Dawn of Science

12. Logarithms

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The invention of logarithms saved much time and effort in astronomical and mathematical calculations. Until the advent of electronic calculating machines, they remained the most effective tool for scientists.

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Resonance, Vol.15: p.498, p.590, p.684, p.774, p.870, p.1009, p.1062. Vol.16: p.6, p.110, p.274, p.304.

"My Lord, I have undertaken this long journey purposely to see your person, and to learn by what engine of wit or ingenuity you came first to think of this most excellent help in astronomy." This is how Henry Briggs (1561–1630) greeted the Scottish nobleman, John Napier (1550–1617), the inventor of logarithms, when they met first. And indeed it was an invention of great practical use not only in astronomy but also in other branches of computation. Until the advent of electronic calculating devices, logarithms remained the most effective tool for every scientist.

The inventor of such a device, John Napier, was quite an enigmatic persona. He was born in the Scottish aristocracy and travelled widely in Europe during his youth. This was the time when Europe was in total turmoil, split into warring camps by the Protestant reformation. His native country, Scotland, was fast turning into a Calvinist state. This influenced Napier and he became a very vocal Protestant gentleman. In 1593, he expressed his anti-Catholic views against the Church of Rome in a book entitled *A Plaine Discovery of the Whole Revelation of Saint John*. The book was an overnight bestseller and ran 21 editions of which at least 10 were during his lifetime! So engrossed was he in his religious sentiments that he definitely expected his claim to fame to rest on this book. (He also spent considerable amount of time thinking out different kinds of ingenious war machines to be used against Philip II of Spain, in case he should attack Scotland; none of them was needed.)

Keywords
John Napier, logarithm, Henry Briggs, astronomical computation.

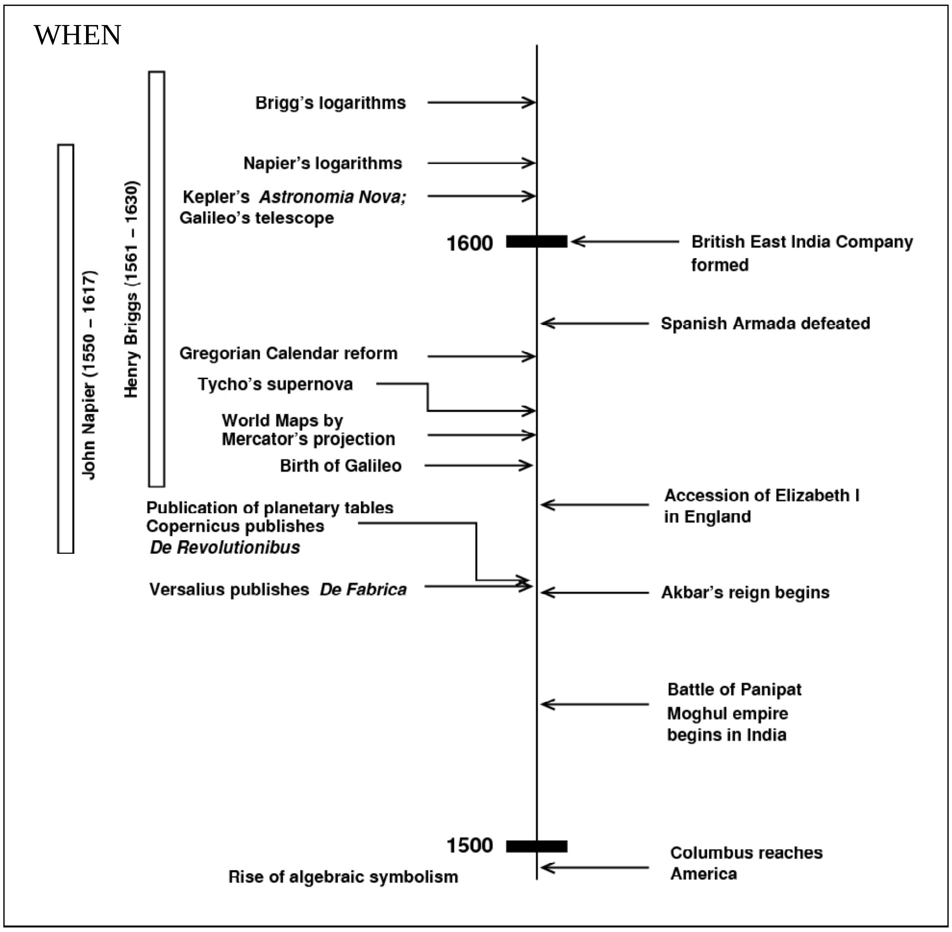

Figure 1.

Figure 2.

Figure 3. John Napier.
Courtesy: http://en.wikipedia.org/wiki/John_Napier

It is therefore rather surprising that he had the time and energy to ponder over mathematics. He was particularly concerned with the amount of labour involved in the multiplication and division of numbers. In fact, most scientists of his time spent a large part of their working day doing routine, boring mechanical calculations. This was especially so in astronomical computations involved in the preparation of planetary tables, etc. Napier's logarithms changed the situation completely; it replaced multiplication by addition and division by subtraction. As Laplace said years later, the invention of logarithm effectively 'doubled the life of the astronomer'.

Incredibly enough, the idea behind logarithm is extremely simple. To understand the basic concept, let us consider the numbers: $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32...$. Suppose we need to multiply the numbers 4 and 8. Since 4 is 2^2 and 8 is 2^3, the product can be written as $4 \times 8 = 2^{2+3} = 2^5$. We know that 2^5 is 32, and this gives us the answer. The crucial point is that the *multiplication* of two numbers 4 and 8 was reduced to the *addition* of the superscripts 2 and 3. Now suppose we have a readymade table of all numbers expressed as various powers of 2. For example, the number 17 can be expressed as $2^{4.087}$ to a very great accuracy and 19 can be written as $2^{4.248}$ (we say that 4.087 is the 'logarithm' of 17 in 'base' 2, etc.). Therefore, to multiply 17 and 19 we only have to add the two powers (4.087 + 4.248) and get 8.335. The product is $2^{8.335}$, which is 323. Of course, we would need a detailed table giving the power for each integer. Once such a table is prepared, any two numbers can be multiplied – or for that matter divided, which requires subtracting one index from the other, within seconds. For large numbers, this saves considerable amount of time and effort.

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In the above illustration, we used the number 2 to express all other integers. One could have used any other positive number in place of 2. Because of a rather complicated reason, Napier used a number which is about 0.3679 which is the reciprocal of a constant usually denoted by the letter 'e' = 2.718. This constant plays a crucial role in all branches of higher mathematics and is called 'the base of natural logarithm'. While such a base may not look 'natural' at first sight, it does provide a simple geometrical interpretation (see *Figure* 4). Consider two line segments AB and DE with AB having unit length. C and F are two moving points on these line segments. They start simultaneously at A and D; F moves with uniform, unit, speed along DE while C moves with a speed which is numerically equal to the length CB. Then DF will give the magnitude of the natural logarithm of CB.

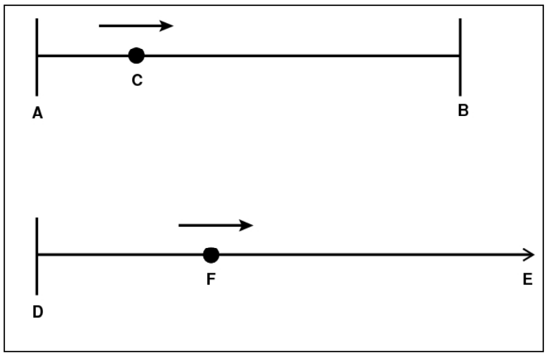

Figure 4.

Napier published his discussion in 1614 in a small brochure titled *Mirifici Logarithmorum Canonis Descriptio* (A Description of the Wonderful Law of Logarithms) which also contained a table giving the logarithms of sines of angles for successive minutes of arc. (This was the most important quantity needed in astronomical computations.) The book roused wide interest. The news travelled fast and in the next year Henry Briggs, a professor of mathematics at London, travelled all the way to Edinburgh (it was quite a distance to cover in those days!) to greet Napier. At their meeting, Briggs convinced Napier that it would be much more useful to use 10 as the base for logarithms. In other words, all numbers will be expressed as powers of 10, instead of 2. Since $100 = 10^2$ and $1000 = 10^3$, the logarithm of 100 will be 2 and that of 1000 will be 3; and that of any number between 100 and 1000 will be a number between 2 and 3.

Briggs started the construction of such a table on his return to London and in 1624 published his *Arithmetica Logarithmica* containing 14-decimal place tables of logarithms for all numbers from 1 to 20,000 and from 90,000 to 100,000. The gap between 20,000 and 90,000 was filled in later by a Dutch bookseller, Adrian Vlacq (1600–1666). Incidentally, these tables remained in vogue for nearly three centuries and were superseded only around the 1940s when extensive 20-decimal place tables were calculated.

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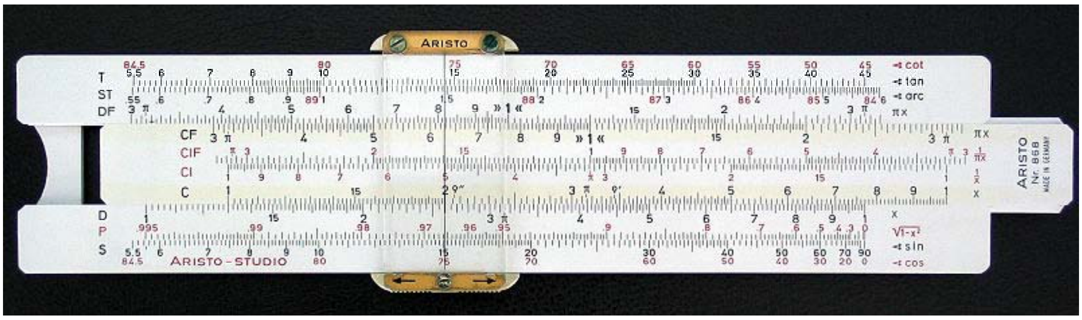

Figure 5. The slide rule.
Courtesy: http://en.wikipedia.org/wiki/Logarithm

Meanwhile, in the 1620s, the English mathematician William Oughtred, came to realise that even the process of looking up logarithmic tables can be eliminated by a simple mechanical device (*Figure* 5). It consisted of two sliding scales in which the numbers are marked in such a way that the distance from the left end of the scale is numerically equal to the logarithm of the number. Numbers could now be multiplied and divided by merely sliding the scale on one another. It is difficult to estimate how much the world of engineering and science owes to this gadget called the 'slide rule'.

Napier also made some contributions to other branches of mathematics – for instance, the perfection of the decimal notation which we now use so frequently. The idea of the decimal fraction had been worked out by the Dutch mathematician, Simon Stevin (1548–1620) earlier, but it was Napier who made the notation compact and convenient to use.

Suggested Reading

[1] **Howard Eves, Great Moments in Mathematics Before 1650, *Dolciani Mathematical Expositions*, Mathematical Association of America, No.5, 1983.**

[2] **Isaac Asimov, *Asimov's Biographical Encyclopedia of Science and Technology*, Doubleday, 1982.**

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