The Errors of Feynman and Hibbs

Daniel F Styer

In 1965, Richard Feynman and his former graduate student Albert Hibbs published a textbook on *Quantum Mechanics and Path Integrals*. This text – based on Feynman's teaching of graduate-level quantum mechanics courses at CalTech – is full of remarkable insight and excruciating errors. The errors have been corrected through an emended edition. This article investigates the source of those errors.

Daniel Styer is Professor of Physics at Oberlin College, Ohio. He is the author of several books that include *Relativity for the Questioning Mind, The Strange World of Quantum Mechanics,* and *Quantum Mechanics Simulations*. He has authored very helpful web pages that include 'Solving Problems in Physics', 'Study Tips for Introductory Physics Students', 'The Whys and Hows of Physics Problems'. His teaching and research interests are in statistical mechanics, quantum mechanics, and the use and abuse of computers in physics education.

1. Introduction

To err is human. Richard Feynman made numerous errors simply because he was so smart. He would write down equations that got to the gist of the difficult part of the question under investigation, but that ignored factors of 3 or π, or of $\hbar$ or c, because it was "obvious" (at least to him) what those factors should be. Why write in the limits of integration when it's transparent what those limits should be? For those of us (all of us) who work at a level somewhat below Feynman's, these factors and limits and so forth are not obvious, and their absence can be a real source of distress and of failure to understand.

Most of the 879 errors in *Quantum Mechanics and Path Integrals* [1], however, are not due to the authors. Many errors of typesetting were present in the 1965 edition. Here I give a brief history of the textbook, then give illuminating examples of both types of errors.

Make no mistake: In saying that Feynman made errors I am not attempting to minimize or denigrate Feynman. Instead, I take the attitude that Donald E Knuth took

Keywords
Quantum mechanics, path integrals, Albert Hibbs, Feynman's errors, mathematical rigor.

when he wrote a list of the errors he made while writing the computer program TEX ... that we learn as much through our errors as through our successes [2].

At Caltech Feynman often taught graduate courses in quantum mechanics .... often using the unconventional path-integral formulation. At one point a representative of McGraw-Hill urged him to turn this approach into a textbook...It is easy to see how some of the errors could have been made by typesetters.

2. History

At CalTech Feynman often taught graduate courses in quantum mechanics and advanced quantum mechanics [3], and in those courses he often used the unconventional path-integral formulation of quantum mechanics that Feynman himself had developed as a graduate student. At one point a representative of the McGraw-Hill Book Company urged him to turn this approach into a textbook. Feynman declined, but his graduate student Albert Hibbs convinced him to change his mind, which he did provided that Hibbs took care of the details [4, p.399].

It is not known exactly when work on the book commenced. Feynman started work at CalTech on 1 July 1950[4, p.332]. On 30 December 1952 the television show 'You Bet Your Life' (Season 1, Episode 3) aired an interview with Hibbs in which he said he's working on a book to be titled *Quantum Mechanics and Path Integrals.* The book did not appear in print until 1965, underscoring Hibbs's remark that "Neither of us was dedicated to getting the thing out in a hurry"[4, page 399].

3. Typographical Errors

Some of the errors in the 1965 edition would never have been made by physicists, but it is easy to see how they could have been made by typesetters. In 1965, the words in manuscripts were typed in by secretaries, but the equations were written in by hand by physicists. Typesetters then turned the manuscripts into (hopefully) beautiful and (hopefully) accurate printed pages.

Sometimes it didn't work out that way. For example, equation (9-92) on page 261 of the 1965 Edition is printed as

$$X' = \frac{\pi}{k} \int e^{(i/\hbar)(S_{int}+S_{rad})} \mathcal{D}a_{1k} \, \mathcal{D}a_{2k}.$$

When I first encountered this equation, I was mystified. It didn't even have the proper dimensions! (Not to mention that there was one integral for two differentials.) How could Feynman and Hibbs have made such an elementary error? After sufficient study, I figured out that Feynman and Hibbs had intended to write

$$X' = \int\!\!\int e^{(i/\hbar)(S_{\rm int}+S_{\rm rad})} \prod_{\mathbf{k}} \mathcal{D}a_{1,\mathbf{k}} \, \mathcal{D}a_{2,\mathbf{k}}.$$

The typesetter undoubtedly saw a handwritten 'product over $\mathbf{k}$' and misinterpreted it as 'π divided by k'.

Similarly, on page 318, above equation (11-90), is the in-line equation $(41n^2 - 1)$. This equation should be $(4 \ln 2 - 1)$; the typesetter confused the l of 'ln' with a 1.

There are many more errors of this sort, all of which are corrected in the emended edition. And while they give us some clues concerning Hibbs's handwriting, they provide little insight into Feynman's thinking.

When I first encountered this equation, I was mystified. It didn't even have the proper dimensions! There are many more errors of this sort … and while they give us some clues concerning Hibbs's handwriting, they provide little insight into Feynman's thinking.

4. Physics Errors

Other errors (perhaps better called 'omissions') in the 1965 Edition could only have been made by physicists.

Equation (3-60) on page 63 asserts that a certain factor in the kernel for the simple harmonic oscillator is

$$F(T) = \left(\frac{m\omega}{2\pi i\hbar \sin \omega T}\right)^{1/2}.$$

This is correct but ambiguous. Which branch of the square root is to be used? That is, should we take $\sqrt{i} = e^{i\pi/4}$ or $\sqrt{i} = e^{i\,5\pi/4}$? Perhaps Feynman considered the answer to be obvious, because it's not in the book. But it's not obvious to me. The proper phase was

Some of the most delightful points made in the book fall in the 'Remarks on Mathematical Rigor'. Yet again, we see an instance of Feynman writing what is technically an error, not because of sloth or sloppiness, but because he had worked clever ways to avoid tedious tasks.

determined by Thorber and Taylor [6] and the emended edition references this determination.

Some of the most delightful points made in the book fall in the 'Remarks on Mathematical Rigor' presented on pages 93–94. The immediate issue concerns the perennial question of whether it's easier to think of wave vectors $\mathbf{k}$ as falling within a continuum (in which case they are to be integrated over) or as taking on certain discrete values (in which case they are to be summed over) that satisfy periodic boundary conditions for a large box of volume 'Vol'. Feynman understood both cases perfectly, and would jump back and forth adroitly depending upon which point-of-view was most insightful for the particular equation in question. In such cases the sum over $\mathbf{k}$, divided by Vol, corresponds to the integral over $\mathbf{k}$, divided by $(2\pi)^3$. Feynman rarely bothered writing in the division by Vol – to him it was obvious that any sum over $\mathbf{k}$ implied a box of volume 1, and you don't need to explicitly write out a division by 1. This convention, however, violates dimensional analysis. Equation (9-68) is particularly jarring, because it contains both a sum and an integral, and the clash of dimensions is almost palpable.

Here is one last example: Equation (3-40) claims that a certain probability distribution is

$$P(x) = \frac{m}{2\pi\hbar(T+t')} \times (\text{a dimensionless function of } x).$$

This expression cannot be a probability density: it has the dimensions of [1/length^2], while a probability density would have dimensions [1/length]. Furthermore, this expression isn't normalized! Finding the normalization constant is a chore, but once found it produces an even simpler (and correct!) expression for $P(x)$:

$$P(x) = \frac{1}{2b_1} \times (\text{that same dimensionless function of } x).$$

Part of the reason that Feynman failed to normalize his probability densities is that he had worked out a number of ways of finding the desired answers from an unnormalized probability: The emended edition has an index entry for 'normalization: avoiding' which references 11 uses of such tricks. Yet again, we see an instance of Feynman writing what is technically an error, not because of sloth or sloppiness, but because he had worked out clever ways to avoid tedious tasks.

Suggested Reading

[1] R P Feynman and A R Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, 1965. Emended by D F Styer, Dover, Mineola, New York, 2010.

[2] D E Knuth, The errors of TEX, *Software – Practice and Experience*, Vol.19, pp.607–685, 1989.

[3] David L Goodstein, 'Richard P. Feynman, Teacher', *Physics Today*, Vol.42, No.2, pp.70–75, February 1989.

[4] Jagdish Mehra, *The Beat of a Different Drum: The Life and Science of Richard Feynman*, Clarendon Press, Oxford, UK, 1994.

[5] See http://www.youtube.com/watch?v=NooenChxGyQ at time 6:40.

[6] N S Thorber and E F Taylor, Propagator for the simple harmonic oscillator, *American Journal of Physics*, Vol.66, pp.1022–1024, 1998.

Address for Correspondence
Daniel F Styer
Department of Physics and Astronomy
Oberlin College, Oberlin
Ohio 44074, USA
Email: Dan.Styer@oberlin.edu