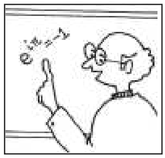

In this section of *Resonance,* we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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Charge Meter: Easy Way to Measure Charge and Capacitance

Some Interesting Electrostatic Experiments

A simple and inexpensive method of building a charge meter is discussed and as an application two experiments are suggested. A charge meter can be used for identification and measurement of charges on dielectric media such as teflon, nylon, silk and rubber balloon obtained by method of friction. Teflon and rubber balloon are found to be negatively charged while silk and nylon are found to be positively charged. A charge meter can also be used to measure capacitance of parallel plate capacitors and hence calculate dielectric constant of the medium between its plates. The values of dielectric constant of transparency plastic sheet and plain paper so obtained are 2.71 ± 0.08 and 4.8 ± 0.29. In the last part, the effective capacitance of a capacitor obtained by placing paper and transparency plastic sheet placed one above the other is shown to be equivalent to a series combination of individual paper and transparency plastic sheet capacitors. These experiments can be pedagogical tools for teaching electrostatics.

Keywords
Charge meter, capacitance meter, parallel plate capacitor, dielectric constant, capacitance, electrostatic experiment.

1. Introduction

The basic concepts of electrostatics are generally introduced at higher secondary school level and later on developed at the undergraduate and postgraduate levels. Students study about charge, electric field and the potential it produces and apply it to understand the working of capacitors and resistors. They also learn about simple circuits consisting of resistors and capacitors. Even though there is substantial theoretical discussion, experiments on these concepts are very few in the curriculum. It is through experiments that the basic concepts should be reinforced. In this article an inexpensive way of building a charge meter is described along with simple electrostatic experiments that can be done with it. In the first experiment, identification and measurement of charges carried by dielectric media such as silk, glass rod, teflon, plastic and nylon is explored. In another experiment, a parallel plate capacitor is built and its capacitance is measured. From this the dielectric constant of the medium is calculated.

1.1 *Charge Meter*

Consider a conductor which carries an unknown charge $Q$and a corresponding potential V with respect to ground (*Figure* 1a). The capacitance C of the conductor is given by $C = Q/V$. For typical objects of laboratory scale (dimensions of few cm), C will be of the order of hundreds of femto farad (fF) or few pico farad (pF). Since C is

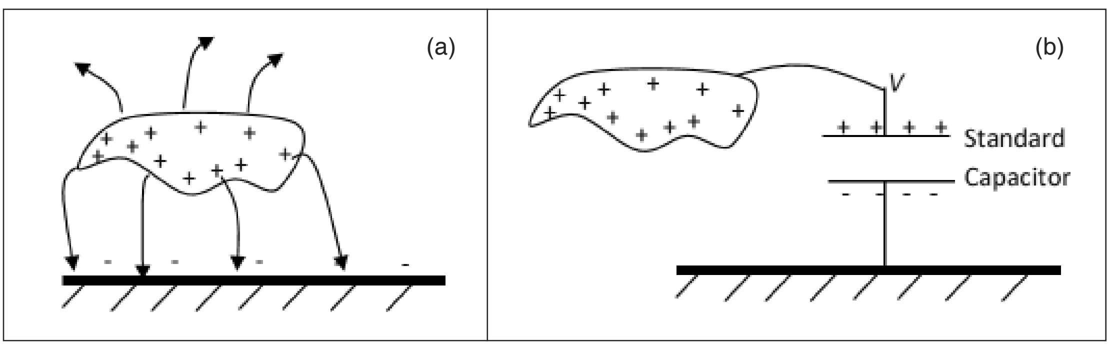

Figure 1. (**a**) Conductor with charge Q and potential V. (**b**) Conductor connected to standard capacitor.

Cheap MOSFET input op-amps are readily available which can be used to construct a high impedance buffer for the multimeter.

also unknown, it is not possible to measure Q simply by measuring V. Therefore the conductor is shorted to a known capacitor of sufficiently large value ($\sim$100 nF) as shown in *Figure* 1b. This is like connecting the unknown capacitance of the charged conductor in parallel with the standard capacitor.

When capacitors are in parallel, their potential is the same and the total charge is shared in proportion to the capacitance. Hence the larger standard capacitor stores almost the entire charge Q. By measuring its potential V and using its known capacitance C_0, the unknown charge $Q = C_0V$ is obtained.

However, measuring the voltage across a standard capacitor of $\sim$100 nF with a conventional multimeter will quickly discharge it since the input impedance is not sufficient. Very high impedance meters known as *electrometers* are available but they are often too sophisticated and expensive. Fortunately, cheap MOSFET input op-amps (such as CA3140) are readily available which can be used to construct a high impedance buffer for the multimeter.

Figure 2 shows a complete charge meter circuit with a 100nF low-leakage polypropylene capacitor connected to non-inverting terminal of an op-amp CA3140 (MOSFET with input impedance of $10^{12}\Omega$) through a resistor. The

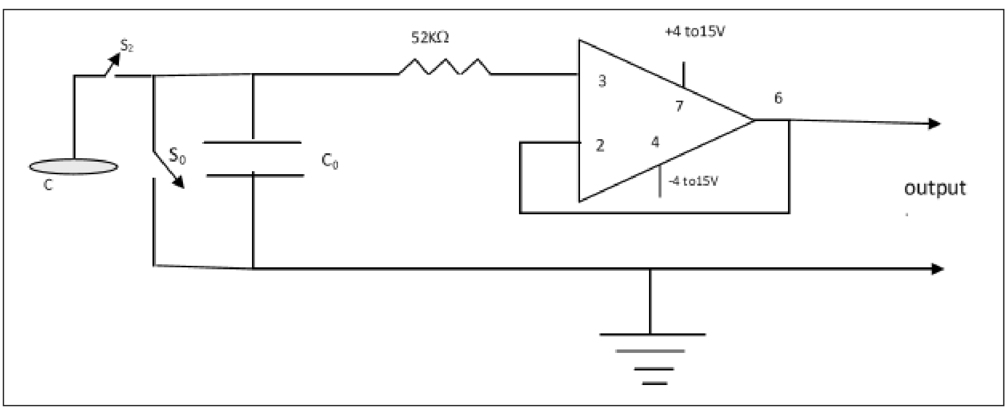

Figure 2. Circuit diagram for charge meter.

inverting terminal is connected to the output to form a unit buffer amplifier. The resistor of 52 kΩ is connected to protect the op-amp from high voltage. C is a charged conductor whose charge has to be measured. S_0 and S_2 are momentary push button switches. Intially S_0 is pressed for a few seconds and released to ensure that C_0 is not holding any charge. Now if switch S_2 is pressed, C gets connected to a standard capacitor C_0 in parallel combination and almost all the unknown charge Q is transferred to it. By measuring the output voltage V, the charge can be estimated using the equation $Q = C_0 V$.

The cost of building the charge meter (excluding DMM) is about Rs.150.

The conductor C can be charged by bringing another charged body (for example, a glass rod rubbed with silk cloth) in contact with it. If switch S_2 is pressed, the charge can be transferred and measured. This way it acts as a charge meter capable of measuring the amount of charge. By noting the sign of the output voltage, the type of charge (positive or negative) can be identified. For this purpose, it is necessary to operate the op-amp with dual supplies.

The entire setup is shown in *Figure* 3. In this circuit a single 9 V battery is used with a resistive voltage divider to obtain ± 4.5 V split supply for the op-amp. It is also possible to use two independent batteries. The cost of building the charge meter (excluding DMM) is about Rs.150.

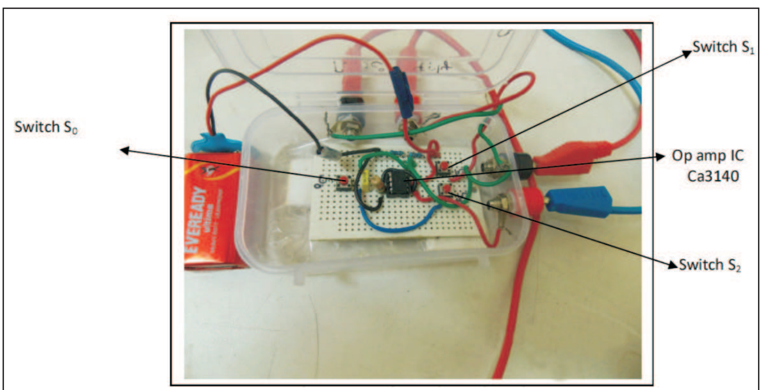

Figure 3. Charge meter housed in a plastic box.

1.2 *Capacitance Meter*

The charge meter can be modified to measure small capacitances accurately. In the modified circuit, capacitor C whose capacitance has to be determined is connected to the circuit as shown in *Figure* 4. When switch S_1 is pressed and released, the capacitor gets charged to the potential V_0 (equal to emf of power supply). Now if we press switch S_2, C gets connected to a standard capacitor C_0 in parallel combination and almost all the charge is transferred. The common potential attained by the combination can be written as

$$V = \frac{V_0 C}{C + C_0} \approx \frac{V_0 C}{C_0} \ . \qquad (1)$$

The common potential appears as output of the op-amp and can be measured using a digital multimeter (DMM). Since C_0 is known, the charge ($V_0 C$) and hence capacitance C can be determined.

A large standard capacitor will help in better transfer of unknown charge and lead to less error. However, a smaller capacitance will give rise to higher voltage, which is easier to measure. In the present circuit, a capacitor of 100 nF is used and in order to increase the voltage, the process of charging and transferring the charge is repeated several times. This helps in building up of the charge V_0C over and above the existing

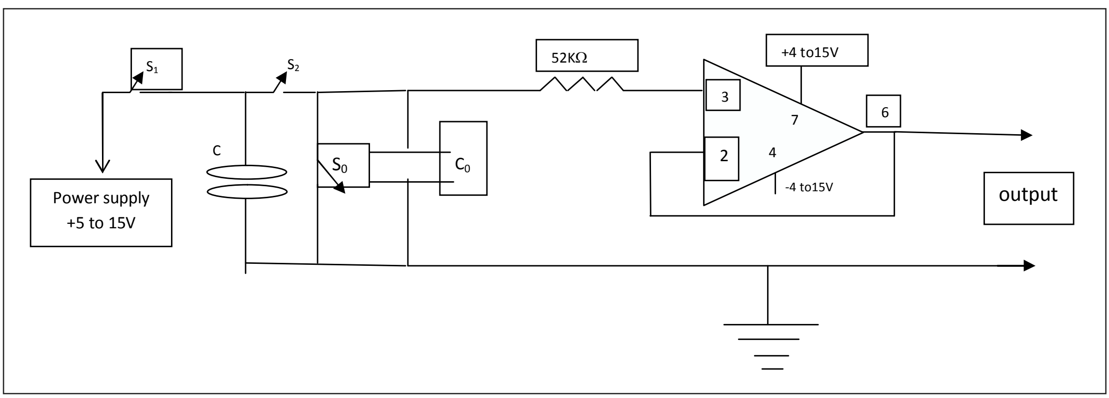

Figure 4. Circuit diagram for capacitance meter.

charge. The charge on standard capacitor can be removed by pressing switch S_0. Before starting the measurement, it is advisable to press switch S_0 to remove stray charges.

In the Triboelectric series, materials which acquire negative and positive charges are given in an order depending on their electron affinity.

2. Experiment to Determine Charge Carried by Dielectric Objects

The concept of charge is introduced to students by demonstrating how suitable objects when rubbed against each other acquire charge (say, a silk cloth rubbed against a rubber rod or a balloon). But the demonstration neither tells us about the amount of charge nor its type. Using the charge meter, the type of charge carried by these objects can be demonstrated along with measuring the amount of charge. In the Triboelectric series (*Table* 1) materials which acquire negative and positive charges

Table 1.
Source: http://en.wikipedia.org/wiki/Triboelectric_effect

Triboelectric Series			
Polyurethane foam	***Most Positively charged***	Rubber balloon	
Hair, oily skin		Resins	
Nylon, dry skin		Hard rubber	
Glass		Nickel, Copper	
Acrylic, Lucite		Sulfur	
Leather		Brass, Silver	
Rabbit's fur		Gold, Platinum	
Quartz		Acetate, Rayon	
Mica		Synthetic rubber	
Lead		Polyester	
Cat's fur		Styrene (Styrofoam – a polystyrene foam)	
Silk		Orlon	
Aluminium		Plastic wrap	
Paper	(***Small positive charge***)	Polyethylene (like Scotch tape)	
Cotton		Polypropylene	
Wool	(***No charge***)	Vinyl (PVC)	
Steel	(***No charge***)	Silicon	
Wood	(***Small negative charge***)	Teflon	
Amber		Silicone rubber	
Sealing wax		Ebonite	***Most negatively charged***
Polystyrene			

are given in an order depending on their electron affinity. From this series the following objects are chosen keeping in mind the availability in the market: a) nylon rod and teflon sheet, b) silk cloth and rubber balloon.

Before the charge meter is used, switch S_0 is pressed to remove stray charges on the standard capacitor. A nylon rod is rubbed against a teflon sheet and brought in contact with an aluminum plate (conductor C in *Figure* 2). The charge is transferred by induction. Switch S_2 is pressed to transfer all the charges from Al plate to standard capacitor. The DMM displays the voltage on the standard capacitor as shown in *Figure* 5a. The charge on the Al plate which is nothing but charge carried by the nylon rod is given by $Q = C_0 V = 100\,\mathrm{nF} \times 0.247\,\mathrm{V} = 24.7 \pm 0.7\,\mathrm{nC}$.

Similarly, charge on the teflon sheet is also measured using the voltage reading shown in *Figure* 5b and its value is -25.7 ± 0.7 nC. Note that teflon is negatively charged while nylon is positively charged and their magnitudes are equal within the experimental uncertainty. The experiment was repeated for the pair of silk cloth and rubber balloon. The voltage readings are shown in *Figures* 5c and 5d respectively. The corresponding values for

Figure 5.
(**a**) Voltage displayed by charge meter when nylon rod is in contact.
(**b**) Voltage displayed by charge meter when teflon sheet is in contact.
(**c**) Voltage displayed by charge meter when silk cloth is in contact.
(**d**) Voltage displayed by charge meter when rubber balloon is in contact.

the charge are 41.2 ± 0.9 nC and -49.6 ± 1.1 nC. The comparison between the electron affinity of teflon and rubber balloon cannot be made easily since the charge acquired by them depends on the shape, size, cleanliness and nature of surface of the objects.

3. Experiment on Parallel Plate Capacitor

A parallel plate capacitor is formed when two conductors of the same size and shape are placed close to each other but not in contact. If a dielectric medium is present between the conductors then it is called a dielectric capacitor. The one used in the experiment is shown in *Figure* 6a. The capacitance C of a parallel plate capacitor depends on the dielectric constant K of the medium between the plates besides the area of the plates (A) and separation between the plates (d) as given below.

$$C = \frac{A\varepsilon_0 K}{d}\,. \tag{2}$$

The capacitance of such a parallel plate capacitor constructed in the laboratory is of the order of nanofarads (nF) and can be measured with the help of capacitance meter described above. The dielectric constant K of the medium can be determined if the capacitance is measured as a function of separation between the plates.

The parallel plate capacitor is made up of two Al plates of length 27.8 ± 0.1 cm and breadth 10.05 ± 0.05 cm separated by thin transparency plastic sheets of known thickness. To reduce the air gap between the sheets, a heavy object is placed on the top plate as shown in *Figure* 6b. The two plates must be connected to the

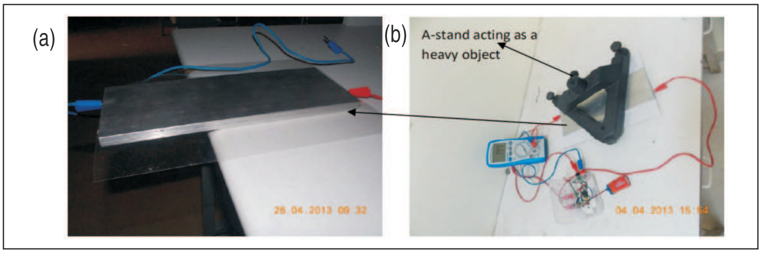

Figure 6.
(a) Parallel plate capacitor consisting of two Al plates with a transparency sheet between them.
(b) The experimental setup for measuring capacitance showing the heavy object placed on the capacitor plates.

Separation between plates (d) in μm	Voltage across capacitor after accumulating the charge 3 times during each trial (mV)			Average voltage per charging process (V) in mV	Capacitance $C = VC_o / V_o$ (nF)	Inverse of capacitance $1/C$ (nF)$^{-1}$
	Trial 1	Trial 2	Trial 3			
88.75	365	365	365	121.67	2.76	0.36
177.50	256	256	255	85.22	1.94	0.52
266.25	203	205	204	68.00	1.55	0.65
355.00	173	172	171	57.33	1.30	0.77
443.75	148	149	149	49.55	1.13	0.88
532.50	131	132	131	43.78	0.996	1.00
621.25	112	112	114	37.56	0.855	1.17
710.00	100	101	101	33.56	0.763	1.31

Table 2.

charge meter in place of C (circuit in *Figure* 4). The separation can be increased by inserting more plastic films. The capacitor is charged to a voltage of 4.39 ± 0.05 V. The charge is transferred to a standard capacitor by pressing switch S_2. The output of the op-amp is measured using a DMM. The charge has been acquired three times for each case (i.e., each time using S_1 to charge and S_2 to transfer). This process should be done sufficiently fast (within 10 s) to ensure negligible charge loss through leakage currents in the capacitor and op-amp. The data recorded is shown in *Table* 2.

3.1 *Data Analysis*

According to equation (2), a graph of C as a function of $1/d$ should be linear and the slope should be equal to $A\varepsilon_0 K$. Since A is known, the dielectric constant K can be determined. The graph is shown in *Figure* 7.

It can be observed that the graph is not linear. This is because the capacitance due to the air gap is neglected. In spite of placing a heavy object on the plates, there is

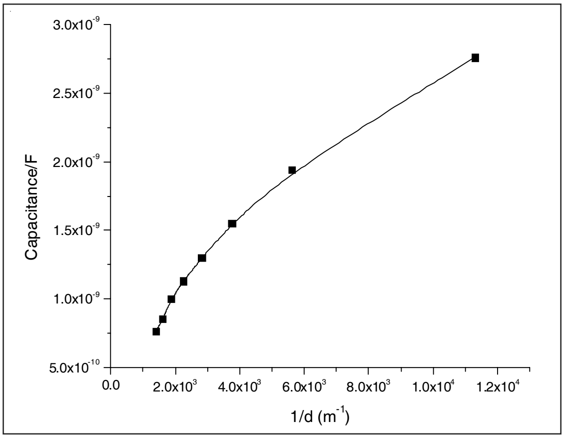

Figure 7. Graph of capacitance as a function of 1/*d*.

still some air gap. The capacitance due to the air gap can be assumed to be in series combination with that due to the dielectric medium. The measured value of capacitance is the effective capacitance across the plates given by

$$\frac{1}{C} = \frac{1}{C_d} + \frac{1}{C_{\rm air}} = \frac{d}{A\varepsilon_0 K} + \frac{1}{C_{\rm air}} \, . \qquad (3)$$

The capacitance due to the air gap can be assumed to be almost a constant. If a graph of $\frac{1}{C}$ as a function of d is plotted, then we will get a straight line with slope equal to $\frac{1}{A\varepsilon_0 K}$ and intercept as $\frac{1}{C_{\rm air}}$. This plot is shown in *Figure* 8. The calculations for error bars indicated in the graph are discussed in *Box* 1.

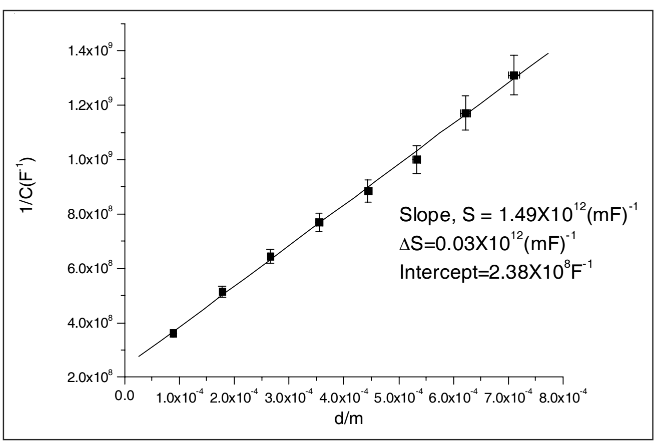

Figure 8. Graph of inverse of capacitance as a function of *d*.

Box 1. Error Analysis

A detailed error analysis has been done for all the cases. As a sample the details for the case of capacitor with transparency plastic sheet is given here.

Uncertainty in measurement of thickness of sheets: Eight sheets of transparency plastic sheets are taken together and its thickness is measured using a micrometer screw gauge with least count of 10 μm. This implies that the uncertainty in measurement of 8 sheets is 10 μm. For one sheet the uncertainty in d is therefore given by

$$\Delta d = \frac{\Delta D}{8} = \frac{10}{8} = 1.25\,\mu\text{m} .$$

Uncertainty in voltage measurement: The voltage across the standard capacitor is measured in 2V range of DMM. The operational manual gives the uncertainty in this range as 0.5 % of the reading + 3 digits, i.e., $\Delta V = 0.5\,\% + 3\text{mV}$. The uncertainty values are shown in column 1 of *Table* A. The voltage used for charging the capacitor has been measured in 20V range and its uncertainty is 0.5% + 3 mV. That is $\Delta V_0 = 0.052$.

Uncertainty in capacitance **(ΔC)**: The expression for estimation of capacitance is given by equation (1). Its uncertainty is obtained by the method of propagation given below.

$$\frac{\Delta C}{C} = \frac{\Delta V}{V} + \frac{\Delta C_0}{C_0} + \frac{\Delta V_0}{V_0}.$$

Hence ΔC_0 is assumed to be 1nC. The values of estimated uncertainty are given in column 2 of *Table* A.

Error in average voltage per charging process (mV)	Error in capacitance ΔC (nF)	Error in inverse of capacitance Δ (1/C) (nF)$^{-1}$
1.61	0.0968	0.0127
1.43	0.0748	0.0199
1.34	0.0644	0.0268
1.29	0.0576	0.0341
1.25	0.0531	0.0416
1.22	0.0495	0.0499
1.19	0.0457	0.0625
1.17	0.0432	0.0742

Table A. Uncertainty Estimation.

Box 1. Continued...

Box 1. Continued...

Uncertainty in $\boldsymbol{1/C}$: The uncertainty in the inverse of capacitance is given by

$$\Delta\left(\frac{1}{C}\right) = \frac{\Delta C}{C^2}.$$

The values are given in column 3 of *Table* A.

Uncertainty in slope (ΔS): The data points in the graph shown in *Figure* 7 are plotted along with error bars given in column 3 of *Table* A. The error in the slope of the graph is obtained using the least squares method. Its value is given by:

$$\Delta S = 0.03 \times 10^{12} (\mathrm{mF})^{-1}.$$

Uncertainty in estimation of K is obtained by the method of propagation given below.

$$\frac{\Delta K}{K} = \frac{\Delta A}{A} + \frac{\Delta S}{S} = \frac{\Delta b}{b} + \frac{\Delta l}{l} + \frac{\Delta S}{S},$$

$$\frac{\Delta K}{K} = \frac{0.05}{10.05} + \frac{0.1}{27.8} + \frac{0.03}{1.49} = 0.038,$$

$$\Delta K = 2.71 \times 0.029 = 0.078.$$

Here b and l are the breadth and length of the plates of the parallel plate capacitor. Thus, the uncertainty in the dielectric constant of transparency plastic sheet is 0.08. A similar analysis for the case of paper capacitor gives the uncertainty in K as 0.29. The uncertainty analysis for the case of two dielectric media gives

a) $\Delta S = 0.53\,\mu\mathrm{m}$; b) $\Delta\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right) = 2.46\,\mu\mathrm{m}$.

This time the graph is linear indicating that capacitance due to air gap does contribute to the effective capacitance. The straight line shown is the least square fit to the data and the value of slope and intercept are given in the graph. The dielectric constant obtained using the value of slope is found to be 2.71 ± 0.08. From the value of intercept, the capacitance of the air gap is found to be 4.2 nF. The experiment has been repeated with a paper of thickness 63.38 ± 1.25 microns as a dielectric medium. The graph is shown in *Figure* 9 and the dielectric constant obtained is 4.8 ± 0.29.

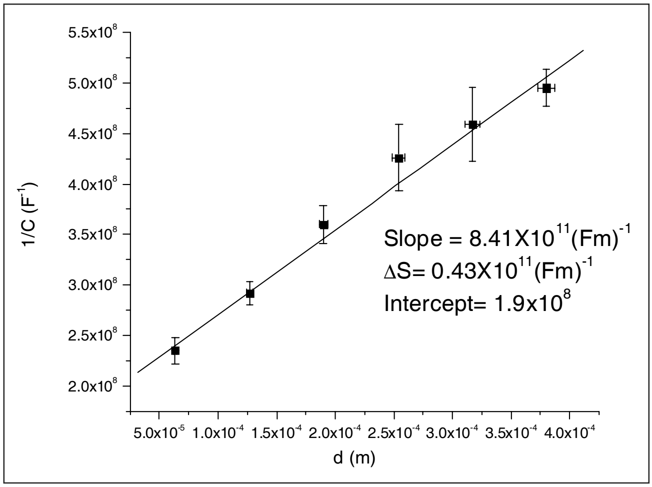

Figure 9. Graph of inverse of capacitance as a function of *d* in the case of paper.

3.2 *Capacitor with Two Different Dielectric Materials*

A parallel plate capacitor can also be obtained by placing two dielectric media (transparency plastic sheet and paper) one above the other as shown in *Figure* 10.

This arrangement can be assumed to be a series combination of two parallel plate capacitors, one with dielectric medium of constant K_1 and thickness d_1 and the other with constant K_2 and thickness d_2. The effective capacitance can be written as

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{\text{air}}}.$$

or

$$\frac{1}{C} = \frac{d_1}{A\varepsilon_0 K_1} + \frac{d_2}{A\varepsilon_0 K_2} + \frac{1}{C_0} = \frac{1}{A\varepsilon_0}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right] + \frac{1}{C_0}\,.$$

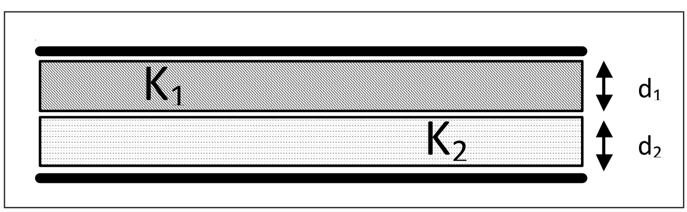

Figure 10. Schematic representation of two different dielectric media.

If n sets of both the dielectric media are placed one above the other, the effective capacitance can be written as

$$\frac{1}{C} = \frac{n}{A\varepsilon_0}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right] + \frac{1}{C_0}\ . \tag{4}$$

An experiment similar to the above was carried out with two dielectric media to verify that the arrangement shown in *Figure* 7 can be considered as a series combination of two capacitors. In this case a graph of $\frac{1}{C}$ is plotted as a function of $\frac{n}{A\varepsilon_0}$ and is shown in *Figure* 11. The slope obtained by the least squares fit method is $43.07 \pm 0.53\,\mu$m. If (4) is correct then this slope must be equal to $\frac{d_1}{K_1} + \frac{d_2}{K_2}$. Substituting the values for d_1, d_2, K_1 and K_2 we get the slope to be equal to

$$\frac{88.75 \times 10^{-6}}{2.79} + \frac{63.375 \times 10^{-6}}{4.8} = 45.0 \pm 2.46\,\mu\text{m}\,.$$

We observe that the slope agrees with the above value within the experimental uncertainty (discussion in the next section). This establishes that the capacitor can indeed be considered as a combination of two capacitors in series.

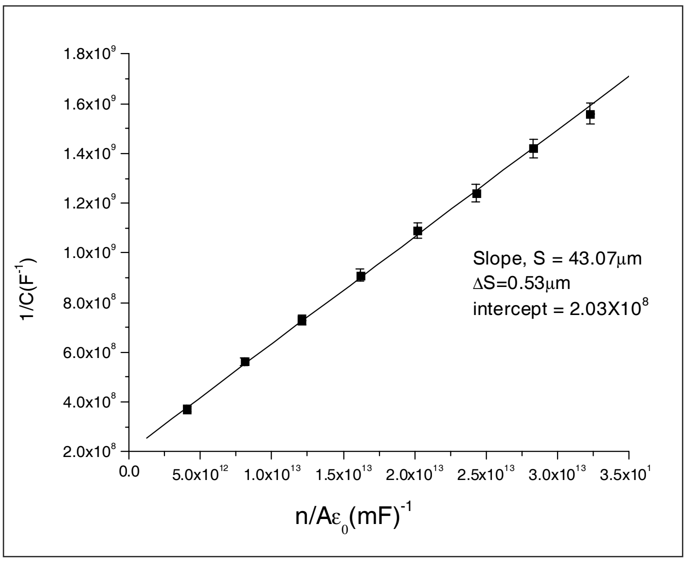

Figure 11. Graph of inverse of capacitance as a function of *d* in the case of combination of two dielectric media.

4. Conclusions

• The charge acquired by teflon, nylon, silk and rubber balloon agrees with those given in triboelectric series. Though the amount of charge can be determined for a particular case of charging, the value of charge depends on the cleanliness of surface, its size and shape. Therefore comparisons of charge on different bodies are not possible.

• The dielectric constant of transparency plastic sheet is 2.71 ± 0.08 and that of paper is 4.80 ± 0.29. It is difficult to compare these values with the literature since compositions of paper and plastic sheet may vary. However they are in the range of values quoted by different sources.

• In the case of a capacitor made up of two dielectric media, the assumption that they can be treated as two separate capacitors combined in series is shown to be correct within the experimental uncertainty. The parallel combination can also be investigated if the two dielectric media of same thickness can be obtained. In fact several types of combinations can also be considered.

• It is interesting to note that the stray capacitance due to the air gap is almost the same for all experiments.

Acknowledgements

One of us (MKR) thanks Prof. M S Hegde, Convener, Talent Development Center, IISc, Kudapura for the constant encouragement and inspiration to do new experiments.

Suggested Reading

[1] *Class XII Physics Part 1,* **NCERT Publication, New Delhi, Chapters 1 & 2, 2006.**