Anybody interested in the laws of nature must be quite dissatisfied with the appearance of *inertia* – the resistance of any object to acceleration – in the equations of motion. The obvious question is, acceleration with respect to what? Is the mass of a particle an *a priori* constant, in which case its value is arbitrary, or is it something that is determined by the environment? Perhaps the greatest idea in human thought, due mainly to the scientist/philosopher Ernst Mach, is that acceleration and inertia are relative, defined with respect to all the matter in the universe. They exert an influence on every scientific experiment with no possibility of shielding! This fundamental principle, called the *Mach Principle*, and how the related ideas of relativity affect our formulation of field theories, form the basis of the following article by Dicke. The article appeared in the *American Journal of Physics*, a journal devoted to the teaching of physics. Although the paper gets a bit technical in some parts, it is worth the effort in trying to understand what Dicke has to say.

Vasant Natarajan and Rajaram Nityananda

Cosmology, Mach's Principle and Relativity*†

R. H. DICKE

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 25 November 1962)

Abstract

The significance of Mach's principle (and the implicit relativity principle) for field theory is discussed, also the significance of zero-mass boson fields for the geometry of the physical space. The significance of such fields for Mach's principle within the framework of cosmology is also discussed. It is suggested that the distant matter of the universe generates one or two zero-mass boson fields, very likely a tensor field and perhaps a scalar, and that each of these fields

* This research was supported by the National Science Foundation and the US Office of Naval Research.

† Reprinted from the Special Cosmology Issue of the *I. C. S. U. Review,* January 1963.

propagates, carrying into the laboratory a quasistatic influence having its origin primarily in the distant matter of the universe. The observable effects of these “Machian fields” are described.

Cosmology is one of the most enchanting of the sciences, for man cannot contemplate the tremendous stretch of the universe, its origins and evolution, without feeling a bit humble. Having its roots in philosophic speculation, cosmology evolved gradually into a physical science, but a science with so little observational basis that philosophical considerations still play a crucial if not dominant role.

From observations made on galaxies in the unobscured 70% of the sky, made mostly in the northern hemisphere, observations which show a decided tendancy for galaxies to cluster, it is concluded, in spite of the clustering, that the universe is basically isotropic (after ignoring “small scale” inhomogeneities). Although these observations were made on a limited volume of space only, and from a single vantage point, we surmise that we would see this same idealized isotropic distribution from any point of observation.

Other observations are even more uncertain. Our knowledge of the density of matter in space is obtained from the visible matter, the stars in galaxies. Virtually nothing is known about intergalactic matter. The observationally determined Hubble expansion age of the universe is believed to be fairly reliable, but the fact that it has doubled twice in a decade shakes ones confidence in this number. While the new radio observations are beginning to provide badly needed supplementary information, the primary need of cosmology continues to be more observations.

Despite the deficiencies in the observational basis of cosmology, it has been possible to lift it above the level of conjecture. This is due largely to the use of a powerful theoretical tool, that of relativistic mechanics.

Perhaps the most important thing to be said about the principles of relativity in relation to the cosmological problem is that these principles provide a rigid frame work, a formal structure that delimits and helps to define the conclusions to be derived from the observations. In a very real sense the host of laboratory experiments performed by physicists, mostly with high-energy particle accelerators, experiments that help to establish the validity of the relativistic principles, are cosmological observations, for the general principles thereby established are directly applicable to the cosmological problem.

While relativity is a strong tool provided by physicists for dealing with the cosmological problem, its early orgins are actually to be found in cosmology. In the early 18th century Bishop G. Berkeley,[1] the British philosopher, in commenting upon Newton's concept of an absolute physical space remarked that this concept was without a physical basis, for a vacuum, devoid of all physical objects, was divested of physical properties, points, lines, and positional relations being meaningless for such an empty *physical* space. Thus Newton's idea of the motion of a body with respect to such an absolute space was a concept devoid of physical significance. Around 1710 Bishop Berkeley wrote, "Let us imagine two globes and that besides them nothing material exists, then the motion in a circle of these two globes round their common center cannot be imagined. But suppose that the heaven of fixed stars suddenly created and we shall be in a position to imagine the motion of the globes by their relative position to different parts of the heavens."[2]

Bishop Berkeley's insistence that the only meaningful motion of a body was motion *relative* to other matter is the *relativity principle.* This idea recurs in the writings of Mach[3] and in the theory of relativity as it was developed by Einstein[4] and others.

Mach asserted that, assuming the validity of the relativity principle, the inertial forces appearing in an accelerated laboratory must have their origin in the distant matter of the universe, for the accelerated motion could with equal right be considered to be that of distant matter relative to the laboratory. Note the cosmological significance of this idea, and the dominant role assigned by Mach to the distant matter of the universe. According to him, this influence of distant accelerated matter penetrates the electrically shielded walls of the laboratory, affecting the physicist's experiment. This is an idea of grand proportions for, if it is right, the universe is much more than the sum of independent parts. The laboratory is tied to and influenced

1 G. Berkeley, *The Principles of Human Knowledge* (1710), pp.111–117; De Motu (1726).

2 D. Sciama, *The Unity of the Universe* (Doubleday & Company, Inc., Garden City, New York, 1959).

3 E. Mach, *Conservation of Energy* (Open Court Publishing Company, Chicago, 1872), Note No. 1; *Science of Mechanics* (Open Court Publishing Company, Chicago, 1883), Chap. II, Sec. VI.

4 A. Einstein, *The Meaning of Relativity* (Princeton University Press, Princeton, New Jersey, 1955), p. 99.

by matter in the remote parts of the universe. If Mach's interpretation of inertia is correct, it might be expected that inertial effects would depend upon the distribution of matter about the point in question. The physicist could ill afford to ignore cosmology under these conditions.

PHYSICAL SPACE

The concepts of an absolute physical space and an absolute time are deeply entrenched. Thus it is difficult to conceive of a physical space, not necessarily Euclidean, a space devoid of points and lines determined *a priori,* a space in which the geometrical properties are physical in origin being derived from the matter contained in the space. Points, lines, and planes are to be considered as meaningless concepts for a physical space devoid of matter (particles), if Berkeley's assumption is correct. Physical points are to be associated with physical events.

Consider a situation in which space is flooded with particles of various kinds, interacting with each other, some perhaps being created or annihilated during these interactions. It is clear that, ideally at least, physical events such as the collision between two particles could be used to define in a meaningful (i.e., invariant) way a point in a 4-dimensional space-time. An electron interacting with a host of photons could define a geometrical point at each such interaction. The sequence of points (events) could define the space-time trajectory of the electron. Each such point could be labeled with a 4-fold set of numbers (coordinates) almost arbitrarily chosen.

Alternatively, if photons were present in a region of space in sufficiently large numbers, their effects, on the average, might be described by an electromagnetic field, the two Maxwell invariants of which could be used, in principle at least, to help locate a point anywhere in the space occupied by the photons. For example, if $\mathbf{E}$ and $\mathbf{B}$ represent electric and magnetic field components, the Maxwell invariants $\mathbf{E}\cdot\mathbf{B}$ and $B^2 - E^2$ could be measured at a given point and their values used as two out of 4 parameters required to physically label the point. Other fields could provide more invariants or alternate invariants.[5] This scheme obviously fails if the various fields are null fields for which the invariants are all zero.

[5] The use of tensor field invariants as invariant coordinates labels is due to A. Komar, Ph.D. thesis, Princeton University (1956) (unpublished).

It should be noted that in this illustrative example it was the Maxwell *invariants,* not field components, which were used to help characterize the points in space physically. These numbers are independent of the orientation and motion of the observer who makes the measurements. Thus they are free of conditions imposed *a priori* by a geometrical coordinate system. This is essential if the concept of a geometrical point is to be divorced from the characterization of an absolute space.

The key idea to be wrung from the relativity principle is the following: Observables, results of measurements by a specific apparatus on a specific physical system are invariants, independent of coordinate system, being dependent only upon the relation of the measuring apparatus to the physical system. The appropriate mathematical formulation of the kinematic and dynamical relations of physics is provided by tensor analysis, for the language of tensors is a language based on coordinates in general, not specific coordinate systems. This has proved to be the key idea; physical events take place in a 4-dimensional space-time continuum and the laws of physics are to be stated in invariant form using the language of tensor analysis.

FIELDS

As was remarked above, when photons are present in space in a sufficiently high concentration, their effects may be considered in bulk, being characterized by a classical field, electromagnetism. The Pauli principle forbids the crowding of fermions to such an extent, and quantum fluctuation effects are never negligible for a fermion field. However, a dense bath of bosons is always possible and can be characterized by a classical field.

By requiring that boson fields, such as gravitation or electromagnetism, be represented by tensors, the range of possible fields is very much limited, and these possibilities are well classified. Thus fields (classical) may be classified as scalar, vector, tensor, and higher-rank tensor. We assume, perhaps without good reason, that higher-rank tensor fields do not exist.

The scalar field is given by a single invariant function of coordinates $\varphi(x)$. If the coordinate system is changed, φ is defined as unchanged at the new coordinates corresponding to the old point in space. A contravariant vector B^i is a set of four functions which transform under coordinate transformations like the coordinate differentials dx^i, namely, $dx^i = (\partial x^i/\partial \bar{x}^j)d\bar{x}^j$ and

$B^i = (\partial x^i / \partial \bar{x}^j)\bar{B}^j$, (note that $i = 1, 2, 3, 4$ is not a power of B but an index label. The sum over the repeated index j=1,. . . , 4 is to be understood.) In similar fashion the covariant vector B_i transforms like the phototype of all such vectors $\varphi_{,i} = \partial\varphi/\partial x^i$, where φ is a scalar, namely, $B_i = (\partial \bar{x}^j/\partial x^i)\bar{B}_j$. It should be noted that the inner product $A^i B_i$ is a scalar. The extension to tensor fields is straightforward. The tensor g_{ij} transforms like

$$g_{ij} = (\partial \bar{x}^k / \partial x^i)(\partial \bar{x}^l/\partial x^j)\bar{g}_{kl}. \tag{1}$$

If the path of a particle is parametrized by an arbitrary chosen monotonically increasing variable χ (invariant), the "velocity" $u^i = dx^i/d\chi$ is a contravariant vector, where x^i refers to coordinates in an arbitrary coordinate system. If there exists a covariant vector field A_i, the product $A_i u^i$ is an invariant.

It should be remarked parenthetically at this stage that the only geometrical concepts so far employed are those of the 4-dimensional space-time continuum and the labeling with smoothly varying but otherwise arbitrary chosen coordinates of events in this 4-dimensional manifold. Nothing has been said about a metric, geodesic curves, or curvature, for these concepts are meaningful only after units of length and time are defined. In a very real sense, the "metry" in geometry is to be taken seriously. The geometry of a physical space is not a property of the space alone. It also involves the means of "measuring" the space. As meter sticks and clocks are physical objects, they are affected by physical fields and the "geometry" based on such objects are affected by these fields. In order to avoid the concept of an absolute space and a geometry introduced *a priori,* the question of the space-time measure, and spatial metric, should be ignored at this stage. As there is a possible arbitrariness in the units and methods of measure, the dynamical behavior of a physical system must be independent of the choice made. Hence it is both possible and desirable to discuss the dynamical behavior of a physical system without specifying closely the geometry of space.

Without a measure of time, a *proper time* cannot be defined in terms of measurements with a moving clock and a four velocity cannot be defined in the usual way. None the less, just as coordinates may be arbitrarily assigned to physical points in space, the trajectory of a particle may be parametrized by arbitrary monotonically increasing parameter (invariant under coordinate transformations) and this parameter χ can be used to define a "four velocity" as shown above. As the parametrization is arbitrary, the space-time trajectory of the particle must be independent of the choice made. This leads to two invariance properties of the equations of motion, invariance under both coordinate transformations and transformations of the "proper time" parameter.

If it is assumed that the motion of a particle interacting only with a vector field is derivable from a variational principle, the field must be covariant and the variational equation is uniquely defined for, the integrand $A_i u^i$ is the only invariant which can be constructed such that the integral over χ, namely,

$$\int A_i u^i d\chi, \tag{2}$$

is independent of the choice of the arbitrary invariant parameter χ. By requiring that this integral between fixed limits take on a value which is an extremum one finds that the motion of the particle is such that

$$0 = (dA_i/d\chi) - A_{j,i}u^j = (A_{i,j} - A_{j,i})u^j. \tag{3}$$

At this point one cannot fail to be struck by the power of the relativity principle, for a close examination of Eq.(3) shows that the right side of this equation has the same form as, and might be taken to represent, a measure of the electromagnetic force acting on a charged particle.

Thus, if this vector field be interpreted as electromagnetism, Eq.(3) states that the particle "moves" in such a way as to make the electromagnetic force (Lorentz) acting on the particle zero. Charged particles do not actually move this way, but this is another story to which we shall return shortly. Here we should simply note that the complex electromagnetic forces fall in a natural way out of the basic assumptions of invariance plus the assumption that electromagnetism is described by a covariant vector field. The covariant vector A_i has four components, essentially the three components of the "vector potential" of classical electromagnetism and the "scalar potential," a total of 4. These names refer to 3-space transformation properties, not to be confused with the more general 4-space general coordinate transformations.

COSMIC FIELDS

The electromagnetic field describes the properties of large numbers of photons in bulk. These particles have zero mass, traveling always with the same speed (of light). The only type of field capable of a long range quasistatic interaction is that associated with such a zero-mass particle. Thus the slowly changing mass distribution in distant parts of the universe could make its presence felt in the laboratory only through zero-mass fields and from the above these fields may be of at least three basic types : scalar, vector, and tensor. (Spinor fields are ignored for reasons which cannot be discussed here.)

Assuming the validity of relativistic principles, these fields, which can be conveniently called cosmic fields, are of the type which in the past, and to this day, determine the gross features of the evolution of the universe, for these are the only types of quasistatic interactions which can extend over great distances. For simplicity we assume that there is at most one field of each type. (The evidence for this is fairly good but cannot be discussed here.)

If it be assumed that Maxwell's electromagnetic equations are valid and that the universe over large volume averages is isotropic, the vector field electromagnetism cannot play a role on the large cosmological scale. The reason for this is the assumed isotropy of the matter distribution which would require both the electric and magnetic fields to vanish over large volume averages and, consequently, the vanishing of electric charge and current (over large volume averages).

Several years ago R. A. Lyttleton and H. Bondi[6] proposed modified electromagnetic equations, nongauge-invariant, such that charge was not strictly conserved. With this assumption cosmological influences of distant matter through the vector field, was possible. However, the extremely precise charge equality measurement of J. G. King[7] can be used to exclude the Lyttleton–Bondi hypothesis.

[6] R. A. Lyttleton and H. Bondi, *Proc. Roy. Soc.* (London) **A252**, 313 (1959).

[7] J. G. King, *Phys. Rev. Letters*, **5**, 562 (1960).

Apparently, we should limit ourselves to scalar and tensor fields in looking for cosmic interactions. Returning for the moment to Eq. (3) it is evident that the element that is missing in order to obtain a sensible equation of motion is an inertial force, for as formulated by Lorentz, a charged particle moves in such a way as to make the sum of the Lorentz force and the inertial reaction zero.

It is evident from Eq. (3) that the vector force is velocity dependent. It is evident that this force is independent of the acceleration of the particle because the variational equation (2) contains the four-velocity linearly only. If the variational equation contains the velocity quadratically, the Euler equation contains a term with a derivative of the velocity. To form an invariant quadratic in velocity a tensor is needed. Apparently, inertial forces can occur only after a tensor field is introduced. We assume that there exists a long range tensor field (associated with chargeless zero-mass particles with a spin angular momentum of $2\hbar$).

From the viewpoint of Berkeley and Mach, it is this tensor field, generated by matter in distant parts of the universe, which must supply the local inertial effects. The precession of a gyroscope, relative to the rotating earth continuing to point always in some fixed direction relative to distant matter of the universe, may be considered the result of its interacting with a tensor field having its origin primarily in this enormous amount of matter ($\sim 10^{56}$g) at great distance (up to light $\sim 10^{10}$ years).

In order to modify Eq. (2) to include the interaction with a tensor field, we replace it by the variational equation

$$0 = \delta \int [m(g_{ij}u^i u^j)^{\frac{1}{2}} + eA_i u^i] d\chi, \tag{4}$$

where e and m are constants introduced to characterize the "charge" and "mass" of the particle. Equation (4) is the only one which can be written depending upon a linear combination of the two interactions, and such that the parameter χ may be chosen arbitrarily. The Euler equation obtained from Eq. (4) [equivalent to Eq. (3)] is

$$\frac{d}{d\chi}\left[\frac{mg_{ij}u^j}{(g_{kl}u^ku^l)^{\frac{1}{2}}}\right]-\frac{1}{2}\frac{mg_{jk,i}u^ju^k}{(g_{kl}u^ku^l)^{\frac{1}{2}}}-eF_{ij}u^j=0, \tag{5}$$

with

$$F_{ij}=A_{i,j}-A_{j,i}=(\partial A_i/\partial x^j)-(\partial A_j/\partial x^i). \tag{6}$$

It is apparent that the first term of Eq. (5) contains an acceleration dependent force. In a sense, the whole term represents the inertial force since it is in the form of a rate of change with respect to "time" of the "momentum" of the particle. The last term is the electromagnetic force already discussed, which has its origin in locally produced electric fields rather than in distant parts of the universe. The seconαd term is a bonus. It represents a force quadratic in velocity and is found upon closer examination to represent the gravitational force.

In a sense, both of the first two terms represent gravitational forces. The first term contains a gravitational force (inertial) which acts upon the body only when it is accelerated. The second term represents a gravitational force present only if there are nonzero gradients in one or more tensor components. The dichotomy between inertial and gravitational forces is artificial, being coordinate-dependent. It can be shown that there always are coordinate systems such that

$$\partial g_{jk}/\partial x^i=g_{jk,i}=0, \tag{7}$$

for any one point in the space. For this choice of coordinate system the gravitational force is zero at this point. There are also coordinate systems in which a given particle is not accelerated, and for which the inertial force is zero.

The condition given by Eq. (7) allows considerable arbitrariness in the coordinate transformation and it is possible in addition to impose the condition that at the point in question, after the coordinate transformation,

$$\begin{aligned} g_{ij} &= 0, \quad \text{for} \quad i\neq j, \\ g_{\alpha\alpha} &= \pm 1, \quad \text{for} \quad \alpha=1,2,3. \\ g_{44} &= 1 \end{aligned} \tag{8}$$

The first three diagonal terms are associated with space-like intervals and because of spatial isotropy have the same sign, not yet specified. The 4th is to be interpreted as associated with time.

There are numerous reasons, mainly obtained from experiments with high energy particles, for believing that there is an upper limit to the velocity of a particle. As the velocity approaches this upper limit its momentum must approach infinity. It can be seen from Eq. (5), after substituting Eqs. (7) and (8), that the proper choice of sign in Eq. (8) is minus, and with this choice

$$mu^j/(g_{kl}u^k u^l)^{\frac{1}{2}} \to \infty$$

as

$$v^2 = \sum_{\alpha=1,2.3} (dx^\alpha/dx^4)^2 \to 1. \qquad (9)$$

Clearly, for this choice of coordinate system $v^2 = 1$ is the upper limit for the square of the speed of a particle. This upper limit can be reached with a finite momentum [see Eq. (5)] only for a massless particle ($m = 0$), hence massless particles such as photons and gravitons (associated with the tensor field g_{ij}) must travel with the same characteristic velocity, that of light.

If the above substitutions are made in Eq. (5) and the arbitrary parameter χ is replaced by dx^4, one obtains for the 4 equations after writing the Lorentz force in conventional form and introducing vector notation

$$(d/dt)[mv/(1-v^2)^{\frac{1}{2}}] = -e(\mathbf{E} + \boldsymbol{v}\times\mathbf{B}) \quad i = 1,2,3 \qquad (10a)$$

$$(d/dt)[m/(1-v^2)^{\frac{1}{2}}] = e\mathbf{v}\cdot\mathbf{E} \quad i = 4. \qquad (10b)$$

The 4th equation is recognized as stating that the work done on the particle by electromagnetic forces is equal to the change in its energy. It is clear from Eqs. (5)–(10) how the tensor field g_{ij} is the source of the prosaic inertial force [the left side of Eq. (10a)].

It is clear from the above that the long-range tensor field, through the acceleration-dependent force which it generates, is a cosmic force par excellence. For its influence, propagated from distant parts of the universe, permeates insidiously the thickest and best shielded walls of the physicist's inner sanctum, affecting all his measurements.

But one cosmic field remains for discussion, the scalar field φ. For reasons, which to this writer are inadequate, it is usually considered to be nonexistent. If it be assumed tentatively to exist, and to act on a particle, the variational equation (4) must be modified to include its effect. A close examination of the effect of a scalar interaction shows that it destroys the constancy of m, making m a function of φ. The only change to be made in Eq. (4) therefore is to assume that m is a function of φ. The same functional dependence must be assumed for all matter, or the composition independence of the gravitational acceleration is destroyed.

For laboratory physics the observable effects of a scalar field, externally provided, are almost nonexistent. If the mass m is of the form $m = m_0 f(\varphi)$ with m_o constant, the function f may be absorbed under the square root in Eq. (4) and combined with g_{ij} to give a new effective $g_{ij}, \bar{g}_{ij} = f^2 g_{ij}$. However, there is always a choice of coordinate system for which locally Eqs. (7) and (8) are satisfied with $\bar{g}_{ij}$ in place of g_{ij}. Then equations of the form of Eq. (10) are satisfied with m replaced by m_0. The only place that the scalar φ would creep into laboratory experiments would be in connection with gravitational measurements, for the gravitational coupling constant

$$Gm^2/\hbar c = (Gm_0^2/\hbar c) f^2, \tag{11}$$

depends explicitely upon f. [In Eq. (11) m refers to the mass of some elementary particle such as a proton.] It is conceivable that the weak coupling constant associated with β decay is also explicitely dependent on particle mass, hence on f.

COSMIC FIELDS AND GEOMETRY

It is to be recalled that geometrical concepts, beyond a simple coordinate labeling of space-time points, have not yet been interjected. The introduction of a metric into the geometry can be carried out as soon as definitions of units of length and time are given. The concept of measure is dependent upon means and units of measure.

If, for the moment, the scalar field is assumed to be nonexistent, the choice of units is natural and almost unambiguous. Characteristic atomic radii and periods (meter sticks and clocks built from atoms) may be chosen to represent units of measure. With this choice of geometrical measure it is found that the tensor g_i, represents the metric tensor of the geometry. This may be readily seen by noting that for the special point in the special coordinate system, given by the conditions of Eqs. (7) and (8), the classical equations of motion of electrons take on a uniquely specified standard form (10). (The same would be true of the quantum mechanical equation.) Thus, in terms of a measure based on these coordinates, the atom assures some definite fixed size and period. But from Eq. (8) (sign minus) the invariant $ds^2 = g_{ij}dx^i dx^j$ represents a measure of the diameter or period of the atom for this case. That is to say, if a diameter of an atom is given by $dx^1 = D$, $dx^2 = dx^3 = dx^4 = 0$, one has $ds^2 = -D^2$, hence ds^2 measures the diameter. As ds is an invariant, it represents a measure of length (or time) for all coordinate systems. Thus g_{ij} represents the metric tensor of a Riemannian geometry.

When a scalar field is present, the choice of units of length and time is not nearly so unambiguous. If measurements are made in such a way as to treat the mass of an elementary particle, Planck's constant, and the velocity of light as constant by definition, the gravitational constant becomes Gf^2 and is variable. Under these conditions, a geometrical measure of space carried out with the atom providing units of length and time would find that the tensor $\bar{g}_{ij}$ can be interpreted as the metric tensor of a Riemannian geometry. If, however, units of measure are provided by defining $G, \hbar$, and c as constant, the tensor g_{ij} plays the role of the metric tensor.[8] The first gravitational theory combining the effects of a tensor and scalar field was formulated by Jordan.[9] This was a theory which attempted to give a formal basis to the cosmological ideas of Dirac.[10] Later the same type of formalism was used by Brans and Dicke in connection with a discussion of Mach's principle.[11] The equivalence of the Brans–Dicke formalism with a "general relativity" theory was later shown by using a units transformation.[8]

[8] R. H. Dicke, *Phys. Rev.* ,**125**, 2163 (1962).

[9] P. Jordan, *Schwerkraft und Weltall* (Vieweg und Sohn, Braunschweig, Germany, 1955); *Z. Physik* **157**, 112 (1959).

[10] P. A. M. Dirac, *Proc. Roy. Soc.* (London) **A165**, 199 (1938).

[11] C. Brans and R. H. Dicke, *Phys. Rev.*, **124**, 925 (1961).

The scalar field, if it exists, carries into the laboratory a cosmological effect, additional to the easily observed inertial effect. Within the framework of a cosmology with a scalar field, it was found that the gravitational coupling constant $Gm^2/\hbar c$ would be expected to have a value of the order of magnitude the age of the universe (expressed in atomic time units) divided by the mass of the visible part (expressed in particle mass units)." Expressed in physical terms, it has the extremely small value of 1.8×10^{-38} (expressed in terms of the proton mass) because it varies inversely as the scalar field which has a large value being generated by the enormous amounts of matter in the universe. This value $\sim 10^{-38}$ agrees roughly with the value calculated from the above relation using the observed mass density of the universe. It would be expected to vary as the universe expands. Dirac[10] was the first to suggest a connection between the mass content of the universe, its size, and the gravitational coupling "constant."

Of particular significance is the fact that the gravitational "constant" would be expected to decrease with time. Assuming an evolving universe, the scalar field would increase with time because of the increasingly larger amounts of matter visible in an expanding universe. This would decrease the strength of the gravitational interaction.

To return briefly to the relativity theme of the first part of this paper, Einstein's "general relativity," a rather special relativity theory for which the scalar field is assumed not to exist, appears to be more nearly a theory of an absolute space than that of a relativistic space. This has been emphasized by Synge in his new book.[12] There are several reasons for this: First, Einstein's field equation for g_{ij} has solutions for empty space in the form of flat space metrics. From the point of view of Mach such solutions are meaningless. Second, the space about a localized mass distribution, becomes asymptotically flat at infinity and there is, in principle, nothing to keep one from journeying arbitrarily far into this twilight zone where inertial effects should disappear. Third, the gravitational acceleration of the earth toward the sun, according to Einstein, is independent of the amount of distant matter isotropically distributed about the sun. According to Mach's ideas concerning the origin of inertia, this acceleration should be dependent upon the total mass distribution. In the opinion of this author, Synge is correct,

[12] J. L. Synge, *Relativity, The General Theory* (North–Holland Publishing Company, Amsterdam, 1960), see preface.

Einstein's theory (in its usual form) is actually not strictly a relativity theory; it is more nearly the theory of an absolute 4-dimensional geometry. It is beyond the scope of this article to discuss these matters in detail, in particular to consider the significance of boundary conditions and space closure with respect to Mach's principle. (See discussions by Wheeler[13], also by Hönl,[14] on this problem.)

The addition of a scalar field, with a suitable boundary condition on the scalar, changes the situation.[11] (1) There is no solution at all for an empty space. (2) Space closes about a localized mass configuration making it impossible to journey into the never-never land of inertialess space. (3) The acceleration of the earth toward the sun depends upon the value of the scalar at the solar system, in turn depending upon the amount of matter at great distance.

COSMIC FIELDS AND COSMOLOGY

We have seen that from Mach's viewpoint the physicist can ill afford to ignore cosmology, for cosmological effects penetrate the walls of his laboratory, affecting his experiments. The two long-range cosmic fields, certainly tensor and possibly scalar, determine also the gross evolutionary history of the universe. The third long-range field, electromagnetism, probably plays an important role on a smaller scale only, with distances of the order of a galactic diameter being the characteristic size of electromagnetic elements.

In turning to more specific cosmological questions, we first write the Einstein field equation for the tensor g_{ij}, (assuming no scalar field exists).

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -(8\pi G/c^4)T_{ij}. \tag{12}$$

Here R_{ij} is a curvature tensor, a tensor measure of the curvature of space with g_{ij} interpreted as the metric tensor of the space. In similar fashion, R is a scalar measure of curvature. T_{ij} is the energy–momentum tensor of matter.

[13] J. A. Wheeler, to appear in *Gravitation and Relativity,* edited by H. Y. Chiu and Wm. F. Hoffmann (W. A. Benjamin, New York, 1963).

[14] H. Hönl, *Physikerlagung Wien,* edited by E. Brüche (Physik Verlag, Mosbach/Bader, Germany, 1962).

The third term on the left is a bone of contention among relativists and cosmologists. The term Λ is the reciprocal of the square of a characteristic length, a length which must be huge $\sim 10^{28}$ cm. The cosmological term was first introduced by Einstein, *ad hoc,* in an attempt to obtain equations describing a static universe. When it became apparent from Hubble's observations that the universe was actually expanding, he dropped the term. Because of the enormous characteristic length which this term introduces, and the inelegant variational principle from which Eq. (12) is derived, most specialists in general relativity also drop this term. On the other hand, some cosmologists, pointing to the discrepancies in age patterns[15] with stars apparently older than the Hubble expansion age of the universe, retain this term to help obtain consistency. The author is inclined to drop the cosmological term until its existence is clearly forced by observations. It is evident, however, that decisions such as this are based more on formal considerations and philosophy than upon observations. Λ is assumed to be zero in the remainder of this article.

The next important problem which a cosmologist must face involves the question of isotropy. If the universe is everywhere isotropic, it is uniform and the field equations[12] are enormously simplified by this uniformity. Without this simplifying assumption the equations are too general and the observations too few to limit the class of possible solutions to a reasonable number. But still it is difficult to support the assumption with observations. Thus Dr. Heckmann has pointed out that a certain type of departure from isotropy could have had drastic effects early in the expansion of the universe and now be essentially unobservable.

Here, again, a decision must be based primarily on matters of philosophy. One assumes the simplest situation compatible with the observations, namely a roughly uniform and isotropic universe. Without this assumption it is essentially impossible to choose between the large number of possible alternatives. However, we stretch the fibres of credulity too far if we believe that this assumption is valid all the way back to the start of the expansion of an infinitely dense universe of zero radius. One important question to answer eventually is, "How far back in time, if at all, is the assumption valid?"

[15] R. H. Dicke, *Rev. Mod. Phys.* **34**, 110 (1962).

If the assumption of isotropy and uniformity is made, several important conclusions can be based on this essentially geometrical fact, conclusions which are based on the kinematics of the universe and are independent of dynamical considerations. There exists a coordinate system, time orthogonal, such that the metric tensor has the form such that the expression for interval is

$$(1/c^2)ds^2 = -[a^2/(1+kr^2)^2] \times [(dx^1)^2 + (dx^2)^2 + (dx^3)^2] + dt^2 \,. \tag{13}$$

This describes a system of coordinates such that galaxies, in the mean, occupy fixed space points $x^1 ... \, x^3$, but that the mesh of this three dimensional coordinate system keeps expanding with time, distances (measured in time units) being proportional to a, a function of time. The time coordinate t is sometimes called cosmic time. It would be measured by a clock on one of these idealized galaxies, stationary in the coordinate mesh. The 3-dimensional subspaces with x^4 constant are of uniform curvature (a function of time). As an example of a curved space that can be easily visualized consider a 2-dimensional surface. A 2-dimensional surface of constant curvature may be a spherical surface (positive curvature), a plane (zero curvature), or a hyperbolic surface (negative curvature). There are also 3 different types of 3-dimensional spaces of constant curvature, namely positive, zero, and negative curvature.

While the observational evidence is poor, it is beginning to favor slightly the universe of positive curvature.[16]

If the tensor g_{ij} in the form of Eq. (13) is substituted into Eq. (12), there is obtained the equation

$$(da/dt)^2 + k = (8\pi/3)G\rho a^2, \tag{14}$$

for the expansion parameter a. Here ρ refers to the average matter density of the universe and k takes on the values $+1$, 0, and -1 for closed, flat, and hyperbolic universes.

The Hubble expansion age of the universe (determined from the galactic red-shift observations) is given by the expression

---

[16] A. Sandage, *Astrophys. J.* **133**, 355 (1961).

$$T_h = a/(da/dt). \tag{15}$$

Another parameter of importance because it can be measured, albeit very imperfectly, is the acceleration parameter.

$$q_0 = -a\frac{d^2a}{dt^2}/\left(\frac{da}{dt}\right)^2 = \frac{4\pi}{3}GT_h^2\left(\rho + 3\frac{P}{c^2}\right), \tag{16}$$

where P is the pressure.

Assuming for definiteness (although the observations are not this firm) that the present value of the Hubble age is $T_h = 1.3 \times 10^{10}$ years and that of q_o is ~ 1.0, one finds that the calculated density of matter in the universe is 2×10^{-29} g/cm^3. This calculation is based upon the assumption that the average matter pressure in space is negligible and this result is to be compared with an observed density of visible matter of 10^{-31} g/cm^3.

This vast discrepancy is interpreted by many cosmologists to mean that space contains matter outside the galaxies, perhaps ionized, perhaps neutral but at very low temperature. It has also been suggested that the required energy density may be provided by a vast swarm of the almost unobservable neutrinos. Gravitational waves and scalar waves are another possible source of energy. Such a large amount of energy in the form of massless particles (neutrinos or gravitons) would imply a large pressure in space (2.10^{-8} dyn/cm^2). Some cosmologists interpret the discrepancy as implying that the cosmological term in Einstein's equations, a term which we dropped, must be included.

If pressure in space is negligible, a value of $q_{\rm o} > 0.5$ implies that space is closed. If the energy is supplied by neutrinos, gravitons, or scalarons, the condition for space to close is $q_{\rm o} > 1.0$. It is apparent that the observations tend to favor the universe with a closed space. However, the value $q_{\rm o} = 1 \pm \frac{1}{2}$ obtained from the observations does not inspire great confidence.

It has been explicitely assumed in the above development that the scalar field is nonexistent. If it does exist, surprisingly little modification is needed. Equations (12)–(16) are still valid. However, the energy momentum tensor of matter in Eq. (12) now contains contributions from the scalar field. Also the unit of measure of length is not based on the atom but is taken to be the characteristic length $(G\hbar/c^3)^{\frac{1}{2}}$.

The definition of the metric tensor of the Brans–Dicke theory is based on atomic-standard units of length and time. This (Brans–Dicke) metric tensor is obtained from that for which Eq.(12) is valid by multiplying by the reciprocal of the field scalar. In similar fashion, to obtain the cosmological expansion parameter of the Brans–Dicke theory, the a of Eq. (14) is multiplied by $\lambda^{-\frac{1}{2}}$ (The time variable is also multiplied by this factor.) See Ref. 8 for the details of this transformation.

For the Brans–Dicke theory, the expansion parameter being based on $\bar{g}_{ij}$, the equation for a is a little more complicated. (See Ref. 11.) However, assuming outgoing wave boundary conditions for the scalar the solutions of this equation differ surprisingly little from those without the scalar present. An example is plotted in Fig. 1 for the case of a close space. The parameters have been chosen to correspond with the above assumed values of T_h but with $q_o = 1.15$. The resulting value for the "age" of the universe is approximately 7×10^9 years. Figure 1 includes a curve for λ, the scalar, which is to be interpreted as proportional to the reciprocal of the gravitational constant.

The curve for $a(t)$ in Fig. 1 is nearly the same shape as that without a scalar field present. Also the gravitational constant given by λ^{-1} in Fig. 1 is not sufficiently rapidly varying function to make simple a decision between the two theories on the basis of the effect of a greater gravitational interaction, in the past, on the color and luminosity of distant galaxies.

STEADY STATE COSMOLOGY

Finally, it is necessary to make a few remarks about the steady-state cosmology of Hoyle,[17] and of H. Bondi and T. Gold.[18] This is based on the ingenious idea that a continually expanding universe could exist in a steady state, the continuous rarefaction implicit in expansion being vitiated by the creation of new matter. It is visualized that hydrogen materializes in free space, gathers into galaxies which in turn evolve and disappear through the general expansion process.

Because of the requirements of a steady state, the general expansion process

[17] F. Hoyle, *Monthly Notices Roy. Astron. Soc.* **108**, 372 (1948); **120**, 256 (1960).

[18] H. Bondi and T. Gold, *Monthly Notices Roy. Astron. Soc.* **108**, 252 (1948).

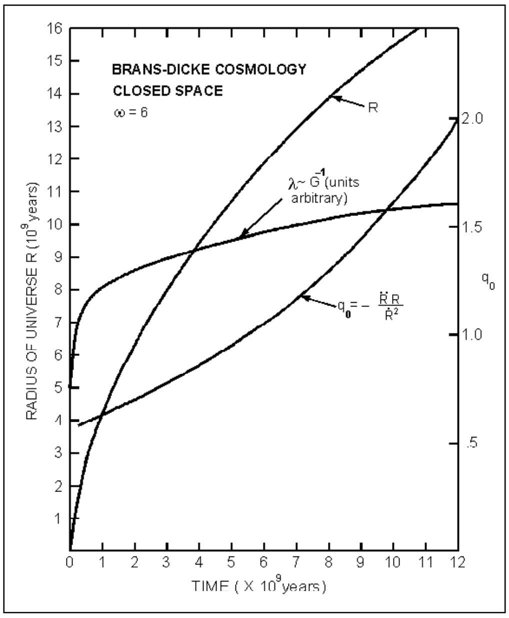

Figure 1. The expansion parameter R (=a) plotted against time for the Brans–Dicke cosmology. (Also given are the scalar and the acceleration parameter.)

results in the abundance of galaxies decaying exponentially with their age. The mean age of long-lived objects, such as galaxies, is $\frac{1}{2}T_h \sim 4 \times 10^9$ years. The probability of a galaxy being older than 2×10^{10} years is under 0.01.

One particularly satisfactory aspect of the steady-state cosmology is that it provides a constant distribution of distant matter and permits Mach's principle to be satisfied with a constant gravitational coupling "constant."

The ideas encompassed in the steady-state cosmology are interesting, but they have not yet been reduced to a satisfactory mathematical formulation complete enough to permit a detailed analysis.

From the standpoint of relativity, the theory appears to be vulnerable to attack on the grounds that it violates fundamental conservation laws, those of energy and momentum. While a quasi-static scalar field, similar to the one introduced by Hoyle,[17] can affect conservation relations by modifying the rest masses of particles, new particles cannot, within the framework of the usual relativistic theories, be created by such a field. New particles can be created only by stealing energy and momentum from other particles. If it is presumed that space is filled with high-energy particles capable of creating protons, this gas of queer particles would also suffer rarefaction from the expansion.