GENERALIZED THEORY OF INTERFERENCE, 
AND ITS APPLICATIONS

Part I. Coherent Pencils

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§ 1. INTRODUCTION

The investigations of which the results are presented in this paper arose during the study of certain specific problems in crystal optics. As investigators in this field are well aware, the simplest procedures for studying the optical properties of anisotropic media (e.g., examination under the polarising microscope) generally involve the use and study of polarised light. The complexity of the peculiar interference phenomena exhibited and also of their customary theoretical analysis (by algebraic methods) become quite
considerable even in the case of transparent optically active crystals like quartz—as may be seen by a reference to the treatises of Mascart (1891) and Walker (1904); this is because the two waves propagated along any direction in such a medium are no longer linearly polarised at right angles to one another, but are elliptically polarised. Nevertheless, the types of ‘oppositely’ polarised waves propagated in such media must be termed simple compared to the elliptically polarised waves propagated in absorbing biaxial crystals.

The remarkable interference phenomena exhibited by absorbing biaxial crystals may be easily studied by looking at an extended source through a plate (cut normal to an optic axis); a suitable material is the mineral iolite—which the author had the opportunity of investigating experimentally (Pancharatnam, 1955). (1) With the incident light polarised, and even without the use of an analyser, interference rings are seen, which are feeble but are nevertheless easily visible. (2) When, in addition, an analyser is also introduced, the biaxial interference figures seen are notably different from those seen in transparent crystals under the same conditions. (3) With the analyser in position and with no polariser—i.e., with the incident light completely unpolarised—feeble interference rings are again easily visible. (4) Finally, even when both analyser and polariser are absent, incipient traces of an interference pattern may be discerned.

Viewing these particular phenomena from a slightly broader perspective we see that their analysis is connected with certain general questions concerning the properties of two polarised beams travelling along the same direction. We shall now formulate these problems since they form the main content of the paper. The study of the effects with a polariser alone leads us to investigate the following questions: the interference of two coherent beams in different states of elliptic polarisation (§ 3); the resolution of any polarised beam into two beams in given states of polarisation—which occurs at the first face of the crystal plate (§ 4); and the composition of two coherent beams of different polarisation—at the second face of the plate (§§ 5, 6). The problem involved when an analyser is also introduced (keeping the incident light polarised) reduces to the following: the interference of two coherent polarised beams which are ‘brought to the same state of elliptic vibration’ by the use of a suitable analyser (§ 8). In § 9 we shall consider the addition of $n$ coherent beams in different states of polarisation.

An attempt to formulate in general terms the problems associated with (3) and (4) leads rather unexpectedly into the subject of the partial coherence of polarised beams. We shall leave the discussion of this subject for Part II.
Some figures from other papers by Pancharatnam (Caption by RN)

**Figure 1.** Two pictures from a later paper showing spiral interference figures. The spiral dark lines represent constant (180°) phase difference between two waves as seen by an analyser, and can best be understood in terms of the Pancharatnam phase.

**Figure 2.** This figure is from an unrelated paper on the mirage, which is the only one co-authored with Raman. The paper has a theoretical part, but also an ingenious experiment in which a hot plate was held vertical so that the layers of varying refractive index were not disturbed by convection. It is an elegant demonstration of multiple imaging resulting in an odd number of images in general, a result which resurfaced in astrophysics much later.