# Israel Moiseevich Gelfand – An Appreciation from a Physicist's Viewpoint

The erstwhile Soviet Union – which collapsed around 1990 – had by all accounts set up an excellent framework for spotting and training really talented young people in the sciences and mathematics from all parts of its enormous territory. As part of this system, many leading scientists and mathematicians also were gifted writers of textbooks and monographs at advanced levels. In particular, many 'pure' mathematicians wrote books in an extremely lucid manner, always keeping in mind readers from the physics and engineering communities. This style of exposition of mathematics, consciously making it accessible to users who may not themselves have been professional mathematicians, seems to have been carefully cultivated. Many classic texts of that era were available at very affordable prices, often brought out by Mir Publishers.

Israel Moiseevich Gelfand was a leading figure in Soviet mathematics in the 20th century, and contributed enormously in the manner described above. The vast range of his work is explained in V S Sunder's article in this issue. One aspect of it is worth mentioning as it has in a sense an 'Indian connection', and that is his work with M A Naimark on the unitary representations of the Lorentz group.

The homogeneous Lorentz group SO(3,1) is fundamental to special relativity; it describes the transformations between the inertial frames of physics, familiar to all students of physics. It is made up of special Lorentz transformations, rotations in three-dimensional space, and their combinations. This group is basic to both classical relativistic physics, and relativistic quantum mechanics.

Many of the familiar relativistic physical quantities belong to various finite dimensional representations of the Lorentz group, and these are necessarily non-unitary. (Unitarity is a concept which every student of quantum mechanics learns quite early on). Examples are space-time position four-vectors of point particles, as well as their energy-momentum four-vectors; the Maxwell electromagnetic field strengths, the relativistic energy-momentum tensor of general field theories; etc.

The famous Dirac relativistic wave equation for the electron, discovered in 1928, also involves quantities belonging to another finite (in fact four) dimensional representation of the Lorentz group, namely the Dirac spinor representation.

In 1945, in a fundamental paper, Dirac constructed some new infinite-dimensional *unitary* representations of the Lorentz group, and suggested that they may be important for physics. This idea itself was new both within physics and within mathematics. Just around that time, Harish-Chandra joined him as a PhD student in Cambridge. Harish-Chandra had completed his MSc in physics from the University of Allahabad in 1943 and then moved to the Indian Institute of Science in Bangalore as a research student. In the period 1943–1945 he did a considerable amount of work in collaboration with Homi Bhabha. His being accepted as a PhD student by Dirac was facilitated by strong recommendations by Bhabha and K S Krishnan who had taught Harish-Chandra in Allahabad.

Dirac proposed to Harish-Chandra that he study the problem of constructing all the unitary (irreducible) representations of the Lorentz group. This challenging problem was solved by Harish-Chandra during 1945–1947, leading to two landmark papers in 1947. As it happened, this same problem was also solved essentially simultaneously and independently by Gelfand and M A Naimark in the Soviet Union, and published by them also in 1947.

Later developments in physics have not made use of these representations of the Lorentz group to any significant extent, but in any case the credit for constructing these goes both to Harish-Chandra and Gelfand and Naimark. All this – starting with Dirac's 1945 work – is justifiably regarded as the origin of a beautiful chapter in modern mathematics.

Going back to the opening paragraph of this article, for the benefit of younger readers it may be useful to mention some of the outstanding books by Gelfand and his collaborators, and by others belonging to the old Soviet School in mathematics as well as in physics. Here is a partial list:

1. I M Gelfand, R A Minlos and Z Ya Shapiro, *Representations of the Rotation and Lorentz Groups and their Applications*.
2. M A Naimark, *Linear Representations of the Lorentz Group*.
3. I M Gelfand and G E Shilov, *Generalized Functions,* Volumes I – V.
4. L S Pontryagin, *Topological Groups*.
5. L D Landau and E M Lifshitz, *Course of Theoretical Physics*, Volumes I – VII.
6. B Gnedenko, *Theory of Probability*.

Each of these, and many others, will attest to the high pedagogical skills and clarity achieved by the authors of that era.

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