Ya B Zeldovich (1914–1987)

Chemist, Nuclear Physicist, Cosmologist

Varun Sahni

Ya B Zeldovich was a pre-eminent Soviet physicist whose seminal contributions spanned many fields ranging from physical chemistry to nuclear and particle physics, and finally astrophysics and cosmology. This article attempts to convey some of the zest with which he did science and the important role he played in fostering and mentoring a whole generation of talented Soviet scientists.

Varun Sahni did his PhD in Moscow State University under the supervision of Ya B Zeldovich and A A Starobinsky.
He is a professor at IUCAA with interests in: the early Universe, gravity waves, dark matter and dark energy, quantum field theory in curved space-time and the formation of large scale structure in the Universe.

Introduction

Yakov Borisovich Zeldovich was exceptionally talented. His active scientific career included major contributions in fields as diverse as chemical physics (adsorption and catalysis), the theory of shock waves, thermal explosions, the theory of flame propogation, the theory of combustion and detonation, nuclear and particle physics, and, during the latter part of his life: gravitation, astrophysics and cosmology [1].

Zeldovich made key contributions in all these areas, nurturing a creative and thriving scientific community in the process. His total scientific output exceeds 500 research articles and 20 books. Indeed, after meeting him, the famous English physicist Stephen Hawking wrote "Now I know that you are a real person and not a group of scientists like the Bourbaki"[1]. Others have compared his enormously varied scientific output to that of Lord Raleigh who, a hundred years before Zeldovich, worked on fields as varied as optics and engineering.

[1] Bourbaki was the pseudonym collectively adopted by a group of twentieth century mathematicians who wrote several influential books under this pseudonym on advanced mathematical concepts.

Keywords
Ya B Zeldovich, cosmology, black holes, expanding universe, cosmic web.

Remarkably Zeldovich never received any formal university education! He graduated from high school in St. Petersburg at the age of 15 after which he joined the

Institute for Mechanical Processing of Useful Minerals ('Mekhanabor') to train as a laboratory assistant. The depth of Zeldovich's questioning and his deep interest in science soon reached senior members of the scientific community and, in 1931, the influential soviet scientist A F Ioffe wrote a letter to Mekhanabor requesting that Zeldovich be "released to science". Zeldovich defended his PhD in 1936 and, years later, reminiscenced of the "happy times when permission to defend [a PhD] was granted to people who had no higher education".

Despite his never having been formally taught (or perhaps because of it!) Zeldovich developed a very original style of doing science, and became, in the process, an exceptional teacher. It is also interesting that in his early years Zeldovich had been an experimentalist as well as a theoretician, and this closeness to complementary aspects of science guided him throughout his later life.

Nuclear and Particle Physics

It is quite remarkable that Zeldovich ventured into a totally new field – Astrophysics – around 1964 when he was nearing 50. By then Zeldovich had already developed a very considerable reputation in fields ranging from physical chemistry (there is a 'Zeldovich number' in combustion theory) to nuclear physics. Indeed, having done pathbreaking work on combustion and detonation, Zeldovich moved to nuclear physics in the 1930's writing seminal papers demonstrating the possibility of controlled fission chain reactions among uranium isotopes. This was the time when fascism was on the rise in Germany, and, in an effort for national survival, the Soviet Union was developing its own atomic program of which Zeldovich (then in his mid 20's) quickly became a key member. According to Andrei Sakharov, "from the very beginning of Soviet work on the atomic (and later thermonuclear) problem, Zeldovich was at the very epicenter of events. His role there was completely

Pioneering work on nuclear fission and fusion.

exceptional" [2]. One might add that Zeldovich's earlier work on combustion paved the way for creating the internal ballistics of solid-fuel rockets which formed the basis of the Soviet missile program during the 'great patriotic war' and after [3]. Sadly, much of Zeldovich's work during this period remains classified to this day.

After the war Zeldovich went on to do pioneering work in several other aspects of nuclear and particle physics including [2]:

[2] Space does not permit me to elaborate on Zeldovich's other seminal work in this area which included the possibility of muon-catalyzed fusion (1954), his work on weak interactions including his brilliant hypothesis on the existence of parity violating neutral currents (1959), his remark (1957) about the existence of a toroidal dipole moment (which has since spawned an active research area) and his prediction about the existence of the η meson (1958) which was subsequently discovered. The reader is referred to [1] for more details.

- In 1952 and 1953 Zeldovich proposed laws for the conservation of lepton and baryon charges.
- In 1955 Zeldovich and Gershtein suggested the conservation of the weak vector current. This idea was independently discovered some years later by Feynman and Gell-Mann and played a key role in the development of the theory of weak interactions. Zeldovich (1959) also suggested the existence of neutral currents which violated parity conservation. He showed that parity violation in weak interactions should lead to the rotation of the plane of polarization of light propogating in a substance not containing optically active molecules. This prediction was subsequently confirmed.
- In 1959 Zeldovich suggested a method of containment (storage) of slow neutrons via total internal reflection from graphite. This method is now regularly used to measure the neutron electrical moment.
- Zeldovich also studied the possible existence of long-lived nuclei with a large isotopic spin. He suggested the possibility of observing an isotope of ^{8}He, soon after which this isotope was, in fact, discovered!

Astrophysics and Cosmology

Instead of resting on his laurels (had he so desired Zeldovich could have easily landed a 'comfortable' job heading a premier research laboratory or institute), Zeldovich decided to change course midstream and, from about 1964, devoted his phenomenal abilities to problems in astrophysics[3]. A change of track can be precarious if made later in life when ones scientific tastes and habits have usually become set. Indeed, each scientific discipline has its own nuances the mastering of which can take considerable time and effort, and history is replete with examples of scientists – exceedingly capable in their chosen field – making grave errors of judgement when moving to another.

[3] By this time Zeldovich was a very well-known figure in Soviet science and had won several major laurels including three gold stars, the Lenin Prize and a fellowship of the prestigious Soviet Academy of Science.

It is therefore quite remarkable that Zeldovich not only plunged deeply into astrophysics, he virtually revolutionized the field, becoming in the process one of the founders of relativistic astrophysics and physical cosmology.

Below is an imperfect attempt to summarize some of Zeldovich's seminal contributions to astrophysics and cosmology.

1. In 1963 Zeldovich and Dmitriev showed that the total energy of a system of particles interacting via gravity (in a universe expanding according to Hubble's law $\mathbf{v} = H\mathbf{R}$) evolved according to the simple formula

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -(2K + U)H, \tag{1}$$

where $E = K + U$ is the total energy of the system with K and U being its kinetic and potential energy respectively. (This relation was independently suggested by Layzer (1963) and Irvine (1961) and is frequently referred to as the Cosmic Energy Equation.) $\dot{E} = 0$ once matter decouples from cosmological expansion. In this case, (1) reduces to the virial equation $2K+U = 0$ which can be used to investigate the amount of dark matter associated with gravitationally-bound systems such as clusters of galaxies.

2. Zeldovich was intrigued by black holes and spent a considerable amount of effort in understanding their properties. In 1962 Zeldovich published a paper in which he showed that a black hole could be formed not only during the course of stellar explosions, as was then widely believed, but by any mechanism which compressed matter to sufficiently high densities. This opened up the possibility of the formation of microscopically small black holes in the early Universe. Were they to survive until today, the smallest black holes would have masses of about 10^{16} grams, roughly that of a large mountain[4]. There has also been some speculation that microscopic black holes may be the *dark matter* that everyone is searching for! (This was Zeldovich's first paper on General Relativity. It was also the last work that he discussed with his teacher, Lev Landau, before the latter's tragic car accident in 1962.)

[4] Smaller mass black holes would have evaporated by emitting Hawking radiation and disappeared.

3. In 1964 Zeldovich suggested that a black hole may be detected by its influence on the surrounding gas which would accrete onto the hole. (The same result was independently obtained by E Salpeter in the USA.) In 1966 Zeldovich also suggested (with Guseinov) that one could look for a black hole in binary star systems through the hole's influence on the motion of its bright stellar companion. These papers helped create a paradigm shift in which black holes were elevated from their earlier status of 'impossible to observe passive objects' to *objects which created very significant astrophysical activity in their vicinity.*

Zeldovich suggested that black holes might 'glow' and be detectable since they accreted matter.

Thus Zeldovich's early work led to what is currently a thriving area in astrophysics, with numerous observational programmes being dedicated to the discovery of black hole candidates in our own galaxy as well as in distant galaxies and QSO's.

4. Zeldovich remained deeply interested in black holes and, in 1971, turned his attention to the issue of particle production and vacuum polarization in the strong gravitational fields that one would expect to encounter near black holes and during the early infancy of the universe. Zeldovich (1971) and his student Starobinsky (1973) showed that under certain conditions a rotating black hole could loose energy via the production of a particle–antiparticle pair. These papers were precursors of a whole body of later work including Stephen Hawking's famous paper on evaporating black holes published in 1975.

5. In tandem with their study of quantum effects near black holes, Zeldovich and Starobinsky began a systematic investigation into quantum effects which occur during the early stages of the universe when it was expanding very rapidly. Quantum field theory informs us that, far from being empty, the vacuum is actually seething with activity in the form of virtual particle–antiparticle pairs in the constant process of creation and destruction. From the uncertainty principle $\Delta E \Delta t \geq \hbar$ we know that such pairs (of mass $2m$) come into existence for a very short time $\Delta t \leq \hbar/2mc^2$. If an electric field is applied to the vacuum then, for a sufficiently large value ($E_{\rm cr}$), the work done (δW) on the virtual pair can become equal to the total rest mass $2mc^2$:

$$\delta W = \lambda_c |eE_{\rm cr}| \simeq 2mc^2 \ , \qquad (2)$$

Zeldovich and Starobinsky demonstrated that in a rapidly expanding universe the vacuum was unstable and spontaneously gave birth to particles and antiparticles .

where the Compton length $\lambda_c = \hbar/mc$ provides an estimate of the separation between particle and antiparticle. The critical field value $E_{\rm cr} \simeq m^2c^3/|e|\hbar$ at which the vacuum becomes unstable to particle–antiparticle production is called the Schwinger limit, after Julian Schwinger who discovered it in 1951. ($E_{\rm cr} \sim 10^{16}$ volt/cm for electrons.)

Zeldovich and Starobinsky showed that a similar effect occurs if the universe is expanding rapidly. In this case the role of the electric field is played by the gravitational field which, in Einstein's general relativity, is an expression of space-time curvature. A rapidly expanding universe literally tears a virtual particle–antiparticle pair apart giving rise to spontaneous particle creation from the vacuum. Near the big bang the universe expands very rapidly, (its rate of expansion being given by the Hubble parameter $H = \dot{a}/a$), and copious particle production from the vacuum is expected when the expansion rate becomes of the order of the particle mass ($H \sim m$).

In an influential paper published in 1971, Zeldovich and Starobinsky showed that particle production from the vacuum becomes extremely significant if the universe expands *anisotropically*[5]. In this case the copious energy density of particles released from the vacuum far exceeds that of any pre-existing matter in the universe! Furthermore, the newly created particles (and their antiparticles) backreact on the universe through the semi-classical Einstein equations, $G_{ik} = 8\pi G\langle T_{ik}\rangle$, isotropising its expansion. The mechanism proposed by Zeldovich and Starobinsky provided an interesting means of making the properties of the universe similar to what we observe today.

[5] The present-day universe expands according to Hubble's law $\mathbf{v} = H\mathbf{r}$, where the Hubble parameter H is a scalar quantity whose value depends only upon time but not upon spatial direction. During anisotropic expansion, the expansion rate is different along the three spatial directions so that $v_i = H_{ij}r^j$, with the Hubble parameter being promoted to a tensor.

It is well known that the most general solutions to the Einstein equations will be both inhomogeneous and anisotropic. Even if one were to restrict oneself to homogeneous models (whose spatial properties did not depend upon the precise location of the observer), a universe which expanded at different rates along different directions was, in a sense, much more likely than our own isotropic universe. This dilemma was substantially ameliorated by the work of Zeldovich and Starobinsky since copious particle production (and vacuum polarization) would ensure that a universe whose expansion was initially aniso- tropic soon isotropized and began to resemble our own!

Even a tiny mass of the neutrino is sufficient to make it an important player in the cosmological scenario .

6. In 1966 Zeldovich and Gershtein showed that, if neutrinos were massive, then they could very easily be the dominant matter component in the universe. The reason has to do with the fact that the theory of weak interactions predicts a relic abundance for neutrinos of roughly 100 particles per cubic centimeter (per species). (By comparison the cosmic microwave background contains ~ 400 photons per cubic centimeter, also of relic origin.) If neutrinos were massive then, for a large enough mass, their density could easily exceed that of visible matter in the universe! Zeldovich and Gershtein placed a limit on the mass of the muonic (and electronic) neutrino from cosmological considerations. Their result, $m(\nu_\mu) < 400$ ev/cm^3, was considerably lower than laboratory bounds at the time, and convincingly demonstrated that the universe could be used as a particle physics laboratory.

Subsequently Schwartzmann, a student of Zeldovich, showed how the number of neutrino species could be constrained from observations of the helium abundance in the universe. (The reason is simple: a larger number of neutrinos speeds up cosmic expansion and, in so doing, alters the primordial nucleosynthesis of light elements taking place during the first few minutes of the big bang.) These early papers by Zeldovich and his students proved to be prescient in defining a vibrant new field, Astroparticle Physics, in which the early universe plays the role of a particle-physics laboratory. In the 1970's, the existence of a large amount of dark matter in galaxies was discovered observationally. Zeldovich's earlier work demonstrated that relic non-baryonic particles left over from the big bang could easily play this role! Non-baryonic dark matter is currently believed to constitute roughly a third of the total matter density in the universe – significantly more than the $\sim 4\%$ contributed by baryons and electrons[6].

[6] The possibility of massive neutrinos playing the role of dark matter was pointed out by Cowsik and McClelland (1973) and independently by Marx and Szalay (1972). However it is currently believed that neutrinos account for only a small fraction of the total dark matter density in the universe.

7. Zeldovich returned to the issue of relic abundances a few years later in a seminal work on field theory. Cosmology, like other disciplines, has frequently been influenced by developments in neighboring fields. An example is provided by field theories including those in which the ground state is degenerate so that the system can settle into different ground states in different regions of space. (In ferromagnetism, at temperatures below the Curie point, the magnetization vector can point in any given direction. The phase transition in this case is of second order.)

Cosmic strings can seed galaxy formation in the universe.

Zeldovich investigated cosmological consequences of phase transitions which could have occured in the early universe when its temperature was exceedingly high. He demonstrated that theories with degenerate ground states predicted a relic abundance of 'new objects', called topological defects, which can be zero-dimensional (such as magnetic monopoles), one-dimensional (such as cosmic strings), or even two-dimensional domain walls. Top -ological defects arise during an early phase transition once the universe has cooled below a critical value. Their properties are similar to the defects seen in laboratory physics such as vortices in a superfluid and flux tubes in a superconductor. Zeldovich, Kobzarev and Okun (1974), and independently Kibble (1976), appreciated the enormous impact that stable topological defects could have within a cosmological setting. Zeldovich and his colleagues demonstrated that whereas monopoles and walls were disastrous for cosmology, cosmic strings might be useful since they would act as 'seeds' onto which matter accreted resulting in the formation of galaxies and other gravitationally-bound systems[7].

8. In 1967 Zeldovich applied himself to the puzzle of the cosmological constant 'Λ'. Originally introduced by Einstein in 1917, the cosmological constant has the unusual property that its pressure is *negative* and equal, in absolute terms, to its density ($P = -\rho$). Hence, while

[7] (i) Zeldovich's paper (with Khlopov) demonstrating that the abundance of 't Hooft–Polyakov monopoles was unacceptably large appeared in 1978, a year before the famous paper by Preskill which highlighted the same problem but within the framework of grand unified theories. (ii) Cosmic strings created during a grand unified phase transition will have a radius much smaller than that of a proton, an enormous length (upto a million light years) and a very high density: a kilometer long cosmic string can be as massive as the earth! These enormously long 'one-dimensional' objects should not be confused with the much smaller superstrings.

The Cosmological Constant naturally arises within the framework of quantum field theory.

the density in normal forms of matter declines in an expanding universe, the density in Λ remains frozen to a constant value $\rho = \Lambda/8\pi G$. After its inception the cosmological constant had fallen into disrepute since it did not seem to be required by observations. Even Einstein distanced himself from it, calling the Λ-term 'my biggest blunder'.

Zeldovich radically changed this perspective by persuasively arguing that, within the context of quantum field theory, the prospect of a non-zero value for the Λ-term should be taken extremely seriously. The reason is that the quantum polarization of the vacuum results in a *vacuum energy* which, quite remarkably, has the precise form of a cosmological constant: $\langle T_i^k \rangle = \Lambda \delta_i^k / 8\pi G$. (The equation of state $P = -\rho$ is, in fact, Lorenz invariant and remains the same in any coordinate system. So the properties of the vacuum do not change from one coordinate system to another, which is indeed a desirable property.) A positive cosmological constant can result in an accelerating universe, and by strongly supporting the Λ-term, Zeldovich paved the way for future advances including cosmic inflation in the 1980's and dark energy in this century [4]. These new results suggest that the universe accelerated both in its remote past (inflation) and at present (dark energy). Thus current observations appear to require a form of matter whose properties are tantalizingly similar to that of the Λ-term and one is reminded of Zeldovich's prescient statement, regarding Λ, made almost half a century ago [5] "the genie has been let out of the bottle, and it is no longer easy to force it back in".

9. The discovery of the cosmic microwave background (CMB) in 1964 resulted in Zeldovich becoming an ardent believer in the hot big bang model of the universe (originally proposed by another gifted Russian physicist George Gamow). During 1968–1971, working with a dedicated team of students and researchers (including

Doroshkevich, Novikov and Sunyaev), Zeldovich showed how the CMB could be used to probe the properties of the early universe. His work in this field focussed on several key issues including that of the cosmological recombination of hydrogen from free protons and electrons[8], the nature of angular fluctuations in the CMB, and how these could be linked to fluctuations in matter. He also addressed the problem of cosmological nucleosynthesis in the hot early universe.

[8] This took place about 100,000 years after the big bang, when the temperature of the universe had dropped to 4000 °K. Prior to this the universe had been opaque to the passage of light.

The Zeldovich Approximation and the Cosmic Web

Zeldovich himself felt that his main contribution to cosmology was in the understanding of how gravitational instability develops in the universe from small initial values.

A realistic model of galaxy formation clearly requires two essential ingredients:

(i) A description of the initial fluctuations in the distribution of matter which might later form galaxies.

(ii) A theory describing the growth of these fluctuations under the influence of gravity or other forces.

In 1972 Zeldovich published a paper which subsequently proved to be rather prescient. In it he discussed how 'seed' fluctuations (which later gave rise to galaxies) could have a *scale invariant* spectrum. Mathematically this meant that the fourier amplitudes of the spatially fluctuating gravitational potential had the form $|\phi(k)|^2 \propto k^{-3}$, so that $\int |\phi(k)|^2 \mathrm{d}^3k = \int \mathrm{d}\log k$. In other words, each logarithmic interval contributed an equal amount of power to the gravitational potential responsible for moving matter into galaxies[9]. (The scale invariant spectrum was independently discovered by E Harrison.) A decade after it was initially proposed on phenomenological grounds, the scale invariant spectrum was shown to be a generic prediction of *inflationary models* of the

[9] Additionally, if the phases of $\phi(k)$ are randomly distributed, then the spatial distribution of the gravitational potential $\phi(\mathbf{x})$ has the properties of a Gaussian random field.

early universe, where its origin was quantum mechanical. The presence of fluctuations in the CMB detected by the COBE satellite in 1992 marked a turning point in our understanding of the universe since it showed that the infant universe had not been featureless and smooth, but had tiny fluctuations (approximately 1 part in 100,000) imprinted on it. The (fourier) spectrum of these fluctuations turned out to have the 'scale-invariant form' suggested by Zeldovich and Harrison more than two decades earlier. (These tiny fluctuations in matter become galaxies several billion years later, after being amplified by gravitational instability.) This milestone discovery made by COBE was awarded the Nobel Prize in Physics in 2006.

An important property of the universe is that it is gravitationally unstable. This means that small initial fluctuations in the density of matter grow and become larger. A theoretical analysis of density fluctuations had been carried out in the 1940's by the eminent Soviet physicist Evgeny Lifshitz. Lifshitz had shown that in an expanding universe density perturbations grow at the rather modest rate $\delta \propto t^{2/3}$. This is very much slower than the exponentially rapid 'Jeans instability', $\delta \propto \exp(\sqrt{4\pi G\rho}t)$ which occurs in a static universe. The reason for the difference is that cosmic expansion moves particles away from one another while gravity pushes them together. Since the two influences oppose each other, gravitational instability becomes weaker if the universe expands. Although Lifshitz's treatment was rigorous it had a fundamental limitation. In order to solve the complicated equations of general relativity Lifshitz had to assume that perturbations were linear, i.e., $\delta \ll 1$, where $\delta = (\rho - \bar{\rho})/\bar{\rho}$ is the perturbation in the cosmic density relative to its mean value $\bar{\rho}$. Although this approach was exceedingly useful in the context of the early universe which was quasi-homogeneous, it broke down at more recent times since density perturbations at the present epoch are exceedingly large, the density contrast associated with a galaxy being close to a million ($\delta \sim 10^6$).

Zeldovich set about rectifying this situation by proposing, in 1970, a remarkably simple and elegant approximation which could be used to follow a perturbation from its initially linear form into the fully nonlinear regime when $\delta \gg 1$. Along the way Zeldovich upset a widely prevailing world view according to which the assembly of the first large astrophysical objects in the universe was spherical in nature. According to this point of view, the spherical globular clusters that orbit our galaxy were the first objects to condense out of an expanding quasi-homogeneous gas of neutral hydrogen. Zeldovich toppled this deeply ingrained notion by demonstrating that gravitational instability was much more likely to proceed in an anisotropic manner resulting in the formation of two-dimensional sheet-like objects which he called 'Pancakes'[10].

[10] The pancake is a popular and tasty Russian dish. One could equally well substitute pizza, chapati or dosa, depending upon one's, favourite cuisine!

The Zeldovich approximation proposes that the final coordinate of a particle is related to its initial coordinate by the transformation

$$\mathbf{r} = \mathbf{q} + \mathbf{v}(\mathbf{q})\delta_\ell \,, \tag{3}$$

where δ_ℓ is the density contrast predicted by *linear theory* and $\mathbf{v}(\mathbf{q})$ is the initial velocity field of a perturbation[11]. ($\mathbf{v}(\mathbf{q}) = 0$ if particles remain at rest with respect to the background cosmic expansion, in which case $\mathbf{r} = \mathbf{q}$.) If the particle flow is irrotational then, under some assumptions, one can relate the velocity field to the linearised gravitational potential

$$\mathbf{v}(\mathbf{q}) = -\nabla\phi \ . \tag{4}$$

In turn ϕ is related to the primordial density perturbation δ_ℓ and the mean density of matter in the universe, $\bar{\rho}$, through the Poisson equation

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho} \times \delta_\ell \ . \tag{5}$$

[11] The *comoving coordinate* **r** is a convenient quantity because, in the absence of perturbations, its value does not change as the universe expands. It is related to the *physical coordinate* **x** by means of the transformation $\mathbf{r} = \mathbf{x}/a(t)$. The Zeldovich approximation in physical coordinates has the form $\mathbf{x} = a(t)[\mathbf{q} + \mathbf{v}(\mathbf{q})\,\delta_l]$. **r** and **q** respectively represent the Eulerian and Lagrangian coordinate of a particle in fluid dynamics.

Figure 1. Caustics of light in a pool of water.

We noted earlier that $\delta_\ell \propto t^{2/3}$. Introducing a new time coordinate $T = \delta_\ell \propto t^{2/3}$ allows us to rewrite (3) as

$$\mathbf{r} = \mathbf{q} + \mathbf{v}(\mathbf{q})T \ . \tag{6}$$

We have thus shown that the Zeldovich approximation is equivalent to the simple inertial motion of particles! An essential feature of inertial motion from random initial conditions is the intersection of particle trajectories leading to the formation of singularities in the density field. A similar effect is seen in the propogation of light as it passes through a medium such as a plate of glass or water. After passing through the plate, neighboring light trajectories intersect to form caustics where the intensity of light is exceedingly bright (*Figure* 1).

The cosmic web seen in the distribution of galaxies is similar to caustics of light which form in a pool of water.

Indeed, the Zeldovich approximation bears a very simple analogy to the propogation of light rays in geometrical optics! Consider a light ray which enters a (two-dimensional) glass plate at the point $\mathbf{q} = \{q_1, q_2\}$. If the thickness of the plate is $h = h(x, y)$ then, after passing through the plate, the light ray will be deflected by an angle $\vec{s}(\mathbf{q})$, which determines the direction of the ray after it passes through the plate [6]. The two-dimensional coordinates of the light ray after it has emerged from the plate will depend upon $\vec{s}$ as well as the location of the screen z, so that

$$\mathbf{R}(z, \mathbf{q}) = \mathbf{q} + \mathbf{s}z \ , \tag{7}$$

where

$$s_i = -(n-1)\frac{\partial h(\mathbf{q})}{\partial q_i} \tag{8}$$

and n is the refractive index of the plate.

A screen kept some distance away from the plate will see an inhomogeneous distribution of light (*Figure* 2). Varying the location of the screen one will see caustics, regions where light trajectories have interesected causing the brightness of light to suddenly shoot up. If we denote the brightness of light by ρ, it is easy to show that

$$\rho(z, \mathbf{q}) = \frac{\rho_0}{[1 - z\alpha(\mathbf{q})][1 - z\beta(\mathbf{q})]} \ , \tag{9}$$

where ρ_0 is the initial intensity of light and $\alpha(\mathbf{q})$ and $\beta(\mathbf{q})$ are the principal curvatures of the surface of the plate $h = h(\mathbf{q})$. In other words $\alpha(\mathbf{q})$ and $\beta(\mathbf{q})$ are eigenvalues of the tensor $\partial^2 h/\partial q_i \partial q_k$. Clearly the optics relation (7) is identical to the Zeldovich approximation given by (6). The similarity between optics and gravitational instability is striking! The role of the plate thickness h is played by the gravitational potential ϕ, and the location

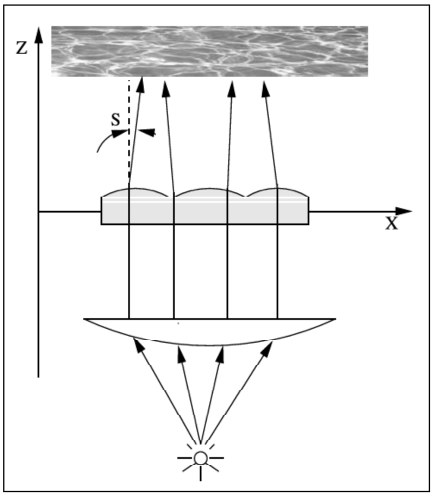

Figure 2. Caustics of light on a screen.
Adapted from [6].

of the screen z is analogous to the cosmic time coordinate T. In optics (gravitational instability) the intensity of light (density of matter) becomes extremely large at caustics where nearby light (matter) trajectories intersect. From (9) we find $\rho \to \infty$ when $z \simeq \alpha^{-1}$ (assuming $\alpha > \beta$). A similar result holds in the Zeldovich approximation when we replace z by T. In this case the law of matter conservation results in the following expression for the density

$$\rho(\mathbf{x}, t) = \frac{\rho_0}{[1 - T\alpha(\mathbf{q})][1 - T\beta(\mathbf{q})][1 - T\gamma(\mathbf{q})]} ,$$
$$T \equiv \delta_\ell \propto t^{2/3} , \quad (10)$$

where α, β, γ are the eigenvalues of the three-dimensional deformation tensor $\partial^2\phi/\partial q_i \partial q_k$ and ϕ is the inhomogeneous primordial gravitational potential responsible for moving particles in (3), (4) and (6). Thus the Zeldovich approximation predicts that matter moving under the influence of a perturbing gravitational potential $\phi(\mathbf{q})$ will get focussed into caustics, at locations specified by the eigenvalues of $\partial^2\phi/\partial q_i \partial q_k$. Whether a given volume element contracts or expands depends upon the specific values of the eigenvalues $\alpha(\mathbf{q}), \beta(\mathbf{q}), \gamma(\mathbf{q})$, in a given region of space and especially on their sign. If, at the point $\mathbf{q}$ one of the eigenvalues, say $\alpha(\mathbf{q})$, is positive, then, since $T \propto t^{2/3}$ is a monotonically increasing function of time, a time will come when $1 - \alpha T = 0$. From (10) we find that the denominator in this expression will vanish as the density of matter aquires very large (formally infinite) values. This signals the formation of a caustic at $\mathbf{q}$. Such a region will be the birthplace of a *pancake*.

The cosmic web is a percolating cellular structure consisting of clusters and superclusters of galaxies separated by giant voids.

Intersections of fully grown pancakes will form filaments and intersections of filaments will result in clumps. Matter will have a multistream flow within pancakes, moving along them towards filaments, and then moving along filaments to converge into clumps. The formation of caustics will be accompanied by the formation of immense

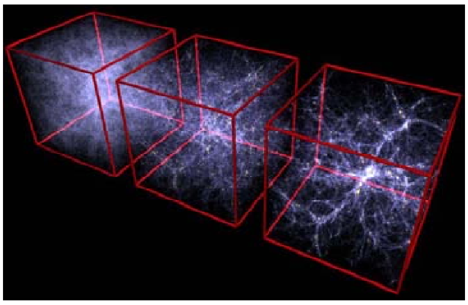

Figure 3. The development of the 'Cosmic Web' is shown in a hydrodynamic N-body simulation from z = 6 (leftmost cube) to z = 0 (rightmost cube) via z = 2 (middle cube). A near featureless density field evolves to produce filamentary and sheet-like superclusters which percolate through the box-volume. These are separated by large voids.

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underdense regions, or voids, from which matter gets drained into pancakes and filaments. Thus the Zeldovich approximation predicts that, as the universe expands, the matter in it becomes concentrated along planar and filamentary 'superclusters', and that neighboring superclusters are separated by 'voids' – vast empty regions virtually devoid of the presence of matter (*Figure* 3). It is remarkable that precisely such a *supercluster-void* network of galaxies, now commonly referred to as the *Cosmic Web*, has been discovered by large galaxy surveys almost 30 years after it was first predicted to exist by Zeldovich [7]. The supercluster which currently holds the record for being the largest contiguous object in the universe is close to a billion light years across and has been dubbed the *Sloan Great Wall* since it was discovered by the Sloan Digital Sky Survey (SDSS) (*Figure* 4).

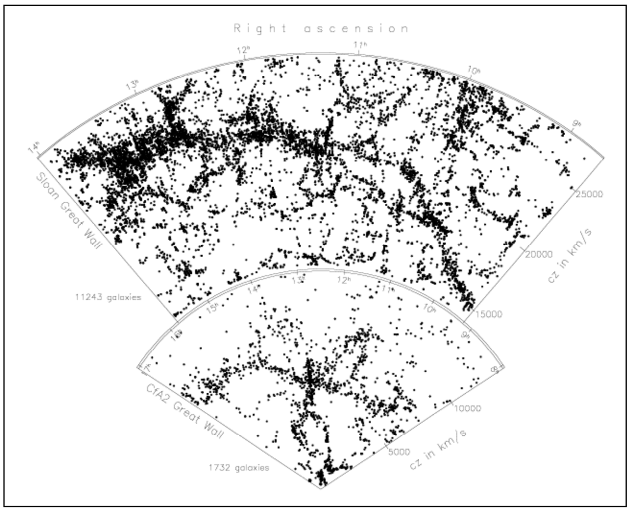

Figure 4. The 'Sloan Great Wall' situated at the redshift z $\simeq$ 0.08 is shown in the upper panel of the figure. For comparison, a relatively nearer structure known as the CfA great wall is also shown. The linear extent of the Sloan Great Wall is about 500 Megaparsec, which is roughly twice as large as the CfA Great Wall.

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The ionized gas in galaxy clusters distorts the Cosmic Microwave Background.

In his later years, Zeldovich, together with students Doroshkevich and Shandarin and the eminent Soviet mathematician Vladimir Arnold, developed a rigorous mathematical understanding of gravitational clustering in the universe involving sophisticated mathemetical methods which included catastrophy theory and percolation analysis. Work in this direction continues to this day [8].

The *Cosmic Web* is spectacular. The 'atoms' of this *web* are galaxies, which gravitationally bind together to form clusters of galaxies (a 'rich' cluster can contain several thousand galaxies) and the much larger superclusters. In 1972, Zeldovich and Sunyaev published a seminal paper in which they showed that photons from the cosmic microwave background would scatter off the hot plasma trapped in the deep potential wells of clusters. This would alter the brightness of the CMB when viewed in the direction of a cluster. Since its prediction almost four decades ago, the *Sunyaev–Zeldovich effect* has been observed in many galaxy clusters and promises to pave the way for a deeper understanding of our universe on the very largest scales, perhaps even throwing light on the elusive nature of dark energy [9].

I hope that this short article has managed to convey Zeldovich's enormous contribution to science in general and to astrophysics and cosmology in particular. His legacy and style of work are enormously inspiring and the directions and vistas opened by him continue to be very actively explored by a new generation of physicists. Some personal reminiscences of Prof. Zeldovich are contained in my second article on him in this issue of *Resonance*, p.462.

Suggested Reading

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Address for Correspondence
Varun Sahni
Inter-University Centre for
Astronomy & Astrophysics
Post Bag 4, Ganeshkhind
Pune 411 007, India.
Email: varun@iucaa.ernet.in