood produced two medical certificates to prove Ramanujan's mental fitness. The main argument placed in favour of Ramanujan was "For a Fellow of Royal Society to be denied a Trinity Fellowship would be a scandal." All these honours encouraged Ramanujan in his mathematical research. According to Srinibas Rao – *These awards acted as great incentives to Ramanujan who discovered some of the most beautiful results in mathematics subsequently*. To appreciate the truth of this statement, we note that shortly before his selection in the Royal Society, Ramanujan jumped in front of a running train to kill himself but was saved somehow. He was arrested by Scotland Yard Police but was released through Hardy's intervention. All these inevitably imply that none other than Hardy should be regarded as the foremost mentor of Ramanujan.

Due to his illness, Ramanujan started his return journey to India on 27 February, 1919 on the ship S S Nagoa and reached Mumbai on 27 March, 1919. In his last letter to Hardy written on 12 January, 1920 Ramanujan described his newly discovered 'Mock Theta Function'. The importance of mock theta function was so great that it became the topic of the farewell lecture of George Neville Watson (1886–1965) in the London Mathematical Society on 14 November, 1936. Watson finished his lecture with the words – *….To his (Ramanujan's) students such discoveries will be a source of delight and wonder until the time will come when we shall make our journey to the Garden of Paradise where*

Pale, beyond porch and portal
Crowned with Calm leaves, she stands
Who gathers all things mortal
With cold immortal hands.

Anyway, even after coming back to India, Ramanujan had no respite from his malady and finally he succumbed to his disease on 26 April, 1920. Dewan Bahadur Ramachandra Rao arranged for his cremation through his own son-in-law and Ramanujan's childhood friend Rajagopalachari.

The news of Ramanujan's demise reached Hardy while he was in Oxford. The news read: *By direction of the [University] Syndicate, I write to communicate to you, with feelings of great regret, the sad news of the death of Mr. Ramanujan, F. R. S., which took place on the morning of the 26th April.* Hardy, the principal mentor of Ramanujan, reacted with the words – *It was a great shock and surprise to me to hear of Mr. Ramanujan's death,…… For my part, it is difficult for me to say what I owe to Ramanujan – his originality has been a constant source of suggestion*

to me ever since I knew him, and his death is one of the worst blows I have ever had. Since at that time, publication of the *Journal of the Indian Mathematical Society* was lagging behind schedule, the news of Ramanujan's death appeared in its December 1919 issue. After seven months, Seshu Iyer and Ramachandra Rao wrote two obituaries in the same issue of the journal. In England, Hardy wrote an obituary note in *Nature* which was complemented by addition of more information by Neville a few months later. Afterwards, a more elaborate version of the obituary written by Hardy was published in the *Proceedings of the London mathematical Society* in 1921. The same article was published some time later in the *Proceedings of the Royal Society* and again in *Collected Papers of Ramanujan*, when it was published in 1927. In that article, Hardy wrote – *One gift [Ramanujan's work] has which no one can deny, ….profound and invincible originality*. He went on writing – *He would probably have been a greater mathematician if he had been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of great importance. On the other hand he would have been less of a Ramanujan, and more of a European professor, and the loss might have been greater than the gain.* The *American Mathematical Monthly* also dedicated a lot of space in one of their issues for describing the discoveries and other aspects of Ramanujan. In 1936, Hardy went to Harvard to deliver lectures on Ramanujan and *Ramanujan*: *Twelve Lectures on Subjects Suggested by His Life and Work* was a result of it.

For evaluating Ramanujan's place in the world of mathematics, Bruce C Berndt has said – *Paul Erdös has passed on to us Hardy's personal ratings of mathematicians. Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100, Hardy gave himself a score of 25, Littlewood 30, Hilbert 80 and Ramanujan 100*. Hardy described Ramanujan as *the most remarkable mathematician I have ever seen*.

Aftermath of Eternal Journey

The patrons and mentors of Ramanujan continued performing their duties even after the genius started his eternal journey. The last paper of Ramanujan, entitled 'Congruence Properties of Partitions' was published in *Mathematische Zeitschrift* in 1921 (Vol.9, pp.147–153) by the effort of Hardy. Three years after Ramanujan's death, Hardy edited Chapter XII of Ramanujan's second *NoteBook*. Soon after learning of Ramanujan's death, Hardy had written to Dewsbury: *Is it possible that Madras would consider the question of publishing the papers in a collected form ? There should be some permanent memorial of so remarkable a genius; and this memorial would certainly be the most appropriate form.* In 1927, *Collected Papers of Ramanujan*, containing 355 pages, was published by the Cambridge University Press under the editorship of G H Hardy, P V Seshu Iyer and B M Wilson. Almost the entire works of Ramanujan are in it including the works done in India, even the questions posed by him in the *Journal of the Indian*

Mathematical Society as well as the mathematical portions of his letters to Hardy. This work was published afterwards by Chelsea and Narosa Publishing House in 1962 and 1987 respectively. Bruce C Berndt (from Spinger) has done a great job by editing and publishing the original *Notebooks* of Ramanujan in five volumes (Parts I to V) since 1985. On the other hand, Ramanujan's *Lost Notebook* was recovered in 1976 by George Andrews of Pennsylvania State University in the estate of G N Watson (1886–1965). A facsimile edition of the book was published by Narosa Publishing House in 1987 on the eve of Ramanujan's Birth Centenary.

Epilogue

It is clear that Ramanujan received generous support from many individuals within India and England, from mathematicians as well as public figures. (In this regard his experience was quite unlike, say, that of Niels Henrik Abel, who died unknown and unrecognized by any great mathematician.) The precise cause of his early demise may never be fully known, nor whether it could have been prevented by better care. But it is obvious that it was a tragic loss, not just to India but to the entire world of mathematics.

Suggested Reading

[1] Robert Kanigel, *The Man Who Knew Infinity*, Abacus, 2010.

[2] Satyabachi Sar, *Ganit Jagater Bismoy Ramanujan*; Jnan Bichitra, Agartala, 2006.

[3] Rabindranath Patra, *Srinibas Ramanujan*, Kishore Jnan-Bijnan, Kolkata, 2010.

[4] Apart from the above three books, the author has consulted various sources on the internet and it is not possible to refer to those sources separately. The author is indebted to all those sources.

Utpal Mukhopahyay is a teacher of Barasat Satyabharati Vidyapith. He obtained his PhD degree in physics from Jadavpur University, Kolkata by working on Lambda Dark Energy. His area of research interest is relativistic cosmology. He is also actively engaged in science popularization and has written several popular and semi-popular articles in various English and Bengali journals including *Resonance*. Recently he has coauthored a book *The Accelerating Universe : Role of Dark Energy* published by Lap Lambert Publishing Company, Germany.