George Pólya – Educator Extraordinaire

If great mathematicians are immortal, George Pólya came close to it – he lived almost to the age of 98!

Pólya is considered the father of mathematical problem-solving in the 20th century. It was his constant refrain that problem-solving was not some innate special ability but can actually be taught to anyone. His famous book *How To Solve It* was published in 1945 after amazingly being turned down by three or four publishers; it has been translated into at least 17 languages. *Mathematical Discovery* – published in 1962 – is also a classic. Teachers of mathematics over the world have reportedly adopted Pólya's suggestions in these two texts with unusually remarkable success. The influence of the delightful volumes *Problems and Theorems in Analysis* by Pólya and Szego on several generations of mathematicians is invaluable (I recommend these to every serious student of mathematics). The Mathematics Geneaology Project lists at least 1739 mathematical descendants for Pólya (his students and his students' students).

George Pólya was born in Budapest, Hungary on December 13, 1887 to Jewish parents who later converted to Catholicism. Due to an injury he suffered during his student days playing soccer, Pólya was deemed unfit for military service. In fact, he was relieved as he had become a pacifist then. In 1914, World War I started, and Pólya moved to Zurich. There he met his future wife Stella Webb and they married in 1918 and remained married until his death in 1985. With the possibility of Germany invading Switzerland, the Pólyas became concerned due to their Jewish background and moved to the United States in 1940, where Pólya was at Brown University for two years before moving to Stanford University. Pólya taught at Stanford until 1978 even though he had retired in 1953. He continued to live in California until his demise on the 7th of September, 1985.

Pólya made fundamental contributions to probability theory (random walks), combinatorics (his enumeration theorem), algebra, number theory, and geometry, but real and complex analysis are considered his main interests. Incidentally, the name 'central limit theorem' was coined by Pólya. Pólya and Szego published a series of problem books on analysis which were different from other problem books in that the problems were listed according to the method of solution and not according to the subject they belong to; to this day, there is not a single problem book in mathematics which comes anywhere close to this gem of a book. The first systematic body of work on 'inequalities' is due to Pólya in collaboration with Hardy and Littlewood which was published as a book. Amazingly, during the preparation of this book, Pólya wrote 31 other research papers in three years (1926–1928). There is a remarkable conjecture due to Pólya (and, independently, to Hilbert) that there is a correspondence between the zeroes of the Riemann zeta function and the eigenvalues of a certain self-adjoint, hermitian operator.

The cover is a reproduction of a page from George Pólya's 1924 paper which displays an illustrative tiling for each of the 17 plane symmetry groups. To explain what the chart means, observe first that patterns which are left invariant by linear combinations of two linearly independent translations repeat periodically in two directions. These planar patterns are classified by means of their groups of transformations which are called plane crystallographic groups – more commonly known as wallpaper groups. A mathematical study of these groups and of their fundamental domains shows that there are exactly 17 of them; this is known from the late 19th century. There are 320 analogous groups in three dimensions. The chart shows how to tile the plane using these 17 groups. The International Union of Crystallography adopted other notations for the 17 wallpaper groups. Pólya's notation is different. In the IUC notation, the symmetry groups associated to the 17 figures on the cover are, respectively, p1, p2, p3, p4, p6, pm, pg, cm, pmm, pgg, pmg, cmm, p4m, p4g, p3m1, p31m, p6m.

Escher was already intuitively aware of the congruence-preserving transformations Pólya spoke of but probably did not understand any of the discussion about symmetry groups. Escher carefully sketched each of these seventeen tilings and studied them, map-coloring some of them. Feeling grateful for the help that Pólya's paper provided, Escher wrote to thank him. He sent Pólya the print of his woodcut 'Development I' and enquired whether Pólya had written a book on symmetry for "laymen" as his article indicated he had hoped to do. A suitcase full of Pólya's notes and other collected letters and papers, now in the Pólya archives at Stanford University, shows that Pólya even sent Escher his own attempt at an Escher-like tiling. Among these papers is Pólya's drawing of a tiling by snakes, inscribed "sent to MCE", at the address where Escher resided from 1937 to 1940. Also, found there is an outline of Pólya's never-completed book *The Symmetry of Ornament and many sketches of tilings*, both for the planned book and for the 1924 article that made such an impression on Escher.

Pólya's enumeration theorem was of fundamental importance in the determination of isomers. Another famous contribution by Pólya is to the random walk problem which can be described as follows. He solved the random walk problem which is described as follows. Here, we walk on an infinite rectangular lattice on the plane, where the probability of walking to any of the adjacent lattice points is equal. Pólya proved that it is almost certain (with probability 1) that we would eventually return to the original point. But, in dimensions more than two, we would almost never (with probabilty 0) return to the original point.

Pólya criticised that many textbooks give students a wrong idea of the way mathematics is done by presenting mathematics as a set of axioms and theorems which follow from them. To alter this, he stated ten guidelines for teachers; these are described in the article by Shailesh Shirali.

Pólya won several accolades and awards for his writings on teaching and learning. In the International Congress on Mathematical Education in 1972, Pólya was honoured along with Jean Piaget. An amusing fact which came to light during that conference was that Stella Pólya and Jean Piaget had been classmates in dancing class as children. The first two volumes of his collected papers appeared in 1974 and the next two appeared in 1984. Pólya had worked on such a spectrum of areas in mathematics that it was a daunting task nigh impossible for any one editor to be acquainted with all the branches. Consequently, the problem was solved by having different editors for different volumes with invited commentaries from appropriate specialists for each volume!

Pólya was one of the most influential mathematicians of the 20th century. Metaphors and precepts he used in his writings have often been quoted; for example:

- If there is a problem you can't solve, then there is an easier problem you can solve; find it.
- Mathematics is not a spectator sport. To understand mathematics means – to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.
- A mathematics teacher is a midwife to ideas.
- What the teacher says in the class is not unimportant but what the students think is a thousand times more important.
- We need heuristic reasoning when we construct a strict proof as we need scaffolding when we erect a building.
- Analogy pervades all our thinking, our everyday speech and our trivial conclusions as well as artistic ways of expression and the highest scientific achievements.
- Solving problems is a practical art, like swimming, or skiing, or playing the piano; you can learn it only by imitation and practice.
- Look around when you have got your first mushroom or made your first discovery; they grow in clusters.

On one occasion, Pólya wrote:

A mathematician who can only generalize is like a monkey who can only climb up a tree and, a mathematician who can only specialize is like a monkey who can only climb down a tree. In fact neither the up monkey nor the down monkey is a viable creature. A real monkey must find food and escape his enemies and so must be able to incessantly climb up and down. A real mathematician must be able to generalize and specialize. There is, I think, a moral for the teacher. A teacher of traditional mathematics is in danger of becoming a down monkey, and a teacher of modern mathematics an up monkey. The down teacher dishing out one routine problem after another may never get off the ground, never attain any general idea. and the up teacher dishing out one definition after the other may never climb down from his verbiage, may never get down to solid ground, to something of tangible interest for his pupils.

In short, one might say:

Said once a mathematician from Budapest
It can be proved if it can be guessed.
His teaching methods
are now bywords.
George Pólya was simply the best!

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