Robert Dicke and Atomic Physics

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In this article, we highlight the contributions of Dicke to atomic physics. The effects, called Dicke narrowing and Dicke superradiance, are contained in two seminal papers published in the 1950's. The description below should be accessible to anyone who has had a brief introduction to quantum mechanics.

1. Line Shapes and Widths

Spectroscopy is the name given to the process of measuring the frequency response of a system – its *spectrum*. In atomic physics, this usually means measuring one of the resonance lines of an atom, i.e., a transition from a ground state to an excited state. It is well known that no resonance line is infinitely narrow, even in theory, because spontaneous emission introduces a finite lifetime to the excited state, and therefore (by the time-frequency uncertainty principle) a natural width to the transition. In practice, a number of other processes, such as collisions, Doppler effect, light intensity (power broadening), field inhomogeneities, etc., act to increase the width of resonance lines. It is important to understand how these effects manifest themselves in the shape of the resonance curve (the 'line shape') so that one can do a curve fit to the observed spectrum and thereby find the resonance frequency ω_0.

Line broadening mechanisms are broadly classified into two, *homogeneous* and *inhomogeneous*, depending on whether the mechanism affects the line shape in a similar way for each particle (homogeneous) or whether the broadening arises from a random shift in the resonance frequency for different particles causing a widening for

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an ensemble of particles (inhomogeneous). Thus, the natural linewidth and collision-induced effects are examples of homogeneous broadening, while the Doppler effect and field inhomogeneities result in inhomogeneous broadening.

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Mathematically, the line shape is quite different for the two broadening mechanisms. Homogeneous broadening arises from random changes in the phase (or coherence) of the radiation from the particles and therefore results in a *Lorentzian* line shape. The normalized Lorentzian function is given by

$$L(\omega) = \frac{1}{\pi} \frac{(\Gamma/2)^2}{(\omega - \omega_0)^2 + (\Gamma/2)^2} , \qquad (1)$$

where Γ is called the full width at half maximum (FWHM). When the only broadening is due to the natural linewidth, Γ is the inverse of the excited-state lifetime.

Inhomogeneous broadening results from random perturbations in the resonance frequency of the particle which follows a Gaussian (or normal) distribution, and thus produces a *Gaussian* line shape. The normalized Gaussian function is given by

$$G(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\omega - \omega_0)^2}{2\sigma^2} \right] , \qquad (2)$$

where σ is the width of the Gaussian.

1.1 *Line Shape of Confined Particles*

Confined or trapped particles offer the ultimate in spectroscopic resolution because:

Homogeneous broadening results in a *Lorentzian* line shape. Inhomogeneous broadening produces a *Gaussian* line shape.

1. The trapped particle can be cooled with lasers or cryogenically to a temperature of order 1 mK, so that the second-order Doppler effect is below 10^{-16}.
2. With proper vacuum, the effect of collisions can be virtually eliminated.

The spectrum of a trapped particle consists of an unshifted central line and sidebands separated by the oscillation frequency in the trap.

3. The first-order Doppler effect can also be suppressed because the spectrum of a trapped particle consists of an unshifted central line and sidebands separated by the oscillation frequency in the trap. In fact, it is possible to just address the recoilless central line, as in the case of the Mössbauer effect or in the use of buffer gases.

To see this last point more clearly, consider a particle trapped in a harmonic potential. If the particle is oscillating in the trap at a frequency ω_t and amplitude x_0, then its instantaneous phase at a far detector is

$$\phi(t) = -kx_0 \sin \omega_t t - \omega_0 t \,, \tag{3}$$

where $k = 2\pi/\lambda$ is the wave vector. The instantaneous frequency of the emitted wave is therefore

$$\omega = -\dot{\phi}(t) = kx_0\omega_t \cos \omega_t t + \omega_0 \,. \tag{4}$$

The first term can be recognized to be the usual Doppler shift of kv. In the language of electrical engineering, the above equations show that the wave is undergoing *frequency modulation* with a modulation index

$$\beta = kx_0 \,. \tag{5}$$

Any standard textbook will tell you that the amplitude of a wave undergoing fm modulation with an index β is

$$a(t) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta) \cos[(\omega_0 - n\omega_t)t] \,, \tag{6}$$

where J_n's are nth-order Bessel functions. Thus the spectrum consists of a central line at ω_0, and an infinite set of sidebands spaced uniformly at multiples of the oscillation frequency ω_t but with progressively smaller amplitude. The spectral intensity of the emitted light is proportional to

$$I(\omega) \propto \sum_{n=-\infty}^{\infty} J_n^2(\beta)\delta(\omega - \omega_0 - n\omega_t) \,. \tag{7}$$

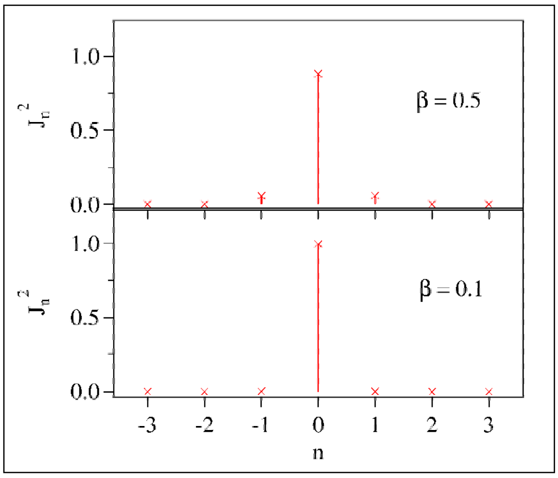

Figure 1. Spectral intensity of center and sidebands in fm modulation for two values of the modulation index β. The sidebands are practically zero for β = 0.1.

The effect of β on the spectral intensity is seen graphically in *Figure* 1. The height of the central peak and three sidebands are shown in the figure for two values of β. For $\beta = 0.5$, only the $n = \pm 1$ sidebands are significant, while the others are practically zero. For $\beta = 0.1$, even this sideband is almost zero.

1.2 *Tight Confinement: The Lamb–Dicke Regime*

The discussion above shows that the most dramatic effects of confinement will be seen when the particle is confined to a size that is smaller than the wavelength of the emitted light, called tight confinement. In other words, $x_0 \ll \lambda$ or $\beta = kx_0 \ll 1$. From *Figure* 1, the sidebands are almost completely suppressed when $\beta \ll 1$ and only the central unshifted line survives. This is called *recoilless emission* because the particle has the same momentum before and after the emission, and the momentum of the emitted photon is provided (or taken up in the case of absorption) by the whole trap. This is analogous to the Mössbauer effect in solids where the momentum of the gamma ray photon is provided by the whole crystal.

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The regime of tight confinement is particularly easy to achieve for microwave transitions in ion traps, where the wavelength is much larger than the trap size. Such

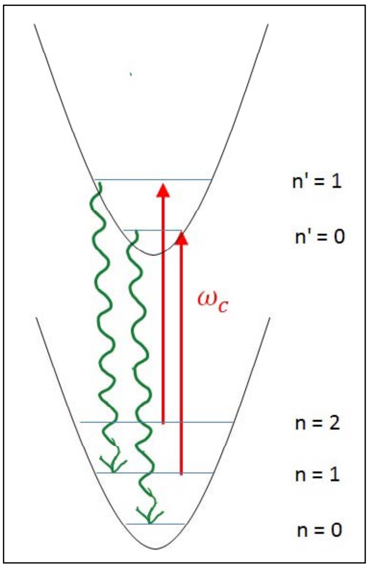

Figure 2. Laser cooling of harmonically-trapped ions by tuning the laser to the lower motional sideband. The ground and excited states of the ion are quantized into equispaced motional levels. The ion loses one quantum of motion in each absorption–emission cycle.

confinement plays an important role in precision spectroscopy, atomic clocks, and quantum computation. Trapped ions can also be laser cooled using what is called *sideband cooling*. The basic idea is seen in *Figure* 2. The external degree of freedom of the ion in the harmonic trap (i.e., oscillatory motion) is quantized to form discrete energy levels. The levels are equally spaced for a harmonic oscillator. On top of this, the internal degrees of freedom form the energy levels of the ion. Thus, the ground and excited states form two sets of harmonic-oscillator levels spaced apart by ω_{ge}, which is the energy difference between the two states. For sideband cooling, the laser is tuned to the lower motional sideband of the transition, i.e., $\omega_c = \omega_{ge} - \omega_t$. After absorbing such a laser photon, the ion will emit by spontaneous emission most probably at ω_{ge}. Thus the ion loses one quantum of trap motion in each absorption-emission cycle. This cooling works best when the natural line width of the upper state is much less than ω_t, so that the uncertainty in the levels of the upper state is much smaller than their spacing. Under these conditions, the ion can be cooled to the lowest quantum state of the trap and one can realize tight confinement.

1.3 *Dicke Narrowing*

The regime of tight confinement is called the Lamb–Dicke regime in honour of Dicke's contribution. But in his seminal paper [1], Dicke went beyond and argued that it is not necessary to confine the particle harmonically to see significant narrowing. He showed that collisions could reduce the usual Doppler width significantly if two conditions were met: (i) the mean free path between collisions is much smaller than the wavelength, and (ii) the collisions do not destroy the coherence between the radiating states. Under these conditions, Dicke showed that the spectrum consists of a narrow 'recoilless' peak on top of a broad Doppler pedestal.

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The essence of Dicke's argument is that the effect of collisions can be viewed as a series of one-dimensional traps of size a, where $a \ll \lambda$, since the scale of a is set by the mean free path. The atom moves back and forth between the two walls, so that the oscillation frequency varies with a. In other words, the radiation is frequency modulated at different frequencies depending on a. Thus all particles would have the same recoilless line and a set of sidebands whose positions varied randomly. The total emission would then show this narrow unshifted line as a prominent feature, and the sidebands would average to a broad spectrum with roughly the original Doppler width.

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Dicke also considered a gas where $a \ll \lambda$ and the atoms diffused randomly. He showed that the line shape was Lorentzian instead of Gaussian. The width of the line was roughly $2.8a/\lambda$ times the normal Doppler width, which is again considerably smaller.

The only thing that Dicke did not say in this paper was that the recoilless peak could be seen more easily for a nuclear transition from nuclei localized in a solid. Otherwise, the Mössbauer effect would have been discovered several years earlier and would have been called the Dicke effect!

2. Coherence

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Coherence is a term that is used to describe a process where amplitudes add with a relative phase and one observes a physical quantity that is proportional to the square of the total amplitude. A familiar example is the interference pattern from two slits one sees in classical optics. The light coming from the two slits is coherent, so that, at any point on the screen, one adds the electric-field amplitudes, while the observed light intensity is proportional to the square of the total E field. In quantum mechanics, coherence and interference arise when the wave amplitudes have to be added.

In atomic physics, the term coherence is used for two distinctly different phenomena: (i) those which occur in a *single* atom when two states have a relative phase between them, and (ii) those which occur when there is more than one atom radiating so that radiative coupling between the atoms becomes important. The first kind of coherence leads to phenomena such as quantum beats (in which the radiation rate oscillates in time) and level crossing (in which interference between two nearly-degenerate states modifies the fluorescence intensity and polarization). The second kind dominates when the atoms are localized to a spatial region that is smaller than a wavelength of the radiation. Then, radiative coupling between the atoms becomes important and the phase of the field between the radiators does not vary much, i.e., retardation effects can be neglected.

If the radiation rate of a single particle is I_1, one expects that the rate for the entire system would be just NI_1. But this is incorrect because it ignores the fact that all the particles are interacting with a common radiation field.

In his paper [2], Dicke discussed this second kind of coherence, namely the collective behaviour of N particles decaying by spontaneous emission. If the radiation rate of a single particle is I_1, one expects that the rate for the entire system would be just NI_1. But this is incorrect because it ignores the fact that all the particles are interacting with a common radiation field.

2.1 *Coherence Between Two Spin-1/2 Particles*

To illustrate this point, Dicke considers the coupling between two two-level systems. The canonical example of a two-level system is a spin-1/2 particle (e.g., neutron) in a magnetic field. (In fact, there is a theorem called the Feynman–Vernon–Hellwarth theorem which states that the dynamics of any two-level system can be mapped to the spin-1/2 in a magnetic field.) If a neutron is placed in a uniform magnetic field in the higher of the two spin states, then in due course of time it would decay to the lower state by spontaneously emitting a photon via a magnetic dipole transition. Thus the probability of finding it in the upper state would decay exponentially to zero.

The Feynman–Vernon–Hellwarth theorem states that the dynamics of any two-level system can be mapped to the spin-1/2 in a magnetic field.

Now consider what happens if a second neutron in the *ground state* is placed close to the first excited one. Here, close means at a distance small compared to the radiation wavelength, but large compared to the interparticle dipole–dipole interaction length. If the above independent radiator hypothesis is correct, the radiation from the first particle would not change. But, in reality, the radiation would be strongly affected because the transition probability would fall exponentially to one-half and not zero.

To see this, consider the combined states of two spin-1/2 particles. Their individual spin-angular momenta add to form what are called singlet and triplet states. The singlet state has total angular momentum of zero, and therefore a single magnetic sublevel. On the other hand, the triplet state has total angular momentum of 1, and hence three sublevels. Remember that the total angular momentum characterizes the eigenstate of the square of the spin operator ($\mathbf{S^2}$), while the sublevels are eigenstates of the $\hat{z}$-component of the spin operator ($\mathbf{S_z}$). If we denote their simultaneous eigenstate by

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If there is no photon emitted after a long time, we can be sure that the neutrons are in a singlet state but cannot say which neutron is in the excited state.

$|s,m\rangle$, then

$$\begin{aligned} \mathbf{S^2}\,|s,m\rangle &= s(s+1)\,|s,m\rangle\ , \\ \mathbf{S_z}\,|s,m\rangle &= m\,|s,m\rangle\ . \end{aligned} \tag{8}$$

One can think of the singlet state as one where the two neutrons are anti-aligned, and the triplet state as one where the two neutrons are aligned.

Therefore, when the second unexcited neutron is added, the initial state (with $m_s = 0$) is an equal superposition of the singlet and triplet states, namely the $|0,0\rangle$ and $|1,0\rangle$ states. The singlet and triplet sets do not couple to each other, thus only the triplet state can decay to the lower energy state while the singlet state is stable. Hence, after a long time, there is still a probability of one-half that no photon has been emitted. Indeed, if there is no photon emitted after a long time, we can be sure that the neutrons are in a singlet state but cannot say which neutron is in the excited state. One can also show that the radiation rate is double that for just a lone excited neutron.

2.2 *Dicke Superradiance*

From the above discussion, it is clear that coherence between two radiators can strongly influence the spontaneous radiation rate. Dicke now extends this discussion to the coherent radiative behaviour of N two-level systems. Unlike in his paper, we will consider these systems to be spin-1/2 particles so that what he calls the cooperation operator $\mathbf{R}$ becomes the familiar angular momentum operator. We have

$$\mathbf{R} = \sum_{i=1}^{N} \mathbf{S_i} \tag{9}$$

as the total angular momentum operator. The eigenvalue of $\mathbf{R^2}$ is (c.f. (8))

$$r(r+1) \leq \frac{N}{2}\left(\frac{N}{2}+1\right)\ .$$

The eigenvalue of the z component of $\mathbf{R}$ is

$$m = \sum_{i=1}^{N} m_i = \frac{1}{2}(n_+ - n_-) ,$$

where n_+ and n_- are the numbers of up and down spins, respectively. Thus, r represents how much the spins are aligned while m represents the projection of the total spin along the z direction.

r represents how much the spins are aligned while *m* represents the projection of the total spin along the *z* direction.

Using this formalism, Dicke shows that the matrix element governing the interaction with the field (i.e., the raising operator to go from $m-1$ to m) is proportional to $[(r+m)(r-m+1)]^{1/2}$. Hence, the radiated intensity is

$$I = I_1(r+m)(r-m+1) , \qquad (10)$$

where I_1 is the rate for a single system. This is much larger than NI_1 when r is big (which can be as large as $N/2$ if all the spins line up) and m is small (which is 0 if the total spin precesses in the x–y plane). Such states with enhanced radiation rates are called *superradiant* states. For example, for the case when $r = N/2$ and $m = 0$, $I \approx N^2 I_1$, which is much larger than NI_1 and is in accord with our earlier discussion on the effect of coherence for N particles. On the other hand, if r is small (can be $1/2$ when N is odd and 0 when N is even), then $I = I_1$ or 0 depending on whether N is odd or even. These states are *subradiant* because they radiate less rapidly compared to N independent systems. The intermediate case when both r and m are big, say $N/2$ and r, the rate is $I = NI_1$, the same as for a set of N independent radiators.

States with enhanced radiation rates are called *superradiant* states. States are *subradiant* if they radiate less rapidly compared to *N* independent systems.

Dicke suggests two ways of producing a superradiant state with $r \approx N/2$ and $m \approx 0$. The first method is to cool the system so that it is in its ground state of $r = N/2$ and $m = -r$. Then give it a pulse to bring it to the x–y plane where $m = 0$. Such a pulse is called a $\pi/2$ pulse in NMR since it changes the angle of the magnetic moment by $\pi/2$. The second method is to put the system in its extreme excited state with all the particles in the upper level, i.e., with $r = N/2$ and $m = r$, and then just *wait*. At first, the system will decay as independent radiators, with $I = NI_1$. Since the selection rule for the transitions is $\Delta r = 0$ and $\Delta m = -1$, the system will radiate faster and faster until $m \approx 0$. At this point, the intensity will be $N/2$ times larger, which can be quite dramatic if N is large.

In his paper, Dicke also points out that the rate of stimulated emission/absorption is not enhanced by N^2, but only by N, even for the state with $r = N/2$. Thus, his beautiful analysis demonstrates a surprising result – the rate of *spontaneous emission*, which is a self-induced and random process for a single atom, shows coherence effects for a suitably localized ensemble of atoms.

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Suggested Reading

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[2] R H Dicke, Coherence in Spontaneous Radiation Processes, *Phys. Rev.*, Vol.93, p.99 1954.