# India's Arrival on the Modern Mathematical Scene

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**Modern mathematics, and modern science in general, was embraced enthusiastically in the Indian subcontinent quite early, thanks to a large extent to the tradition of learning going back to the ancient times. By the early decades of the 20th century the Indian mathematical community had made important research contributions in diverse areas of mathematics, including number theory, real and complex analysis, differential geometry, differential equations, algebra, combinatorial mathematics and applied mathematics. Interaction with world leaders in the field, reforms in the educational system, establishment of societies for actively pursuing study and discussion of mathematics, publication of mathematical journals, were some of the major progressive steps taken. Apart from the indigenous leadership, participation of a few enlightened foreigners in the process also paved the way for the advancement of the subject in the country. We recount here the story of the early developments of the mathematical scene in India, and of the various players involved.**

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Thanks to a unique combination of circumstances India turned out to be one of the first countries outside of Europe to enter the realm of modern science in a big way. Though the modern educational system set up by the British colonial rulers was aimed primarily at training Indians to serve in the subordinate civil service positions in India, the intellectual urges in the Indian society, deeply rooted in tradition and waiting to manifest themselves at the first opportunity, broke through in the new environment and soon transformed the scien-

tific scene in the country. This transformation process was also fostered by some enlightened foreigners, though few in number.

Modern educational institutions began to emerge in the first half of the 19th century. The Hindoo College was founded in Calcutta in 1817 with the expressed intention of providing modern education to the Indian students; in 1855 the college was renamed as Presidency College and brought under government control. The Presidency College in Madras was started in 1840 by Eyre Burton Powell, a professor of mathematics. In 1857, the first modern universities were established, by the British goverenment, in Calcutta (the then capital), Bombay and Madras. Though the universities were modelled on the University of London, with several decentralised colleges where the faculty was largely engaged, onerously, in dissemination of curricular material, and had little opportunity to influence students in to an enlightened approach to knowledge, in course of time various individuals and institutions steered the academic life of the country on a positive track. As the capital, Calcutta offered Indians the best and sustained opportunities to be exposed to modern science, leading eventually to a stream of eminent scientists, J C Bose, P C Roy, C V Raman, S N Bose, M N Saha, ... . Around 1920 Calcutta was one of the most intense sites of scientific activity outside of Europe ([1], p.37).

Mathematics progressed in parallel, in various parts of the country. Everyone knows of the legendary genius Srinivasa Ramanujan (1887–1920), and given the mystique of the prodigy, to many minds it may seem that modern Indian mathematics began with him. While no doubt the indirect impact of the great man on Indian mathematics is immeasurable, the development of the subject in the country has a separate tale of its own, manifesting a collective and conscious effort by the then emergent society, starting well ahead of the advent of

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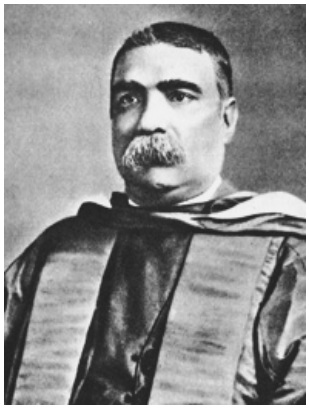
Ramanujan. The founding of two major mathematical societies, the Indian Mathematical Society and the Calcutta Mathematical Society, before Ramanujan had left the shores of India for Cambridge where he received his first professional recognition, stands as ample testimony to this. Indeed, the two societies played an important role in the early development of mathematics in India (see [2] for a detailed account in this respect).

## 1. The Mathematical Societies

The membership of the Indian Mathematical Society from the early times shows a broad spread in terms of geographical locations and vocational pursuits of the members.

The Indian Mathematical Society was established in 1907 (as 'The Analytic Club', which in 1910 was turned into the 'The Indian Mathematical Society') by V Ramaswamy Aiyar, a civil servant who was the Deputy Collector of Gooty (in the Anantapur district of the present state of Andhra Pradesh). The Club subscribed to various international journals to promote an interest in mathematics, and also ran a magazine of its own, which in 1909 was formally styled as a journal. The membership of the Club listed in the first issue, as also in later issues, of the journal (which at that time had the name *Journal of the Indian Analytic Club* which was later changed to *Journal of the Indian Mathematical Society*) shows a broad spread in many respects: geographical locations of the members ranged all the way from Lahore and Amritsar in the north, Trichinapally in the south and Rangoon in the east, many cities in (present) Maharashtra, Gujarat, Madhya Pradesh, Uttar Pradesh, Karnataka, Andhra and Tamilnadu; in terms of vocation the members included teachers and administrators of mathematics and education, engineers, students, as well as civil servants and other bureaucrats (these too in significant numbers). Posing problems and presenting solutions in the subsequent issues was one of the notable features in the early volumes. There were also many short notes on a variety of topics in geometry, trigonometry, arithmetic, etc.

Many members contributed regularly to this activity, including M T Narayaniyengar and S Narayana Aiyar who edited the journal, and R P Paranjapye (known also as Wrangler Paranjapye with his Cambridge title acquired at the Tripos examination) who was an Honorary Member of the Society, and Honorary Librarian of the Society's Library, located at Poona (now Pune). At the end of the 4th issue in Volume 1 there is an extract included from *The Times of India* of 10th July 1909 titled 'Mathematics in India' and it is hard to resist the temptation to quote the following segment from the article: "In these days no science can be considered to deserve the name unless it takes the neat form of mathematics; and the very presence of some masters of mathematical science at Bangalore would have produced there such a mathematical atmosphere. Without mathematics science is very often apt to degenerate into pure empiricism or a barren record of facts." How contemporary!



**Asutosh Mukherjee**
Courtesy:
http://en.wikipedia.org/wiki/Ashutosh_Mukherjee

While the situation had not matured enough to help Ramanujan with his mathematics very much, the interest in mathematics and the formation of a fraternity of mathematically inclined people helped Ramanujan materially until he found his way to G H Hardy, and onward to fame.

The Calcutta Mathematical Society was founded in 1908. Asutosh Mukherjee (1864–1924) who was the Vice Chancellor of the Calcutta University from 1906 to 1924, was instrumental in the Society being established, and he was the President until he passed away in 1924. Though a lawyer by profession, who later served also as a High Court judge, he had profound interest in mathematics, and published some papers on algebraic curves and differential equations. The Calcutta Mathematical Society also started a journal, *Bulletin of the Calcutta Mathematical Society* in 1909. The early issues of the Journal show considerable contact with continental European, especially French, mathematics of that time, which is

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**S Mukhopadhyaya**
Courtesy: Calcutta Mathematical Society

notable especially in the backdrop of the British colonial context of the era. The first issue, dated April 1909, opens with a translation of an article of Emile Borel 'The Method of Poincaré' from *La Revue du Mois* of March 1909. The issues carried a section 'Notes and News' in which many interesting pieces of news were reported, including Hilbert's solution of Waring's problem on representability of positive integers by a fixed number of $n$th powers, a report on the Fourth International Congress of Mathematicians held in Rome (in April 1908), information on award of prizes, prize problems announced by the Academies of Sciences from France, Belgium, Denmark, The Royal Society of Naples, The Prince Jahlonowski Society of Leipsic. Each issue had a section 'Societies and Academies' listing talks held at various learned bodies in Europe and America.

## 2. The Early Achievers

Asutosh Mukherjee is also recognised for promoting mathematics in Calcutta by bringing in several talented mathematicians. One of his finds was Syamadas Mukhopadhyaya (1866–1937), well-known for his theorem in global differential geometry, the 'four vertex theorem', that a simple closed strictly convex curve in the plane, other than the circle, must have at least four points (called vertices) at which the curvature has a local extremum (maximum or minimum) [3]. He obtained his MA degree from the Presidency College, Calcutta, and was later awarded the first Doctor of Philosophy in mathematics from the Calcutta University, for a thesis titled, 'Parametric Coefficients in the Differential Geometry of Curves in an $N$-space', which earned him the Griffith Memorial Prize for 1910. Hadamard is known to have had a high opinion of the work of Mukhopadhyaya ([4], p.246), and stated in the *Memoirs of Collège de France* that the new methods introduced by Mukhopadhyaya would be the subject of discussion in their Colloquia. Mukhopadhyaya also made some interesting contribu-

Mukhopadhyaya was a regular contributor of research papers to the *Bulletin of the Calcutta Mathematical Society* and in particular, his well-known paper on the four vertex theorem was published there.

tions in non-Euclidean geometry. A collection of his papers on geometry was published and its review in Nature [5] states that the papers in it are "of special interest both on account of the original methods employed and the results obtained." Mukhopadhyaya was one of the founding members of the Calcutta Mathematical Society. He served on its Council and was elected Vice President in 1917 and President in 1935; (his tenure as President was short as he passed away in 1937). He was a regular contributor of research papers to the *Bulletin* of the Society. Practically all his work until 1927, including in particular his four vertex theorem mentioned above was published in the *Bulletin*. In later years he published also in *Mathematische Zeitschrift* and the *Tohoku Mathematical Journal*. After his retirement in 1932, Mukhopadhyaya went to Europe to study methods of education, and on returning to India wrote a series of memoirs about this. (See [6] for more details.)

Another notable name from this period is Ganesh Prasad (1876–1935). He was the first DSc from the Allahabad University. After his initial work in India in 1899 he went to Cambridge, England and then to Göttingen where he came in contact with Klein and Hilbert. Soon after his return to India, between 1905 and 1912 he published 3 papers in *Mathematische Annalen* on differential geometry, concerning surfaces of positive curvature. Later on he made several important contributions to potential theory and summability of Fourier series, and also wrote 11 books, including *A Treatise on Spherical Harmonics and Functions of Bessel and Lame* which became a classic. He worked in Allahabad and Benaras before moving to Calcutta in 1923, where he was to continue until his death. He was a keen promoter of research and had guided many students. Apart from mathematical research he had also a great interest in history of mathematics. His book *Some Great Mathematicians of the Nineteenth Century: Their Lives and Their Works*

**Ganesh Prasad**
Courtesy: Indian Mathematical Society

**S Ramanujan**
Courtesy: Bruce Berndt

(Benaras, 1933–1934), in two volumes, adding to over 700 pages, containing portraits of 16 celebrated mathematicians of the nineteenth century is a valuable contribution. This book as well as his earlier book *Mathematical Research in the Last Twenty Years* (Berlin, 1923) have been cited in [7] (p.186 and p.216 respectively). He endowed a fund to enable the Calcutta Mathematical Society to award a regular prize for work related to the history of Indian mathematics; (however the organization in-charge of it never actually made an award). Under his influence two of his doctoral students Avadhesh Narayan Singh and Bibhutibhushan Datta who, even though their doctoral work was in pure and applied mathematics respectively, turned to the study of ancient Indian mathematics, to which they made profound contributions and in particular wrote a book which is an important reference in the area, even today. While in Benaras, he founded the Benaras Mathematical Society. In 1924 he succeeded Asutosh Mukherjee as President of the Calcutta Mathematical Society.

### 3. Ramanujan Appears on the Scene

These are but a few pointers to the upward trajectory in the development of mathematics in India during the early decades of the twentieth century. Then came Ramanujan. The rather unique life story of Ramanujan, with his remarkable mathematical abilities, his finding a benefactor in G H Hardy, his unhappy family life, and his sad demise at the early age of 32 due to a mysterious disease, have all been oft quoted and well documented, covering a variety of aspects and nuances; we shall not go into it here, save for a few remarks, mainly in relation to the Indian mathematical community; for a general reading on Ramanujan [8], [9], and [10] are especially recommended. Discovering Ramanujan is considered one of the proudest achievements of the Indian Mathematical Society (see [11]). Ramanujan's contributions to the Society's journal began to appear in 1911 and his first ar-

ticle 'Some properties of Bernoulli's numbers' attracted considerable attention. Ramanujan contributed several papers to the journal, including some after he went to England. Interestingly an account of his work done in England, written by G H Hardy was published in the *Journal of the Indian Mathematical Society* in 1917. He returned to India in 1919, after a long delay caused by the first world war, to receive a hero's welcome, with the newspapers publishing special articles announcing his arrival. A stipend of 250 pounds had been arranged for him at the University of Madras. There he continued to work on $q$-series and produced his 'lost notebook' (see [9]). However, his health deteriorated rapidly and he passed away in April 1920, within a little over a year after his return from England. Much as he would have indeed liked to, his condition after his return from England did not permit him to play a direct role in the development of mathematics in the country; however he left a huge legacy inspiring generations of students in India by infusing a romanticism in the pursuit of mathematics.

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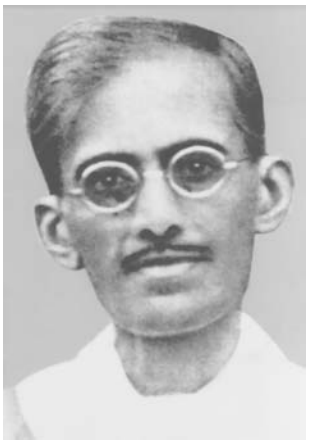
## 4. The Coming of Age

While Madras was not destined to have the benefit of Ramanujan in a guiding role, within a few years of his passing away a duo of mathematicians was to jointly build a congenial atmosphere for the advancement of mathematics there: K Ananda Rau (1893–1966) and R Vaidyanathaswamy (1894–1960). Being only six and seven years younger than Ramanujan, they were both, interestingly, products of the times when career choices were yet to be influenced by the saga of Ramanujan. Like Ramanujan, Rau worked with Hardy at Cambridge, and as a student he won the coveted Smith Prize for his results extending H Bohr's theorem on orders of functions defined by the Dirichlet series. He was an outstanding analyst and had important results to his credit on the summability theory and functions of a complex



**K Ananda Rau**
Courtesy: Ramanujan Institute for Advanced Study in Mathematics, University of Madras

variable. He established very general 'Tauberian theorems' which had resisted the efforts of some of the best analysts. In his later years he worked on modular functions and representations of integers in terms of quadratic forms. A theorem named after him appears in Hardy's classic book *Divergent Series*. He served as professor at Presidency College, Madras from 1919 until 1948 when he attained the then mandatory age for retirement. He was from all accounts a very inspiring teacher, and gave clear and impressive expositions of various topics (see [12]). Many of his students went on to do good work and achieve renown: T Vijayaraghavan, S S Pillai, K Chandrasekharan, M V Subbarao, V Ganapathy Iyer.

Vaidyanathaswamy also went to UK for higher studies, but it was after a period as a teacher and four years as a research scholar at Madras University. He obtained his PhD from St. Andrews University, and worked with H W Turnbull, E T Whittaker and H F Baker during a stay of three years. He ventured into new areas such as symbolic logic, lattice theory, and topology. Returning to India in 1925 he joined the University of Madras in 1927, after a stint at the Benaras Hindu University. Based on his courses of lectures delivered at the University of Madras, he published in 1947 the book *Treatise on Set Topology*, the first book on the topic in English; the first edition was published by the Indian Mathematical Society; a second edition with some improvements was published in 1960 by Chelsea Publishing, New York, and it was reprinted in 1999 by Dover Publications, NY. He retired from the university in 1952 but worked later for a few years at the Indian Statistical Institute (ISI), Calcutta and Sri Venkateswara University, Tirupati. He was actively involved with the Indian Mathematical Society. He was the Editor of the *Journal of the Indian Mathematical Society* over the prolonged period 1927–1950, and his concerted efforts and innovations went a

**K Vaidyanathaswamy**
Courtesy: Indian Mathematical Society

long way in establishing the journal. He was President of the Society during 1940–1942. He was also a founding Fellow of the Indian Academy of Sciences. Besides mathematics he was keenly interested in Carnatic music and yogic sadhana. He was a follower of Sri Aurobindo and held weekly sessions interpreting certain Vedic texts (see [13]).

Annamalai University, situated on the outskirts of a small town called Chidambaram, an ancient pilgrimage site of Hindus about 150 miles south of Madras, was home to A Narasinga Rao (1893–1967) during the years 1929–1946; he then moved to Andhra University at Waltair, on the east coast, and later to the Indian Institute of Technology (IIT), Madras in 1951. Rao worked on Euclidean geometry and was also interested in other fields such as aerodynamics. He was very inspiring to young mathematicians. He was the founder editor of the *Mathematics Student*, a journal founded by the Indian Mathematical Society in 1933, following a decision taken at the Silver Jubilee of the Society in 1932. The journal, which continues to be published to-date, is aimed at mathematics teachers and students, who are also encouraged to contribute articles to it. Rao worked as the editor almost single-handedly with great devotion for 18 years, until 1950, placing it on a firm footing. He also promoted good research. He instituted a medal, named after himself, which is awarded through the IMS in recognition of quality research work by young authors.

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Elsewhere in India too mathematics was flourishing simultaneously, with contributions to various topics. B N Prasad (1899–1966) who was at the University of Allahabad through most of his career, published widely on Fourier analysis. He went to England, where he was a student of Titchmarsh, and to Paris where he interacted with Denjoy and his work was well thought of. In 1958 he founded the Allahabad Mathematical Society and started a journal. At Delhi Ram Behari (1897–

1981), a student of J L Synge at Cambridge and Dublin, was engaged in work on differential geometry (he was at St. Stephen's college to begin with and moved to the University Department in 1947). He served as Secretary to the Indian Mathematical Society during 1936–1942. Rabindranath Sen (1896–1974) at Calcutta also made numerous contributions to differential geometry, especially on parallelism. Nikhilranjan Sen (1894–1963), who was brought to Calcutta by Asutosh Mukherjee in 1917 worked on applied mathematics (theory of relativity and also fluid mechanics). During 1921–1924 he was in Europe, on study leave from the university, where he came in contact with stalwarts like Einstein, Planck, Sommerfeld, de Broglie. Apart from applied mathematics he also studied probability theory and topology. Both N R Sen and R N Sen were active in the Calcutta Mathematical Society and both held the position of President during certain years.

## 5. The Indian Statistical Institute

In 1931 the Indian Statistical Institute (ISI) was founded by P C Mahalanobis (1893–1972), himself an accomplished statistician known for the 'Mahalanobis distance' (a statistical measure gauging proximity between group populations), some important techniques relating to sample surveys, and pioneering studies in anthropometry. Mahalanobis also started the journal *Sankhya – The Indian Journal of Statistics*, which also published mathematical papers, especially in areas such as Probability, Design theory, combinatorics, etc. In post-independence India, as member of the Planning Commission, he contributed significantly to the five-year plans. The renowned statistician C R Rao worked at ISI for almost four decades (including as Director during 1972–1976), until leaving for the United States in 1979. There was also a sizable group at ISI actively pursuing diverse areas of statistical practice and theory. Though its primary focus was on Statistics, ISI has contributed greatly since

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R C Bose (1901–1987) is one of the prominent names associated with the Institute, from its early times. Bose started his research under the guidance of Syamadas Mukhopadhyaya and published some papers relating to convex curves. His career course changed after joining ISI, not quite to statistics, but something related. In 1939 he wrote his celebrated paper on the construction of balanced incomplete block designs answering some problems posed by the renowned statistician R A Fisher during his visit in the previous year as President of the Indian Science Congress, in Bombay (see [14] for a discussion on the genesis and impact of this paper, including some quotes from R C Bose). He went on to prove several important results on design theory and the theory of error-correcting codes. This work involves algebra and number theory, concerning especially the general linear groups over finite fields. Many of his early papers (as also later ones) continue to be cited frequently in recent literature. In the 1940's, he was also part of the University of Calcutta, when a post-graduate department of Statistics of the university was started in ISI. In 1947, he left for the United States. He continued to influence mathematics in India; one of his students, S S Shrikhande, made important contributions to combinatorial mathematics, as also to institution-building for mathematics. ISI was to later produce several excellent mathematicians in various areas of mathematics. The renowned Abel Laureate S R S Varadhan is also a product of ISI.

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**S S Pillai**
Courtesy: Indian Mathematical Society

## 6. The Memorable 1930's

The 1930's are also memorable for Indian mathematics for the achievements of S S Pillai and T Vijayaraghavan. S S Pillai (1901–1950) was a student of Ananda Rau, and was later a lecturer at Annamalai University dur-

It was shown by Jing-run Chen in 1965 that $g(5) = 37$ and the remaining open case $k = 4$ was settled in 1986 in the work of R Balasubramanian, a plenary speaker at ICM-2010, jointly with Deshouillers and Dress, proving $g(4) = 19$.

ing 1929–1941, where his creativity is seen in full bloom. He is well known for his work on Waring's problem, concerning determination of the smallest integer $g(k)$, given a natural number $k$ greater than 1, such that every natural number can be written as a sum of at most $g(k)$ integers that are $k$-th powers themselves; the existence of such a number $g(k)$ was proved by David Hilbert in 1909. By the famous classical theorem of Lagrange $g(2) = 4$. Wieferich and Kempner proved in 1912 that $g(3) = 9$. Pillai gave in 1936 a remarkable almost complete solution towards determination of $g(k)$ for $k$ greater than 7; $g(k)$ was proved to take the expected value $2^k + [(\frac{3}{2})^k] - 2$ (square brackets denote the integral part), under a condition on the fractional part of $(\frac{3}{2})^k$ that is valid when $k$ is even, and very likely holds for all $k$. He showed also that $g(7) = 143$, and in a later paper that $g(6) = 73$. Pillai's achievements were overshadowed to an extent by Dickson's work on the problem, carried out around the same time; Dickson gave the value of $g(k)$ including when the condition alluded to above fails; it is however not known even today whether the condition actually fails for any number at all.

Recognition did come eventually to Pillai for his outstanding contributions on the theme. He worked also on other problems in number theory, especially diophantine approximation. In the latter area he proved the beautiful theorem that given natural numbers $a, b, m$ and $n$ and $\delta > 0$, for integers $x, y$ if $am^x - bn^y$ is nonzero then its absolute value exceeds $m^{(1-\delta)x}$ for all sufficiently large $x$. Based on the theorem he proposed a conjecture, now known after him, on the finiteness of the number of integer solutions of exponential Diophantine equations, which still remains unresolved. He had an invitation to visit the Institute for Advanced Study, Princeton for the year 1950–1951, and was to participate in the International Congress of Mathematicians-1950 at Harvard. However, on his way to the US tragedy struck, and Pil-

lai died in an air-crash near Cairo on 31 August 1950; for various details the reader is referred to [15] and [16].

**T Vijayaraghavan**
Courtesy: Indian Mathematical Society

T Vijayaraghavan (1902–1955) was influenced by Ananda Rau and began to work independently even as a student at the Presidency College, Madras. Like Ramanujan he had problems sailing through a formal educational system, and again, like Ramanujan he sent some of his work to Hardy. Indeed, as in the previous case this led, though in a less dramatic fashion, to his proceeding to join Hardy, then at New College, Oxford. His Oxford years, 1925–1928, were very fruitful and greatly influenced his later works. He proved some impressive theorems on summability, especially Borel summability.

On his return to India he held appointments at Annamalai and then at Aligarh Muslim University where he had close interaction with André Weil; the latter spent two years, 1930–1932, there as Professor of Mathematics and Head of the Department. In 1931 Vijayaraghavan left Aligarh in protest when the Vice Chancellor offered him professorship from which he planned to oust Weil [17] and moved to Dhaka (now in Bangladesh). In a paper published in 1932 he disproved, in a very ingenious and elegant manner, a conjecture of E Borel about the growth of solutions of nonlinear ordinary differential equations [18], [19], [4]. Impressed by the work, G D Birkhoff arranged to have Vijayaraghavan invited as Visiting Lecturer of the American Mathematical Society in 1936.

Vijayaraghavan is also well known for his work on Diophantine approximation, from around 1940, relating especially to the asymptotics of the fractional parts of $\alpha\theta^n$ as $n$ tends to infinity, $\alpha$ and $\theta$ being fixed real numbers, in which he generalized a result of Hardy. This work led him to study a class of numbers which came to be known as Pisot–Vijayaraghavan numbers; (these numbers were also studied by Pisot in his thesis and are also referred

Vijayaraghavan's work on Diophantine approximation led to the study of a class of numbers which are now known as Pisot–Vijayaraghavan numbers.

The Ramanujan Institute of Mathematics later became part of the University of Madras, in 1957, following a period of financial difficulties and the demise of Dr. Chettiar; initially it was independent of the Department of Mathematics of the university, but in 1966 the two were merged.

as PV numbers or Pisot numbers). In 1946 Vijayaraghavan moved to Andhra University, but left it in 1949 for Madras (now Chennai) to take over as Director of the Ramanujan Institute of Mathematics newly founded by businessman-educationist Dr. Alagappa Chettiar. Vijayaraghavan served as Director of the Institute until he passed away in 1955, at a young age, following a heart attack. Vijayaraghavan was also active in the Indian Mathematical Society; he was the Secretary during 1947–1951 and President during 1951–1953; he served also as the Librarian of the Society for four years during 1950–1954.

## 7. Moving Ahead

About a decade younger to Vijayaraghavan and Pillai was another stalwart of Indian mathematics. S Minakshisundaram (1913–1968) indeed was a very gifted mathematician and his work has had a lasting impact. He was initially influenced by Ananda Rau and worked on the topic of summability of series, a passion which was to continue through his life. His first paper was a striking generalisation of a result of Ananda Rau. However, he soon came under the influence of Fr. Racine, a Jesuit missionary about whom I will say more later, and another mathematician M R Siddiqi from Osmania University, Hyderabad and pursued the area of differential equations, to which he made many contributions of the highest quality. He obtained his doctorate in 1940 for his work on nonlinear parabolic differential equations. Some of the ensuing period was very arduous for him, and he had to survive doing some private coaching of students for university examinations, but he stuck to pursuing mathematics despite the adverse circumstances. He later got an appointment as Lecturer in Andhra University, Waltair. In 1946 he received a visiting membership at the Institute for Advanced Study, Princeton where he spent two years.

**S Minakshisundaram**
Courtesy: Indian Mathematical Society

The Princeton years resulted in what is considered his best work, in collaboration with the Canadian mathematician Pleijel, on the eigenvalue problem of the Laplace operator on a compact Riemannian manifold. He introduced the idea of using the heat equation in the study, which turned out to be a powerful tool. It featured as an important component in a new approach to the celebrated index theorems for elliptic operators, due to Atiyah, Singer and Patodi. The paper with Pleijel continues to be cited regularly in contemporary literature, truly a feat for a paper over sixty years old. During a visit of a few months in 1950 to the Tata Institute of Fundamental Research, Bombay, Minakshisundaram authored a book jointly with K Chandrasekharan, titled *Typical Means*, which was very well received. He delivered an invited talk at the International Congress of Mathematicians of 1958, held at Edinburgh, on *Hilbert Algebras*. He was however, not happy in the university setting and though he liked teaching and tried hard to generate interest for research in his students, he felt the system to be rather stifling and longed to be at a research institution. He was eventually appointed as a professor at the newly created Institute for Advanced Study at Simla, and was happy to move there. He had embarked on writing a book on spectral theory, but unfortunately, in 1968, passed away before completing it. (See [20] for more details).

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## 8. Spread of the Mathematical Culture

By the 1940's there were quite a few mathematicians around the country, publishing works of high quality in various branches of mathematics. In number theory it would be appropriate to mention Hansraj Gupta (1902–1988) and S Chowla (1907–1995), both from Punjab. Gupta worked at the Government College, Hoshiarpur, which subsequent to the independence of India was incorporated into the present Panjab University. His most important work is on partitions. Chowla studied in

Cambridge under Littlewood. He worked at the Government College at Lahore during 1936–1947, but following partition of India he left for the United States. Apart from these two, the Panjab University, now situated at Chandigarh, was to see in later years another stalwart in Number theory, R P Bambah (born 1925). Thanks to the efforts of these experts a school in number theory flourished at Panjab University and important results continue to be produced there.

The Kerala School originated by Madhava in the late 14th century discovered many important ideas in Calculus. Though initially brought to light by Charles Whish in the 1830's the work came into focus in contemporary scholarship after the work of Rajagopal.

C T Rajagopal (1903–1978) from the Ramanujan Institute, Madras, who succeeded Vijayaraghavan as the Director of the Institute after the latter's death, contributed greatly to the area of summability theory, publishing in several leading journals including the *Annals of Mathematics*. He worked also on history of Indian mathematics, and has been instrumental in bringing into focus the work of the Kerala School of Mathematics that flourished for over two centuries from late 14th century. R S Varma (1905–1970) who was on the faculty of the University of Lucknow from 1938 to 1946 worked on integral transforms and special functions; Varma later moved to Delhi to be part of the Defence Science Organisation where he was engaged in problems of ballistics and operations research. S M Shah (1905–1996), who was at Aligarh (a junior faculty when A Weil was there) until 1958 when he left for the United States, worked extensively on functions of a complex variable, dealing especially with properties of entire functions. Ganapathy Iyer (1906–1987), who was brought to Annamalai by Narasinga Rao was another excellent mathematician working on entire functions. Both Shah and Iyer were knowledgeable on the Nevanlinna theory. Many of Ganapathy Iyer's papers involve application of methods of functional analysis to spaces of entire functions. D D Kosambi (1907–1966), who was also a colleague of A Weil at Aligarh and worked at Fergusson College, Pune from 1932 to 1946, made important contributions dur-

ing the period to differential geometry, dealing especially with path spaces; during 1946–1962 he worked at the Tata Institute of Fundamental Research, Mumbai – in later years he got interested in statistics and also numismatics and history which became his mainstay (see [21] for more details).

Some women also attained prominence on the Indian mathematical research scene in the 1940's and 1950's.

Notably, there was also at least one woman to have attained prominence on the Indian mathematical research scene by the 1940s. S Pankajam, a student of Vaidyanathaswamy who worked at the University of Madras, published many interesting results in the 1930s and 1940s on logic and foundations of mathematics; another woman student of Vaidyanathaswamy, K Padmavally, worked on Analysis and Topology in the 1950's.

Considerable work was also done in areas of applied mathematics. B R Seth who worked at Delhi during 1937–1949 made may contributions relating to questions of elasticity and fluid dynamics. In 1950 he moved to the Indian Institute of Technology (IIT), Kharagpur where he developed a school of applied mathematics. V V Narlikar (1908–1991), who was at Benaras Hindu University, worked on Relativity; he was well-known around India and popularised the subject. One of his early students P C Vaidya is also well-known for his achievements in the area. B S Madhava Rao (1900–1987) most of whose professional life was spent at the Central College in Bangalore worked on various topics in mathematical physics; after retirement from the Central College he went to the Defence Science Organisation at Pune where he is said to have been very effective. P L Bhatnagar (1912–1976) was at Delhi during 1940 to 1955, and was engaged during that period in pursuing astronomy and astrophysics; a PhD from the University of Allahabad, he holds the distinction of a two-part PhD thesis, one on Astrophysics under A C Banerjee and the other on Summability under B N Prasad. He later moved to the Indian Institute of Science, Bangalore, as Head of the

Levi had a great deal of influence in the introduction of algebra (then called modern algebra) in India, especially in the university curricula; he is sometimes referred as "father of algebra in India".

Department of Applied Mathematics, where he not only made important contributions to fluid dynamics but was also instrumental in developing the subject at the Institute and around the country.

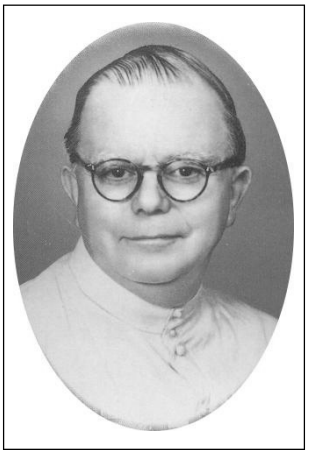
## 9. The Foreign Hand

The picture of the mathematical scenario in India during the 1940s would be incomplete without depiction of the role of a few foreign mathematicians. A Weil's stint at Aligarh during 1930–1932 was noted earlier. On account of the brevity of the stay and the overall context it did not really have much effect, except indirectly through his influence on Vijayaraghavan; Weil's ideas were to have much greater influence on mathematics at the Tata Institute of Fundamental Research during a later stage, which I will not be going into here (see [17]). There were however, two mathematicians from abroad who were in India for a long period and influenced Indian mathematics substantially: F W Levi (1887–1966) and Father Racine (1897–1976).

Levi was a German mathematician who worked at Leipzig during 1920–1935, but had to leave the country on account of his Jewish ancestry when the Nazi's took over. He came to Calcutta as a professor in 1935 and was there until 1948 when he moved to the Tata Institute in Mumbai at the invitation of Homi Bhabha; he later returned to Germany in 1952. Levi's work was in group theory and generalisations and he had a great deal of influence in introducing algebra (then called modern algebra) in India, especially in the university curricula; he is sometimes referred as "father of algebra in India" (see [22]). His seminar at the University of Calcutta on algebra and geometry seems to have influenced the work of R C Bose (see [14]). He was an active promoter of mathematics and was the President of the Indian Mathematical Society for 5 years, during 1942–1947.

Father Racine did his doctorate in Paris in 1934. He

had joined the Jesuit order in 1929 and after his doctorate was sent to India, to work at St. Joseph's College, Tiruchirapally. From there he moved to Loyola College, Madras in 1939 where he stayed until his death in 1976, even though he had retired in 1967. Father Racine had studied under Élie Cartan and Hadamard and had many friends among leading French mathematicians of the time. With that background he greatly influenced students into acquiring a modern view of mathematics. Minakshisundaram was his first great success in this respect, and many more followed in later years (see [17]).



**Rev. Fr. C. Racine**
Courtesy: *Current Science*

## 10. An Evolved Mathematical Community

As this overview of the growth of mathematics in India indicates, mathematics in the country had well and truly arrived on the modern mathematical scene by the 1940's. A large number of papers were published by the researchers from India in leading journals around the world, including the *Annals of Mathematics, American Journal of Mathematics, Duke Journal of Mathematics,* the *Bulletin, Proceedings* and *Transactions of the American Mathematical Society*, the *Journal* and the *Proceedings of the London Mathematical Society*, *Mathematische Annalen,* the *Quarterly Journal of Mathematics (Oxford)*, *Tohoku Mathematical Journal*. Contributions were also made to the *Proceedings of the National Academy of Sciences*, USA as also to *Nature*. Many high quality papers also appeared in journals published from India. Apart from the *Journal of the Indian Mathematical Society*, the *Bulletin of the Calcutta Mathematical Society*, and *Sankhya* mentioned earlier, various other journals had come up and established themselves with good professional standards.

The Indian Academy of Sciences was founded in 1934 and soon started its own journal, the *Proceedings of the Indian Academy of Sciences*; the mathematics papers appeared in Section A of the *Proceedings*, which was

Journal publication activity was carried out with great commitment; for instance it continued even when there was paper shortage during the second world war.

later changed to a series of the *Proceedings* with the name suffixed with '*Mathematical Sciences*'. Many of the established mathematicians contributed regularly to these journals. There were also other journals brought out by various universities or local institutions. The *Journal of the Annamalai University* in which some of Pillai's results appeared, *Proceedings of the Lahore Philosophical Society* in which a large number of notes by S Chowla appeared, *Journal of the University of Bombay* in which some papers of Kosambi and Narlikar are found, *Journal of the Mysore University* to which Madhava Rao contributed, etc.

Many of the journals had also papers published by established mathematicians from abroad. While publishing in local journals precluded reaching out to a wider readership, and in some cases this may have cost the authors much, it contributed greatly in generating a lively mathematical environment. The contributions of the good mathematicians, Indian as well as foreign, were helpful in sustaining the journals. Journal publication activity was carried out with great commitment; for instance it continued even when there was paper shortage during the second world war (see [4]).

## 11. Epilogue

The rest I would say, modifying the common adage, is contemporary history. In 1947 India became independent. Under the leadership of Pandit Jawaharlal Nehru great store was placed on development of science. Tata Institute of Fundamental Research (TIFR) was founded by Homi Bhabha in 1945, and thanks to his enlightened views on the role of mathematics and the efforts of K Chandrasekharan, a strong School of Mathematics was built up at Mumbai. Many new institutions emerged in the post-independence era, including IIT's (Indian Institute of Technology) and new universities. Others were restructured and rejuvenated. TIFR and ISI established

new centres. The impact of all this on the mathematical developments would be a much broader topic, and would deserve to be addressed in detail some day.

I shall conclude this article with a brief mention of some recent developments and events which may be expected to have a major impact on the future of mathematics in India. Apart from the steep increase in the number of universities and IIT's, more recently the IISER's (Indian Institute of Science Education and Research) have come on the scene, with the express objective of promotion of basic sciences and in particular mathematics. Many new prizes and fellowships have been introduced in recent years to encourage pursuit of basic sciences and mathematics. The first International Congress of Mathematicians (ICM) in India (third in Asia) was held at Hyderabad during August 19–27, 2010. The year 2012 has been declared the National Mathematics Year, as a tribute to Srinivasa Ramanujan on the occasion of his 125th birth anniversary, and a variety of activities are being undertaken by various institutions around the country, with substantial financial support from the government, for promotion of mathematics at various levels. On the whole it has been a period of euphoria and commitment to the cause of advancement of mathematics, and one hopes that its impact will be felt in the coming years.

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