Copernican Revolution in the Complex Plane

An Algebraic Way to Show the "Chief Point" of Copernican Innovation

Giorgio Goldoni

Starting from a simplified model of the Ptolemaic system in the complex plane, we show that Copernicus' innovation did not merely consist of choosing a reference frame in which the planetary motions were simpler, but in finding the size of the planetary orbits expressed in what we now call astronomical units. In modern times a misleading appeal to relativity and a comparison only on the grounds of precision has led some to consider the Ptolemaic and the Copernican systems as basically equivalent. This erroneous point of view has resulted in the neglect of the main scientific content of the Copernican theory and has left Copernicus only to historians and philosophers of science. It is time to restore Copernicus to the teachers of physics as an incomparable opportunity to show the formidable power of theoretical investigation.

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Introduction

It is a widespread and firmly established opinion, clearly expressed by some of the well-known scientific figures of the last century (see *Box* 1), that the Copernican system may be basically obtained from the Ptolemaic one through a change in the reference frame. This view which had a dramatic impact on the psyche of medieval man became quite trivial in the eyes of the modern physicist accustomed to the ideas of Einstein's general theory of relativity. Thus, in modern times, Copernicus's achievement has been belittled to such an extent that the name of Copernicus, the founder of modern science, is rarely mentioned in most modern textbooks of

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Ptolemaic system, Copernican system, Tychonic system, retrograde motion, complex plane, superior and Inferior planets, ecliptic, equant point, epicycle–deferent.

Box 1.

"Relatively, not considering the unknown medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally correct; only the latter is more simple and more practical."

Ernst Mach

··· from Einstein's point of view Ptolemy and Copernicus are equally right. What point of view is chosen is a matter of expediency."

Max Born

"It is meaningless to speak of a difference in truth claims of the theories of Copernicus and Ptolemy; the two conceptions are equivalent descriptions."

Hans Reichenbach

"Today we cannot say that the Copernican theory is "right" and the Ptolemaic one is "wrong" in a meaningful physical sense … So the passion loosed on the world by the publication of Copernicus' book, De revolutionibus orbium coelestium libri VI, were logically irrelevant."

Fred Hoyle

"Copernicus's theory was only a modified form of the Ptolemaic system – assuming the same celestial machinery, but with one or two of the wheels interchanged through the transposition of the roles of the Earth and the Sun."

Herbert Butterfield

"Astronomy is easier if we take the Sun as fixed than if we take the Earth, just as accounts are easier in decimal coinage. But to say more for Copernicus is to assume absolute motion, which is a fiction."

Bertrand Russell

physics. At best, we can find a hint that Copernicus only chose a reference frame in which the motions of the planets were simpler and that he persisted in the error of considering circular orbits. So, the Copernican theory, belittled, as we said, in its original scientific content, has inevitably become a subject only for philosophers and historians of science.

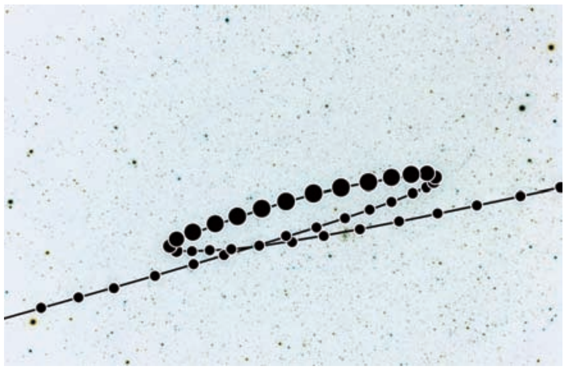
Copernicus, like most of the great scientists, was a unifier.

In this article, we demonstrate that the assumed equivalence between the 'two chief world systems' is completely erroneous and that the idea underlying Copernican innovation goes beyond a mere transformation of reference frame. Starting from a simplified Ptolemaic system in the complex plane, this article demonstrates that the Copernican theory consisted of much more than a change in the reference frame. Indeed, the Copernican theory allowed the determination of the relative sizes of the planetary orbits starting from essentially the same observational data as that used by Ptolemy. Copernicus himself considered this and not the motion of the Earth, the chief point of his theory, as we can infer from a careful reading of some passages in his *De Revolutionibus* and in *Narratio Prima* written by his only disciple, Rethicus.

A Simplified Ptolemaic System in the Complex Plane

Every great scientific theory consists of a basic idea, often very simple, obscured by a lot of technical details. For the expert, technical details are far from being irrelevant and represent the very subject of his work. This is not the case for the teacher, whose primary purpose is to communicate to his students, with the greatest clarity and simplicity, the idea that allowed us to encapsulate in a single conceptual framework, phenomena that previously seemed unrelated to each other. Copernicus, like most of the great scientists, was a unifier, and in order to understand very clearly his brilliant and incomparable contribution, let us begin by analysing a simplified version of Ptolemaic system.

It may be that Ptolemy's main purpose was to provide a means to compile horoscopes, but the really important



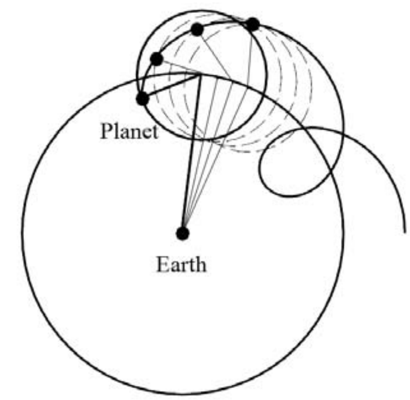


Figure 1. (left) Most of the time the planets move eastward (direct motion) in front of the distant stars, but periodically they reverse their motion (retrograde motion) for a few weeks and trace a loop in the starry sky. Here is the last loop of Mars in Cancer on January 2010. Mars reached the maximum retrograde speed and the maximum brightness on January 29th at the top of the loop.

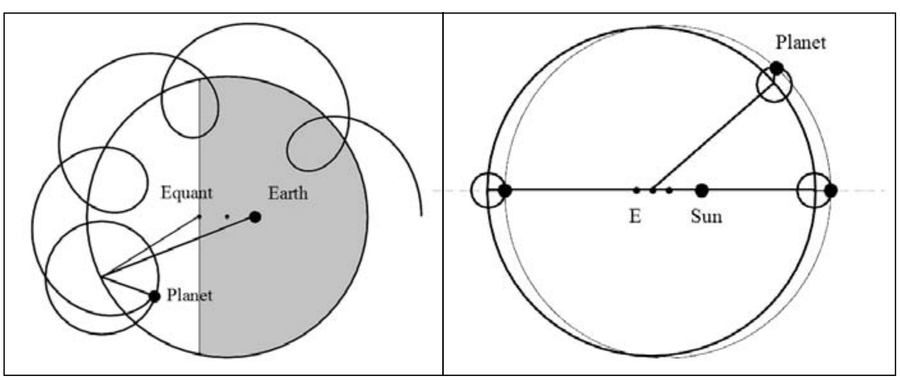
Figure 2. (right) In the model deferent–epicycle by Apollonius, a planet moves on a minor circle (epicycle) whose centre moves on a major circle (deferent) centred at Earth. By adjusting the angular speeds and the ratio of the radii, it is possible to explain the retrograde motion of the planet.

thing is that he developed a powerful mathematical tool to predict the positions of the planets against the background of the fixed stars. Unlike the Sun, that always moves eastward (direct motion) on the Celestial Sphere completing the circle of the ecliptic in one year, the planets periodically reverse their motion for a few weeks (retrograde motion) describing characteristic loops among the stars (*Figure* 1). In the fourth century BC Apollonius explained the motion of the planets by means of the composition of two uniform circular motions. Following the evidence of the senses, the Earth was considered still, while each planet moved on a minor circle (epicycle) whose centre in turn moved on a major circle (deferent) centred in the Earth. With an appropriate choice of the radii and of the angular speeds it was possible to account for the retrograde motions of the planets (*Figure* 2). To reproduce the deviations in latitude it was sufficient to place the epicycle and the deferent on slightly different planes. Ptolemy improved the epicycle–deferent device, introducing the so-called equant point. The deferent was not centred on the Earth and the centre of the epicycle moved with uniform angular speed around a point directly opposite the Earth from the centre of the deferent, the equant point (*Figure* 3). In the case of the Sun there was no need for an epicycle, but only an eccentric circle with an equant point. By means of these mathematical tools Ptolemy obtained a good approximation for the first two of Kepler's laws[1]. Incidentally, Copernicus obtained the same result by adding a small epicycle (epicyclette) to his circular orbits centred in the Sun (*Figure* 4). Therefore it is not true that in the Copernican system the planetary orbits were circles, nor that they were centred on the Sun. They were the composition of two uniform circular motions and the resulting

[1] See *Resonance*, Vol.14, No.12, 2009.

Figure 3. (left) In the Ptolemaic system the deferent of a planet is not centred at Earth and the centre of the epicycle moves at constant angular speed around a point opposite the Earth with respect to the centre of the deferent, called equant point or simply equant. This expedient gives a good approximation for Kepler's law of areas. In fact for an ellipse the difference between the semiaxes goes with the square of the eccentricity and the orbit of a planet with small eccentricity can be approximated by means of a circle eccentric with respect to the Sun. Moreover, with respect to the focus of the ellipse, the planet moves at constant angular speed.

Figure 4. (right) Copernicus resorted to a device similar to Ptolemy's equant, consisting of a little epicycle (epicyclette) revolving in the same direction as the deferent but with double the angular speed and with a diameter equal to the eccentricity e. To a first approximation the planet describes a circle (actually it is an algebraic curve of 5th degree) whose centre is shifted by $e/2$ with respect to the centre of the deferent. The Sun stands at a distance $3e/2$ from the centre of the deferent, while the point on the opposite side at distance $2e$ from the Sun plays the role of Ptolemy's equant.



orbits were not circles but an approximation of Kepler's ellipses. Claiming that the Copernican orbits were circles is as wrong as claiming that a signal that can be decomposed into the sum of sinusoids is a sinusoid.

For the sake of simplicity, henceforth we will make use of deferent–epicycle devices with both circles in the same plane and with deferents centred on the Earth, that is to say, we will ignore the deviations in latitude and Ptolemy's equant point.

In the complex plane, a uniform circular motion centred at the origin is described by an equation of the form

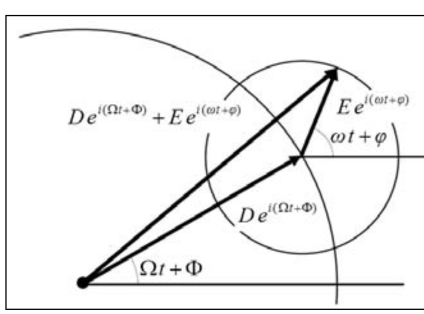
$$z(t) = Re^{i(\omega t+\varphi)} ,$$

where R is the radius, ω is the angular speed and φ is the phase, i.e., the angle corresponding to the initial instant $t = 0$. A deferent–epicycle device is then described by the equation

$$z(t) = De^{i(\Omega\ t+\Phi)} + Ee^{i(\omega t+\varphi)} ,$$

where D and E are respectively the radii of the deferent and of the epicycle (*Figure* 5).

From the commutative law of addition of complex numbers it follows that the path of the planet is the same if we swap the deferent with the epicycle. This means that we may think of the planet as moving on a major circle whose centre moves on a minor circle centred on the Earth (*Figure* 6). Again, if we multiply $z(t)$ by a



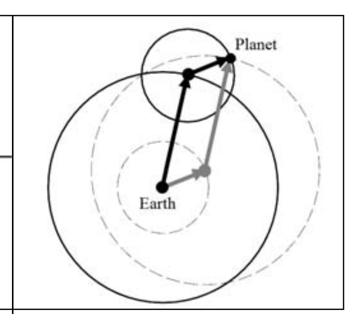


Figure 5. (left) In a simplified Ptolemaic system without the equant and where the planes of the epicycle and of the deferent coincide, the position of a planet is described by the sum of two complex exponentials.

Figure 6. (right) Because of the commutative law for the sum of complex numbers it is possible to swap the deferent and the epicycle without changing the resulting motion of the planet.

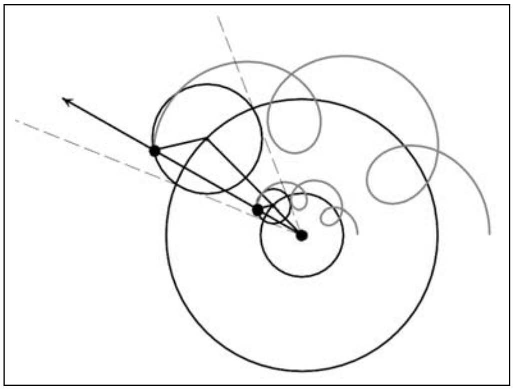


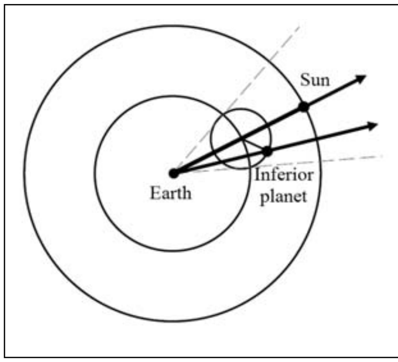
Figure 7. A deferent–epicycle device may be enlarged or shrunk at will without any change in the direction where the planet can be seen from the Earth. In other words, in the Ptolemaic system it is not possible to fix the relative sizes of the planetary orbits.

positive real number, the argument remains the same at every instant and thus only the ratio between D and E is meaningful in order to predict the position of the planet on the Celestial Sphere. Geometrically speaking, this means than we can enlarge or shrink at will the deferent–epicycle device without any change in the direction of the Earth–planet line (*Figure* 7).

But, if we can vary at will the distance of a planet from the Earth, what is the right order of the planets? Ptolemy followed the principle that a slower motion along the zodiac corresponded to a greater distance from Earth, according to our usual experience that distant objects seem to move slower. Applying this principle we find that Saturn is the most distant planet, followed by Jupiter and Mars, but there is a problem with the Sun, Mercury and Venus – they all have a mean zodiacal period of one year. Ptolemy was well aware that only a measure of parallaxes could provide the right answer, but he complained that it was too far beyond his technical abilities. So he followed arbitrarily the ancients, who considered Mercury and Venus below the Sun. For this reason Mercury and Venus were called inferior planets while Mars, Jupiter and Saturn were called superior planets.

Ptolemy followed the principle that a slower motion along the zodiac corresponded to a greater distance from Earth .

Each planet makes loops in the sky periodically but inferior planets, unlike the superior ones that may be seen at



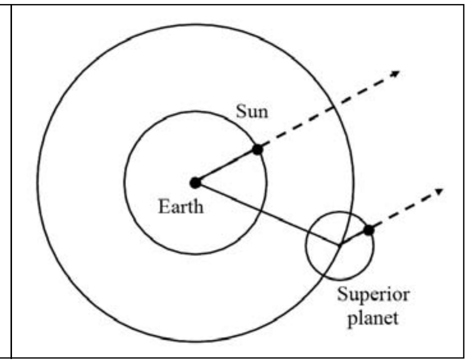


Figure 8. To explain the fact that inferior planets oscillate around a mean position coincident with the Sun, Ptolemy imposed the constraint that the centres of the epicycles had to remain aligned with the Sun.

Figure 9. Concerning the superior planets, Ptolemy imposed the constraint that they had to move along the epicycle in phase with the Sun in order to explain why they were retrograde when in opposition to the Sun.

any angular distance from the Sun, never deviate too far from it. So, Mercury and Venus oscillate around the Sun while it moves along the ecliptic. Ptolemy explained this phenomenon by imposing the constraint that the centres of the epicycles of the inferior planets had to remain aligned with the Sun (*Figure* 8). On the other hand Ptolemy had to impose another kind of constraint for the superior planets. In fact for Mars, Jupiter and Saturn the retrograde motions attain their maximum speed when the planets are in opposition to the Sun, i.e., in the opposite direction of the Sun as seen from Earth. Moreover, in that position their brightness is the highest. In order to explain this behaviour, Ptolemy required that superior planets had to move on their epicycles in phase with the Sun. In other words the line joining the centre of the epicycles with the corresponding superior planet had to remain parallel to the Earth–Sun line (*Figure* 9). This different behaviour of inferior and superior planets was for Ptolemy one more reason to separate the two groups of planets by means of the Sun.

Ptolemy explained the different behaviour of inferior and superior planets introducing two different kinds of constraints.

Let us now number the Earth, the Sun and the planets as follows: 0) Earth, 1) Mercury, 2) Venus, 3) Sun, 4) Mars, 5) Jupiter, 6) Saturn.

In our simplified Ptolemaic system the Earth is still at the origin of the complex plane:

$$z_0(t) = 0 \ .$$

The Sun orbits the Earth on a circle that we may assume with phase $\varphi = 0$:

$$z_3(t) = Re^{i\omega t} \ .$$

Mercury and Venus move on a deferent–epicycle device with the deferent in phase with the Sun:

$$z_k(t) = D_k e^{i\omega t} + E_k e^{i(\omega_k t + \varphi_k)}, \ k = 1, 2 \ .$$

Finally, Mars, Jupiter and Saturn move on a deferent–epicycle device too, but this time with the epicycle in phase with the Sun:

$$z_k(t) = D_k e^{i(\omega_k t + \varphi_k)} + E_k e^{i\omega t}, \ k = 4, 5, 6 \ .$$

About the size of the orbits, the only important thing is that we assume

$$D_1 + E_1 < D_2 - E_2, \ D_2 + E_2 < R < D_4 - E_4,$$
$$D_4 + E_4 < D_5 - E_5, \ D_5 + E_5 < D_6 - E_6 \ .$$

In other words we require that the planets follow the previously established order and that their geometrical devices do not intersect. In the real Ptolemaic system, as we said, orbits were not centred on the Earth, and so the minimum and maximum distances of a planet from Earth were not $D_k - E_k$ and $D_k + E_k$, but slightly different. Moreover, the Sun also had a minimum and a maximum distance from Earth. Ptolemy fixed the size of the orbits in such a way that there was no empty space between an orbit and the next one, starting from the orbit of the Moon, which we have completely ignored in this model. Proceeding in this way Ptolemy deduced that the epicycles of superior planets were smaller than

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Copernicus was in search of a more coherent and unitary arrangement of the orbits of the planets.

the orbit of the Sun, which in our model means $E_k < R, k = 4, 5, 6$.

Copernicus' Innovation

Copernicus was not satisfied by the Ptolemaic system for various reasons, but the main one was the arbitrariness of the order of the planets and of the size of their orbits. He was in search of a more coherent and unitary arrangement. To determine the order of the planets, Ptolemy had to make use of further assumptions, namely he had to resort to the hypothesis that to a greater zodiacal period corresponded a greater distance from Earth. At the same time, this rule did not allow him to determine the relative distances of the Sun, Mercury and Venus. For these Ptolemy made use of entirely arbitrary assumptions. Again, there was no way to fix the size of the orbits, and Ptolemy formulated the further hypothesis that there was no empty space between the area occupied by one planet and the next. Not to mention the mysterious constraints imposed on the deferent–epicycle devices to justify the motion of inferior and of superior planets.

In order to clarify the main idea of Copernicus, we return to our simplified Ptolemaic system in the complex plane. As previously observed, we can multiply the complex number that defines the position of a planet by a positive real number without any change in the argument, that is without any change in the direction in which the planet is seen from Earth. Hence, we have the possibility to change the size of each orbit and eventually the order of the planets with the hope of finding a more coherent arrangement. Let us multiply $z_k(t), k = 1, 2, 4, 5, 6$ by an appropriate value of $\lambda_k > 0$ and then relate the motions of the planets with respect to the Sun by subtracting $z_3(t)$. For the positions of the planets as seen from the Sun we then obtain the following equations:

$$w_k(t) = \lambda_k z_k(t) - z_3(t) = (\lambda_k D_k - R)e^{i\omega_k t} + \lambda_k E_k e^{i(\omega_k t + \varphi_k)}, k = 1, 2,$$

$$w_k(t) = \lambda_k z_k(t) - z_3(t) = \lambda_k D_k e^{i(\omega_k t + \varphi_k)} + (\lambda_k E_k - R)e^{i\omega t}, k = 4, 5, 6.$$

Only by dilating properly each single orbit the planets describe circular orbits around the Sun!

We can readily see that, for an arbitrary choice of the coefficients λ_k, the planets describe epicycloidal orbits relative to the Sun also. But if we choose $\lambda_k = \frac{R}{D_k}$, $k = 1, 2$ and $\lambda_k = \frac{R}{E_k}$, $k = 4, 5, 6$ then one term vanishes in each equation and motions become a lot simpler: they become circular!

$$w_k(t) = \frac{E_k}{D_k} R e^{i(\omega_k t + \varphi_k)},\ k = 1, 2,$$

$$w_k(t) = \frac{D_k}{E_k} R e^{i(\omega_k t + \varphi_k)},\ k = 4, 5, 6.$$

Obviously, in this system the Sun is stationary; therefore,

$$w_3(t) = z_3(t) - z_3(t) = 0,$$

And hence for the Earth, we have

$$w_0(t) = z_0(t) - z_3(t) = -Re^{i\omega t} = Re^{i(\omega t + \pi)},$$

i.e., the Earth is seen from the Sun in a direction opposite to that in which the Sun is seen from Earth.

At this point it is convenient to use the index 0 for the Sun and the index 3 for the Earth.

Let

$$\rho_k = \frac{E_k}{D_k},\ k = 1, 2,$$

If we accept as a principle that the planets and the Earth revolve around the Sun then Ptolemy's constraints on the deferents of inferior planets and the epicycles of superior planets are automatically satisfied.

$$\rho_3 = 1, \omega_3 = \omega, \varphi_3 = \pi,$$

$$\rho_k = \frac{D_k}{E_k},\ \ k = 4, 5, 6.$$

It follows that for $k = 1,...,6$

$$w_k(t) = \rho_k R e^{i(\omega_k t + \varphi_k)}.$$

The Earth and all the planets orbit the Sun! Moreover, the numbers ρ_k express the ratio between the radii of planetary orbits and the radius of the Earth's orbit. In modern language they represent the size of planetary orbits in astronomical units. Hence it is possible to enlarge ($\lambda_k > 1$ for all k) the deferent–epicycle device of each planet in such a manner that all the planets revolve around the Sun. If we accept as a principle that the planets and the Earth revolve around the Sun in circular orbits then we can determine uniquely the relative sizes of planetary orbits without the need for further assumptions. Moreover, referring the motion to the Earth again, Ptolemy's constraints on the deferents of inferior planets and on the epicycles of superior planets are automatically satisfied.

Table 1 shows the present-day values of the semi-major axes of planetary orbits in astronomical units, and the ratio between the radii of the epicycle and the deferent of

Table 1.

k	**Planet**	**Semimajor axes (a.u.)**	$\frac{E_k}{D_k}$	$\frac{D_k}{E_k}$
1	Mercury	**0.3871**	**0.3708**	(2.6969)
2	Venus	**0.7233**	**0.7194**	(1.3900)
4	Mars	**1.5237**	(0.6583)	**1.5191**
5	Jupiter	**5.2028**	(0.1922)	**5.2029**
6	Saturn	**9.5388**	(0.1083)	**9.2336**

each planet and its inverse using the values of Ptolemy. We can immediately see that the numbers representing the relative sizes of planetary orbits lay hidden (with remarkable precision!) in Ptolemy's deferent–epicycle devices, but only with Copernicus do they acquire a clear physical meaning. Moreover, Earth lies between Venus and Mars, because $\rho_1 < \rho_2 < \rho_3 = 1 < \rho_4 < \rho_5 < \rho_6$, and the different behaviours of the inferior and superior planets acquire now a very simple meaning: the inferior planets are those that lie inside Earth's orbit while the superior planets are those that lie outside.

We can immediately see that the numbers representing the relative sizes of planetary orbits lay hidden (with remarkable precision!) in Ptolemy's deferent–epicycle devices, but only with Copernicus do they acquire a clear physical meaning.

Ptolemy's Monster and Copernicus' Adonis

In the Preface and Dedication to Pope Paul III of his *De Revolutionibus*, Copernicus, speaking of his Ptolemaic predecessors, says: " ... *they have not been able to discover or to infer the chief point of all, i.e., the form of the world and the certain commensurability of its parts. But they are in exactly the same fix as someone taking from different places hands, feet, head, and the other limbs – shaped very beautifully but not with reference to one body and without correspondence to one another – so that such parts made up a monster rather than a man.*"

What is this monster? And what are these limbs shaped very beautifully but not with reference to one body and without correspondence to one another? The monster is the Ptolemaic system and the limbs shaped very beautifully are the deferent–epicycle devices of each planet that Ptolemy was able to synchronize in a wonderful way. So why a monster? Because these parts were out of proportion. As Copernicus says, Ptolemy was not able to infer the "chief point of all": the proportion among the parts of the world. Ptolemy's deferent epicycle devices may be enlarged or shrunk at will and he was not able to discover their right size. So, the Ptolemaic world in the eyes of Copernicus was a monster made up of parts beautifully shaped but out of proportion.

Ptolemy built seven independent clocks, one for each planet, whereas Copernicus was able to reassemble the wheels to form a single, coherent, big clock.

Quoting again Copernicus: " ... *if the movements are computed in accordance with the revolution of each planet, not only do all their phenomena follow from that but also this correlation binds together so closely the order and the magnitudes of all the planets and of their spheres or orbital circles and the heavens themselves that nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole.*" So the universe of Copernicus was a single unit. As stated in [5], Ptolemy built seven independent clocks, one for each planet, whereas Copernicus was able to reassemble the wheels to form a single, coherent, big clock.

Georg Joachim Rheticus, the only disciple of Copernicus, in his *Narratio Prima* clearly emphasizes what Copernicus considers the "chief point of all": "*Moreover, ye immortal gods, what dispute, what strife there has been until now over the position of the sphere of Venus and Mercury, and their relation to the Sun. But the case is still before the judge. Is there anyone who does not see that it is very difficult and even impossible ever to settle this question while the common hypotheses are accepted? For what would prevent anyone from locating even Saturn below the Sun, provided that at the same time he preserved the mutual proportions of the spheres and epicycle, since in these same hypotheses there has not yet been established the common measure of the spheres of the planets, whereby each sphere may be geometrically confined to its place?*"

But we find of incomparable beauty and clarity the sentence of Rheticus in which he asserts that, in some sense, his teacher tuned the universe: " ... *my teacher was especially influenced by the realization that the chief cause of all the uncertainty in astronomy was that the masters of this science (no offence is intended to divine Ptolemy, the father of astronomy) fashioned their theories and devices for correcting the motions of the heavenly bodies* *with too little regard for the rule which reminds us that the order and motions of the heavenly spheres agree in an absolute system. We fully grant these distinguished men their honour, as we should. Nevertheless, we should have wished them, in establishing the harmony of the motions, to imitate the musicians who, when one string has either tightened or loosened, with great care and skill regulate and adjust the tones of all the other strings, until all together produce the desired harmony, and no dissonance is heard in any.*" The string of reference for Copernicus was the orbit of the Sun around the Earth, that is the orbit of the Earth around the Sun. Then he adjusted all the other strings, that is the orbits of all the other planets. Compared to Ptolemy's monster, Copernicus' universe was an Adonis.

The transition from the Ptolemaic to the Copernican system did not consist of e simple change of the reference frame.

A Misleading Appeal to Relativity

From the above, it is clear that the transition from the Ptolemaic to the Copernican system did not consist of a simple change of the reference frame. In fact, starting from the Ptolemaic system and referring the motions to the Sun, the planets describe again epicycloidal orbits (*Figure* 10). Copernicus' great insight was that by dilating properly and separately each deferent–epicycle device the planetary system was not only simplified, but the relative sizes of the orbits were completely determined. More precisely, we can enlarge the device of an

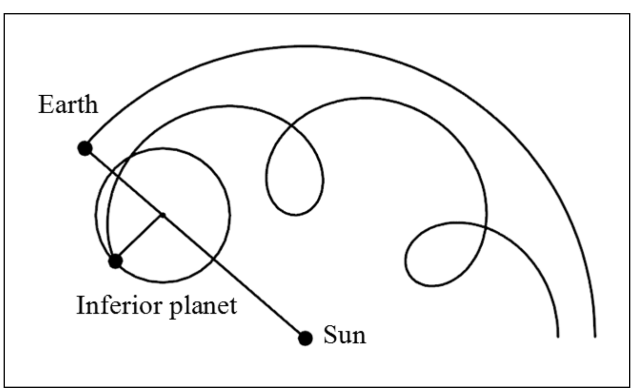
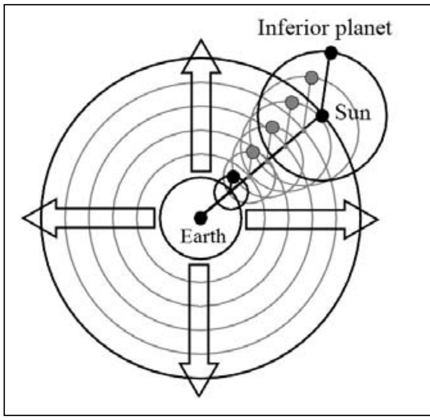


Figure 10. If in the Ptolemaic system we refer the motions of the planets to the Sun then the planets describe again epicycloidal orbits. So it is completely wrong to claim that Copernicus' innovations simply consisted in a change of the reference frame!



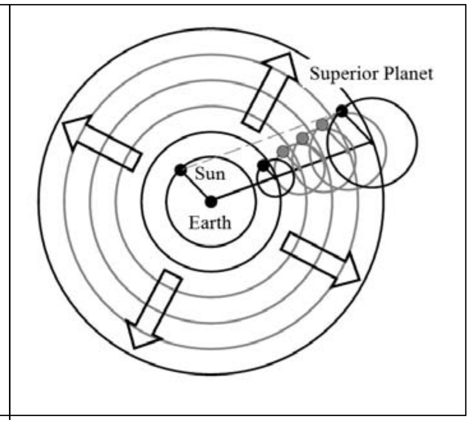


Figure 11. (left) Only if we dilate the deferent-epicycle device of an inferior planet until we overlap its deferent with the orbit of the Sun does the planet orbit the Sun in a simple manner.

Figure 12. **(right)** Only if we dilate the deferent–epicycle device of a superior planet till its epicycle reaches the same size as the orbit of the Sun does the planet orbit the Sun in a circle. In fact in this case the Sun and the Earth form a parallelogram with the planet and the centre of its epicycle, and the distance between the planet and the Sun remains always equal to the radius of the deferent.

inferior planet until it overlaps the centre of the epicycle with the Sun (*Figure* 11). On the other hand, for a superior planet we can first swap the two circles and then enlarge once again the new device until it overlaps the centre of the epicycle with the Sun itself (*Figure* 12). Yet, even great physicists and astronomers claimed the equivalence of the two systems. Whence came this misconception? We believe [5] that it arose from a very diffuse but erroneous approach called 'Ptolemaic involution'. Many authors, in fact, in order to show the above equivalence, analyze the inverse transition from the Copernican to the Ptolemaic system. They show that, as seen from Earth, an inferior planet orbits the Sun while the Sun orbits the Earth. They therefore obtain a deferent–epicycle device where the deferent is the orbit of the Earth and the epicycle is the orbit of the planet. In the case of a superior planet they show once again that, as seen from Earth, it orbits the Sun while the Sun orbits the Earth, but this time the epicycle is greater than the deferent. Finally they show that the two circles can be swapped thus obtaining a correct deferent–epicycle device. This time the deferent is the orbit of the planet while the epicycle is the orbit of the Earth. The conclusion is that one can pass from one system to another by simply interchanging the orbits. In the name of relativity of motion they can hence assert the complete equivalence of the two systems. But the resulting geocentric system is not the Ptolemaic one. In fact, for Ptolemy, the centre of the epicycle of an inferior planet was aligned with the Sun but did not coincide with it. Again, the epicycle of a superior planet was in phase with the Sun but did not have the same size as the orbit of the Sun. The heart of the matter is that infinitely many geocentric systems exist that are able to provide the same angular positions for the planets on the starry sky, each with different relative sizes of the deferent–epicycle devices of the planets and even with a different order of the planets themselves. But there is only one geocentric system equivalent to the Copernican heliocentric one, the so-called Tychonic system. In fact, the great astronomer Tycho Brahe, some decades after the death of Copernicus, suggested a system of the world where the Earth was still and the Sun orbited the Earth, while the planets orbited the Sun. This system, by the principle of relativity, is really equivalent to that of Copernicus.

There exist infinitely many geocentric systems that are able to provide the same angular positions for the planets on the starry sky, but there is only one geocentric system equivalent to the Copernican heliocentric one.

A final question. Why did Copernicus choose to believe in a moving Earth? Would it not be easier to favour a system like that of Tycho? Why believe in such a counter-intuitive idea as the motion of the Earth? The answer is that Copernicus believed, like Ptolemy, in the existence of transparent spheres carrying the planets and the stars. In particular, the spheres were thick enough to contain spherical shells where the epicycles were embedded. In the Tychonic system the sphere of Mars intersects the sphere of the Sun and Copernicus could not believe that such a mechanism could work. So he made the Earth a planet orbiting the Sun like any other planet. And he found this solution to be really wonderful because not only the size of the orbits but also the periods of revolution of the planets were in an increasing order from Mercury to Saturn without any exception: "*Therefore in this ordering we find that the world has a wonderful commensurability and that there is a sure*

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bond of harmony for the movement and magnitude of the orbital circles such as cannot be found in any other way."

Conclusions

From Mach onwards, the leading figures from the world of physics and astronomy such as Max Born, Fred Hoyle and others have reduced the scientific content of the Copernican innovation down to a change in the reference frame. The Copernican theory, deprived of its 'chief point', has then become almost exclusively a matter for philosophers and historians of science, who have had to look elsewhere for a justification of the motion of the Earth. To many of them Copernicus came to believe in a moving Earth because of an aversion against the Ptolemaic equant or because of a hermetic cult of the Sun! The Copernican innovation instead consisted of a prodigious unification. By using basically the same observational data as his ancient predecessors did, Copernicus enclosed in a single conceptual framework, phenomena that previously required different or *ad hoc* explanations; moreover he determined the exact proportions of the planetary orbits. Copernicus believed in Earth's motion because of the greater economy and predictability of his system, thus laying the foundations of the modern criterion on which we measure the value of a theory. It is certainly true that the numbers expressing the ratios of planetary orbits lay hidden in the ratios between the radii of the deferents and epicycles of Ptolemy, but to say that the Ptolemaic system already contained the proportions of the planetary orbits would be like claiming that the macroscopic gas laws already contained Avogadro's number before the atomic hypothesis!

Before Copernicus, astronomy and cosmology were on different levels. There was the cosmology of Aristotle with its spheres, whose purpose was to explain the structure of the cosmos and, in parallel, there was the Ptolemaic astronomy, with its epicycles, deferents and equants – abstract mathematical devices whose purpose was to accurately predict the angular positions of the planets. Copernicus felt the need to unify cosmology and astronomy, something we find obvious today, and found it unacceptable that the Ptolemaic devices did not go beyond the angular positions of the planets, being able to say nothing for sure not only about their distances but also about their order. Even his rejection of the equant was related to the need to give a physical explanation of the cosmos. In fact, Copernicus believed it unacceptable that a sphere could rotate at constant angular velocity around a point other than its centre and in which there was no physical meaning. And that is because he attributed to the rotation of the spheres an importance similar to what Newton gave to the principle of inertia. Hence, for Copernicus the uniform circular motion of a sphere around its centre was a physical principle and he did not consider it acceptable to explain planetary motions by betraying this principle. For consistency, in fact, he replaced the Ptolemaic equant with a small epicycle. Many scholars consider this fact a failure, a kind of fallback, but for Copernicus this was a success because he was able to explain the motion of the planets using a single and clear physical principle. The Copernican innovation represents the greatest and most courageous example of a theory that established itself not because of its agreement with the perception of the senses, but because it translates into a grand mathematical synthesis that can not only explain what is already known but also predict what is not yet known. Copernicus did not live long enough to see the confirmation of his theory and it took almost two centuries for the planetary distances to be measured using the technique of parallax.

Copernicus felt the need to unify cosmology and astronomy, something we find obvious today.

Copernicus' contribution is a scientific theory and not a philosophical one, although its consequences had a huge impact well beyond the scientific environment. We find it scandalous that a quantitative discussion of the Copernican theory, at least in a simplified form like the one used in this article, does not appear in physics courses along with Kepler's laws and Newton's law of gravitation. Simply mentioning that Copernicus said that the planets revolve around the Sun without citing the 'chief point' of his theory would be as trivializing as saying that Newton asserted that all bodies attract each other without citing the quantitative law of universal gravitation and its consequences. I hope this article will motivate the teachers of physics to regain interest in Copernicus, and not leave the father of modern science only to philosophers and historians. Above all, I hope that students will have the opportunity to relive in a conscious way this grand, bold, and unique achievement of human thought.

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Suggested Reading

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