
How Birds Fly Together: The Dynamics of Flocking

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We explain the physical motivation for studying the collective coherent motion of large numbers of self-propelled biological organisms. Starting from a well-known discrete model, we discuss the fundamental microscopic mechanisms leading to collective behavior in bird flocks, and how appropriate behavioral rules for the individuals can determine specific features of the aggregation at the group level. We further discuss the theoretical, experimental, and computational effort undertaken to explore this phenomenon together with possible directions for future research in this area.

Introduction

The intricacies, beauty and grace of collective coherent motion of self-propelled biological organisms namely, flocks of birds, schools of fish, swarms of insects, slime molds, herds of wildebeests, has fascinated humans from ancient times. The sheer elegance of these extraordinary self-organizing events is captured in many paintings, animations and graphics. Its perfect harmony of form and function not only inspired poetry, art and literature, but it has challenged scientists, evolutionary biologists and mathematicians for a long time.

This kind of behavior often referred to as *flocking*, exists in nature at almost every length scale of observation: from human crowds, mammalian herds, bird flocks, fish schools to unicellular organisms such as amoebae and bacteria, individual cells, and even at a microscopic level in the dynamics of actin and tubulin filaments and molecular motors. Despite the huge differences in the scales of aggregations, the similarities in the patterns that such groups produce have suggested that general principles may underlie collective motion. Anyone who has admired the mystical style of flight of a group of birds or the coordinated maneuvers of a school of swimming fish, can appreciate the motivation for the



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Keywords

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study of flocking: How do individual self-driven units maintain the order of a flock or how do they generate large-scale, spatiotemporally complex dynamical patterns? What are the rules which govern this dynamics and how do the principles of physics constrain the behavior of each such unit? These are some of the challenging questions concerning the mechanism of flocking phenomena that has triggered the interest of physicists, mathematicians, and computer scientists. Before discussing the dynamics in detail, let us go through the various experiments on different kinds of flocks and their predictions.

Various Experiments

The dynamics of flocking behavior among living things has long been a mystery. Fascinated by the intriguing collective behavior of bird flocks, fish schools, mammal herds and insect swarms, biologists for many years [1–3] have investigated the underlying biological processes. Biological objects have the tendency to move as other objects do in their neighborhood which raises a question – why should individuals align with their neighbors? A large flock does not seem to have a global leader [4]; instead, the impressive collective flocking phenomenon is caused by individual members of a flock. Individuals can willingly take on leading and following roles and self-organize, given the right incentives and environment. Experiments on plagues of mass-migrating insects such as juvenile desert locusts (*Schistocerca gregaria*) provide evidence that cannibalistic interactions among individuals, and the threat of attack by those approaching from behind, is a principal factor in the onset of collective movement among locusts [5]. Field studies on swarming mormon crickets (*Anabrus simplex*) in the United States [6] indicate that the motion of an individual cricket is driven by the need to find nutrients such as protein and salt. Using computer-aided imaging techniques, Michelle Ballerini and his co-workers on the Starflag project [7] have imaged large flocks of starlings to study the dynamics of individual birds and how this dynamics is influenced by the spatial distribution and behavior of neighboring birds. In this experiment, it has been observed that birds appear to adjust to

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the motion of the flock by measuring the behavior of neighbors. Thus, at each instant, each bird appears to be comparing its position and velocity to those of the birds closest to it, making the adjustments required to maintain the coherence of the flock. An individual must be attracted to its neighbors over a certain threshold distance in order to remain part of the flock. At short distances, however, it is repelled, in order to avoid collisions. For a given distance, these forces of attraction and repulsion will compensate for each other and the net force experienced by the individual will be null: this value becomes ‘*the average nearest neighbor*’ distance of the model. The finding that individuals attempt to maintain a minimum distance between themselves and others at all times is highly intuitive and corresponds to a frequently observed behavior of animals in nature [8]. This is, in fact, one of the main assumptions of numerical models.

It has also been found that the density and the average nearest neighbor distance within flocks do not depend on the size of the group, contrary to the pattern observed in fish schools by Partridge and coworkers [9], and in computer simulations by Kunz and Hemelrijk [10]. In fact, the presence of predators causes a flock to exhibit fast expansions and contractions, indicating that strong perturbations have a direct effect on density. Moreover, some numerical models of fish schools reveal that density is higher at the front of the group [10] as also observed in natural shoals [11]. The evolutionary motivation for grouping has been associated traditionally with its anti-predator function – grouping of similar individuals decreases the probability of being caught by predators [12]. In other words, moving together reduces the ability of a predator to focus on a specific individual and capture it. The response to predators is likely to optimize this ‘confusion effect’¹, as can be seen in the very effective escape manoeuvres displayed by starling flocks under attack. The high density borders that are often observed may represent a feature that enhances such anti-predatory tactics, creating a ‘wall’ effect to increase the predator’s confusion. Given that, in general, animal aggregations are rather stable, it implies that group structure and dynamics

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¹ An inability of predators to visually lock onto one target among many.



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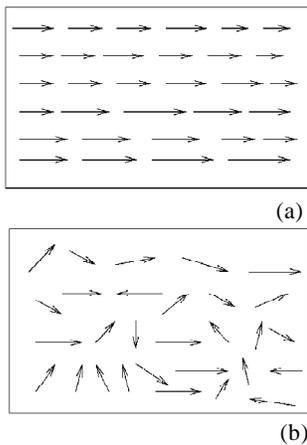
must allow for a systematic redistribution of risk among its members. Individuals must be able to move through the flock and exchange positions, while at the same time maintain integrity of the group. Reproducing such features will be an ongoing challenge for models and theories.

Groups of animals often have to make collective decisions, such as to move together to a specific resource, for example a nest site or food source. Making movement decisions often depends on social interactions among group members. However, in many cases, few individuals have pertinent information, such as knowledge about the location of a food source or of a migration route. The means by which such decisions can be made is very poorly understood, especially in the case of large groups and when individuals are not capable of knowing (i) whether they are in a majority or minority, or (ii) how the quality of their information compares with that of others, or (iii) even whether there are any other individuals in the population with information. Using a simple model, I D Couzin and coworkers [13] were able to provide new insights into the mechanisms of effective leadership and decision-making in biological systems.

Collective Behavior in Other Physical Systems

Spatial structures arise not only in biological systems but also in many other physical systems at thermodynamic equilibrium. When water is cooled down to 0 °C, it turns into ice – a crystalline structure. Below the Curie temperature, the random spins of a paramagnetic substance suddenly align themselves pointing in the same direction to form a ferromagnetic material (*Figure 1*). The originally isotropic nonmagnetic substance turns into a magnetic material associated with a particular direction, namely the direction of the spontaneous magnetization. These transitions to a structured state represent a loss of symmetry, namely continuous spin rotational symmetry. Crystalline solids differ from fluids (liquid and gases) by breaking both translational and orientational symmetry. Other examples are liquid helium, superconductors, etc. Below a certain temperature, liquid helium pos-

Figure 1. Spin configuration in a (a) ferromagnetic and (b) paramagnetic material.



sesses an extraordinary property – a liquid without viscosity, and becomes a superfluid. An ordinary conductor when sufficiently cooled becomes a superconductor where electrons of the metal get into an extraordinary cooperative phase resulting in a near total loss of resistance.

Diverse instances of collective behavior are found in many different fields of science.

Early ideas about the emergence of large-scale complexity from microscopic local rules have been inspired by research in biology. However, recently these ideas are spreading in many different fields contributing to the understanding of diverse processes and phenomena in physics, economy, geology, chemistry, sociology, etc. Interestingly, diverse instances of collective behavior are found in many different fields of science, from the spontaneous ordering of magnetic moments in physics, the coordination of an ensemble of artificial agents with distributed intelligence in robotics, the emergence of herding behavior in financial markets in economics, to the synchronized clapping in a concert hall or the Mexican waves exhibited through the coordinated motion of spectators in a stadium. In all these examples, collective behavior emerges as the result of the local interactions between the individual units, without the need for centralized coordination. The tendency of each agent to imitate its neighbors, can, by itself, produce a global collective state. Whenever this happens, we are in the presence of *self-organized collective behavior* which implies order, regularity, coherence, and coordination within the system itself. Non-equilibrium system takes in energy and matter from the environment, builds up low-entropy structure within the system and through dissipation, releases sufficient entropy to the environment. Self-organization is a manifestation of nonlinearity usually arising from feedback mechanisms.

Self-Organization in Flocks

In the last few decades, many scientists have turned their attention to how birds flock rather than why. While biologists provide strong support to the idea that the limited range of senses is the cause of flocking behavior, further support for this came from computer scientists, mathematicians and both experimental and

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theoretical physicists. Flocks display a spatial coherent pattern on macroscopic scales in length and time, and are capable of very fast, highly-synchronized manoeuvres, either spontaneously, or as a response to predator attacks. It is a self-organized biological system far away from thermodynamic equilibrium. More precisely, in the language of physics, flocking is a familiar example of nonlinear dynamical system with many degrees of freedom far from equilibrium that exhibits self-organizing property at certain critical values of external parameters such as noise and density of individuals. The study of self-organizing systems is a fascinating field of multidisciplinary interest. Extensive theoretical effort has been devoted to realize these non-equilibrium systems. It suggests a new perspective to our understanding of collective motion [14].

Universality is a well-defined concept in physics: the same model and theory can be used to describe quantitatively very different physical systems, provided that they all share the same fundamental symmetries.

Although physicists are mostly concerned with the mechanisms at the origin of collective motion, the ubiquity of the phenomenon at all scales, from intracellular molecular cooperative motion to the displacement in groups of large animals, raises the question of the existence of some universal features possibly shared among many different situations. Universality is a well-defined concept in physics: the same model and theory can be used to describe quantitatively very different physical systems, provided that they all share the same fundamental symmetries. The situation is more complicated in biology because the individuals that form a group are much more complex than particles or spins. For example, J Krause and G D Ruxton, in their book *Living in groups* (Oxford University Press, 2002), mention that despite the fact that fish schools and bird flocks behave similarly, certain collective patterns present in one case are not there in the other.

Mathematical Models

In recent years, the explosion of interest in nonequilibrium statistical problems has essentially ridden on the back of advances in computer development. Models play a crucial role in this respect. Models of real instances of collective behaviour must specify the



minimal conditions necessary to reproduce the empirical observations, so that we can distinguish between general phenomena and those specific to the system. Indeed, it was the modeling exercises that revealed the general principles of how collective behavior can emerge from self-organization. Modeling provides an approach to exploring the ways in which the dynamics of individuals lead to groups, the interactions of groups, and the effects of environmental variability and transport.

One of the main methods used to study the collective behavior is to construct discrete models and study them using computer simulations. In the last few decades lots of models have been proposed for the collective animal behavior that are based upon an abstraction of aggregation tendencies evident in biological systems [4,9]. Some of these were developed for fish schools, some for bird flocks, and some with a non-specific biological target. In all these cases, the models agree on some general behavioral rules. These are based on three distinct features i.e., *alignment of velocities*, *attraction*, and *short-range repulsion*. In all the cases, the models produce cohesive aggregations that look qualitatively similar to the real ones. Here we will concentrate on some specific powerful models pertaining to bird flocks.

Reynolds Model on Boids

It was C W Reynolds [15], who for the first time, created realistic-looking animations of flocks of birds. It was based on three-dimensional computational geometry usually used in computer animation or computer-aided design. He used the term *boid* and bird interchangeably for the particles in his simulations. Each boid follows three simple rules: (1) *collision avoidance*, i.e., maintain a minimum distance from other boids thereby avoid collisions with nearby flockmates; (2) *velocity matching*, i.e., matches its own velocity with the boids in its neighborhood; and (3) *attraction toward boids* within a short radius. These rules follow the constraint that each boid has a maximum velocity and



Figure 2. Large flock of migrating Canada geese – a sure sign of spring and autumn.

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acceleration. Remarkably, these three simple rules, when properly applied, produce realistic-looking flocking behavior. This program clearly indicates that complex behavior, like flocking, need not have complex rules. Indeed, increasingly complex behaviors are created by very simple rules that govern the relationships of individuals to each other. Perhaps this is the most important observation of all.

Vicsek Model

Motivated by Reynolds model, T Vicsek and his collaborators in 1996 [16] proposed a very simple simulation model analogous to a model of a ferromagnet and demonstrated that it exhibits flocking. In this model each particle tends to move in the average direction of motion of its neighbors while being simultaneously subjected to noise. The simulations were carried out in a two-dimensional box of side length L with periodic boundary condition. At $t = 0$, N particles were distributed in random positions (x_i, y_i) . The velocities for each particle were constrained to a constant value v , and initial direction θ_i for each particle were randomized. At each time step, position and direction followed

$$x_i(t+1) = x_i(t) + v \cos \theta_i(t), \quad (1)$$

$$y_i(t+1) = y_i(t) + v \sin \theta_i(t), \quad (2)$$

$$\theta_i(t+1) = \langle \theta_i(t) \rangle_r + \Delta\theta, \quad (3)$$

where $\langle \theta_i(t) \rangle_r$ is the average direction of velocities of particles within a distance r of the i -th particle. Particles further away have no influence on the given particle. The random number $\Delta\theta$, uniformly chosen in the interval $[-\eta/2, +\eta/2]$, is responsible for noise or fluctuation in the velocity direction.

The flock evolves through an iteration of these rules. Equation (3) shows that each flock tends to self organize in the same average direction of movement (first term) while this behavior is randomly perturbed (second term). When simulated with different parameters for density and noise, they found four different behaviors [16]:



- (i) High noise, low density: particles moved independently.
- (ii) Low noise, low density: particles formed groups moving coherently in random directions.
- (iii) High noise, high density: particles moved randomly with some correlation.
- (iv) Low noise, high density: all particles moved in the same spontaneously-selected direction.

These simulations exhibit characteristic collective behaviors, similar to those of natural groups, when certain parameters are changed. Noise plays an important role in group organization. For a finite density of particles in a finite box, perfect alignment is reached easily in the absence of noise or at very low noise level: the onset of collective motion occurs at a finite noise level. In other words, there exists, in the asymptotic limit, an *ordered phase* where the macroscopic velocity of the total population is, on average, finite. On the other hand, strong noise particles² exhibit essentially non-interacting random behavior with zero macroscopic velocity. Thus, there is a transition between an *ordered phase* in which the mean velocity of the flock $\langle \vec{v} \rangle \neq 0$ and a *disordered phase* with $\langle \vec{v} \rangle = 0$ as the strength of the noise is increased. Furthermore, this model also predicts that as the density of particles in the group increases, a rapid transition occurs from disordered movement of individuals within the group to highly-aligned collective motion.

Roughly speaking, Vicsek's model is similar to the model of a ferromagnet. The velocity vector of the individual birds is analogous to the magnetic spin in an iron atom in a ferromagnet. In a flock, all the birds, on average, are moving in the same direction giving rise to a spontaneous long-range order. This is in analogy with the magnetic spins which, on average, point in the same direction. In a ferromagnet, the spins only interact with their nearest neighbors through short-range interactions which is akin to the nearest-neighbor interactions of the birds in a flock. The creatures in the interior of a flock decide their alignment primarily by looking at their immediate neighbors. The level of random perturbation, $\Delta\theta$, is in analogy with temperature. Just as thermal

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² Particles under the influence of very high noise.



³ A non-deterministic element which introduces random effect into the system.

fluctuations introduce randomness at non-zero temperature in an equilibrium ferromagnet, the randomness of the errors $\Delta\theta$ introduces a stochastic element³ to the flocking problem. The transition to a structured state (from paramagnetic to ferromagnetic) represents a loss of symmetry, namely continuous spin rotational symmetry. The same kind of picture is visualized in a spontaneously moving flock in which, due to the development of a nonzero mean center-of-mass velocity $\langle \vec{v} \rangle \neq 0$ for the flock, there is a spontaneous breaking of a continuous symmetry, namely rotational symmetry.

Despite these similarities, there is an important difference between these two interesting cooperative structures. A ferromagnet is an equilibrium thermodynamic structure, whereas a flock is a non-equilibrium dynamical system; structure of equilibrium thermodynamics are static, whereas flock is a dynamic, ever-changing structure. Equilibrium thermodynamics allows for relatively simple structures to emerge when systems are sufficiently cooled (e.g., liquid to solid, liquid helium to superfluid, conductor to super conductor, non-magnetic to magnetic phase transitions). On the other hand, non-equilibrium dynamic structure arises as a result of their inherent nonlinearity – nonlinearity breeds the evolution of rich behavior of systems which are far from equilibrium.

Vicsek's model is a discrete computer model called *cellular automata* [17]. Cellular automata have been extensively used to simulate situations which are otherwise difficult to analyze, such as diffusion, surface growth, aggregation, clustering, pattern formation in plants and animals. Although Vicsek's model is the simplest physical model of flocking, its predictions are as transparent as it is profound: its simple and local rules generate astonishing global patterns and self-organization. These local rules, when applied recursively, produce long-range order. This harmonious coherence of different parts of a biological structure needs no master plan – it arises from recursive application of suitable local rules.

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This model, though simple to simulate, is quite difficult to treat analytically. How do local interactions produce global structures and coherent patterns of characteristic sizes? What explains the long-range order that a flock exhibits in space and time? Vicsek's model seems to offer a natural framework for dealing with these problems, but a detailed model of the process, even for a simple organism, is still a far cry.

Toner and Tu's Model

Due to its simplicity and efficiency, the Vicsek model has been intensively investigated in recent years. Following Vicsek's rule, Toner and Tu [18] have considered the following common features of flocking for a more generalized mathematically sound model:

- (a) Each bird moves over time through a space of dimension d ($= 2, 3, \dots$), following its nearest neighbors at all time of its motion.
- (b) Each flock communicates only with its immediate neighbors by a purely short-range interaction: it is the same as that of any atom with its neighbors.
- (c) Each flock while communicating with its neighbors, makes errors at all times, i.e., the 'following' is not perfect. The errors are modeled as a stochastic noise which is assumed to have only short-ranged spatio-temporal correlations.
- (d) Each organism has no intrinsically preferred direction in which it wishes to move. Rather, it is equally happy to move in any direction picked by its neighbors. Thus it has a complete rotational symmetry. As the flock move together with the same mean velocity $\langle \vec{v} \rangle \neq 0$, this rotational symmetry breaks down. Thus flocking exhibits *spontaneous symmetry breaking*.

Implementing these rules, a generic continuum hydrodynamic theory has been developed by Toner and Tu [18] to describe the behavior of the flock at large scales. The stochastic differential equation of this theory for the velocity and the density fields has been constructed phenomenologically on the basis of general

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symmetry arguments by drawing on analogies with magnetic systems and liquid crystals. This model typically describes the dynamics at large length scales by neglecting the short-length scale details and focuses only on the asymptotic coarse-grained properties. Here, for the sake of compactness, we leave out the details of this model as well as related recent and important work, including numerical calculations. The reader is referred to the excellent article by J Toner, Y Tu and S Ramaswamy for details about various phases of flocks [19]. This work has brought about several important results, including the possibility of long-range order in 2D, the prediction and observation of giant number fluctuations⁴ in the ordered state, and the microscopic origin of the large-scale collective physics. In fact, both the discrete and continuum approaches discussed above, have exposed the dramatically different nature of order-disorder transitions and of the fluctuations in the various phases exhibited by self-propelled systems when compared to their equilibrium counterparts.

⁴ A striking nonlinear effect referring to the large fluctuations in the density of flocks (or flocks). It is an indication of macroscopic phase separations.

Future Directions

Toner and Tu's model is remarkably interesting because of possible applications in a wide range of biological systems involving clustering and migration. This hydrodynamic model of flock produces universal behavior and exhibits relevant characteristics of actual biological systems. It can easily be generalized to account for other more realistic and deeply interesting situations, such as creation and annihilation of flockmates. For example, migration of huge herds of arctic deer over thousands of miles and for many months allow flocks to be born or to die in the dynamics, or in bacterial colonies reproduction and death takes place very rapidly. This model, with some modification, is also capable of reproducing the main observed features of the motion (collective rotation and flocking) of bacteria. In Toner and Tu's model, all the birds are assumed to have the same flying speed. Nevertheless, in a real flock, some birds will be fitter, and hence faster than others. To realize this situation, a Vicsek-type simulation having different species of birds, flying together with different mean speeds would be a possible solution.



Sometimes, on a hot day, we come across a swarm of insects, such as gnats around our head or moths around a light at night wambling around and organizing into a coordinated vortex-like motion. Some fish and birds (namely, hawk, vulture, etc.) also exhibit vortex motion while feeding or looking for food. Such a self-organized vortex state has also been observed in slime molds (*Dictyostelium* cells) when their motion is restricted to a plane. It was found that these cells aggregate and form a two-dimensional round structure. This vortex state, in which the cells are rotating around a common center, arises as a result of self-organization due to local cell–cell interactions, analogous to the alignment interaction in the flocking models.

Conclusion

The problem of animal aggregation is central in both ecological and evolutionary theory. Why do animals group? What proximate cues do they use? What are the ecological consequences? In this article, we investigated the main features of the flock as a whole – density, movement, shape, and structure – and discuss these as emergent attributes of the collective phenomenon.

Although the models discussed in this article visualized the behavior of flocks i.e., in closed or periodic boundary conditions, real flocks are usually surrounded by open space. How do they stay together under these circumstances? What shape does the flock take? How does this shape fluctuate, and is it stable? These are some challenging questions for further work in this fascinating topic. We hope that this article will encourage readers who are interested in biology, physics, and computer science and motivates them for further study in future on this rich, enthralling and still largely unexplored subject.

Acknowledgments

This article is based on various internationally acclaimed literature, a few of them are indicated in the suggested reading.

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