Dawn of Science

7. The Indo–Arabic Numerals

T Padmanabhan

T Padmanabhan works at IUCAA, Pune and is interested in all areas of theoretical physics, especially those which have something to do with gravity.

Previous parts:
Resonance, Vol.15: p.498; p.590; p.684; p.774. p.870, p.1009.

It was traders who tilted the scale in favour of the new number system in the Arab world.

In AD 773, at the height of Arab splendour, there appeared at the court of the Caliph, Al-Mansur, in Baghdad a man from distant India. This traveller had brought with him several volumes of writings from India. Al-Mansur promptly got them translated into Arabic and later several Arabic scholars assimilated their contents. One among them was Abu Jafar Mohammed ibn Musa Al-Khowarizmi (which, freely translated, means 'Mohammed, the father of Jafar and the son of Musa, the Khowarizmian', the last word originating from the Persian province of Khoresem). This man, who lived between AD 780 and 850, was one of the greatest mathematicians of the Arab world and he quickly realised the importance of the number system used in the Indian writings. In fact, he wrote a small book explaining the use of these numerals around AD 820.

The original of this book is lost but there is evidence to suggest that it reached Spain in about AD 1100; there it was translated into Latin by an Englishman, Robert of Chester. And this translation is probably the earliest known introduction of Indian numerals to the West. This manuscript begins with the words, *Dixit Algoritmi: laudes deo rectori nostro atque defensori dicamus dignas* ('Algoritmi has spoken; praise be to God, our Lord and our Defender', the Arab name Al-Khowarirmi having been transliterated into Algoritmi in Latin). In later years, careless readers of the book started attributing the calculational procedures described in the book to Algoritmi; that is how we got the term 'algorithm' for any computational procedure.

Keywords
Indo–Arabic numberls, Al–Khowarizmi, Brahmagupta.

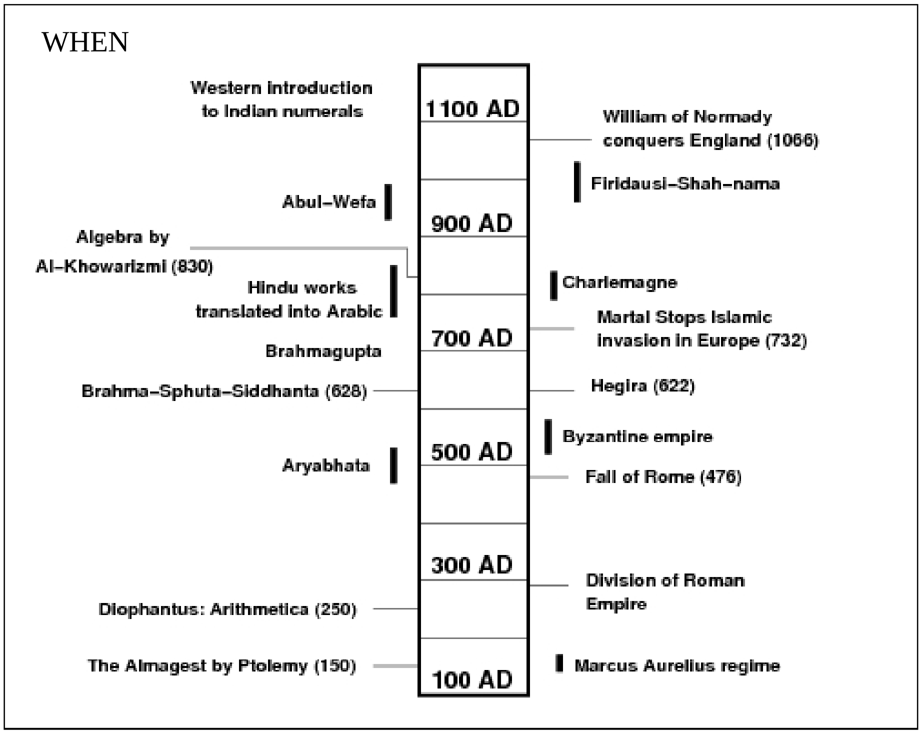


Figure 1.

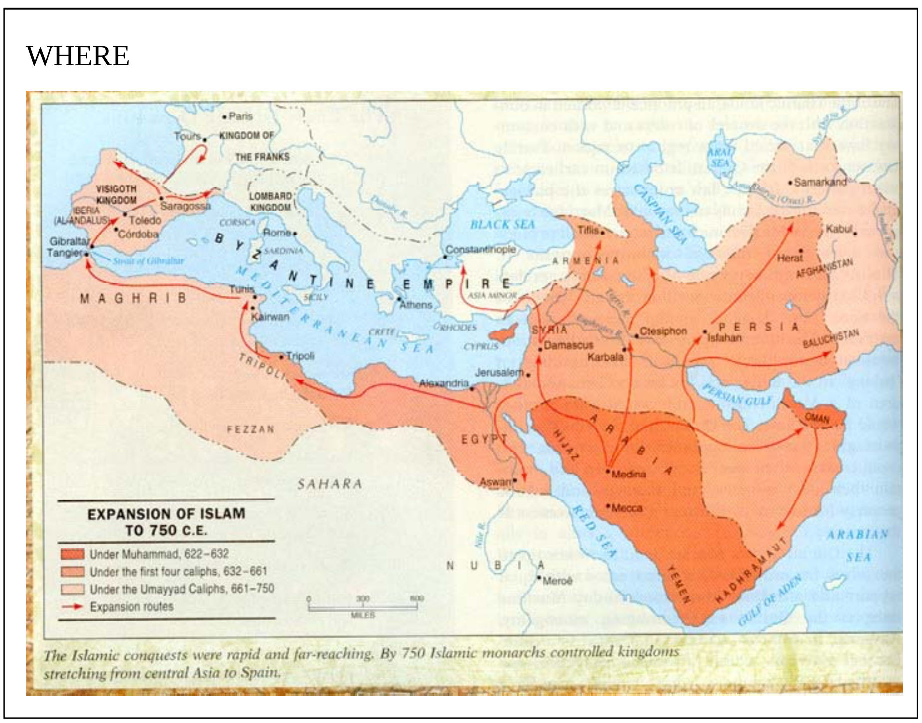


Figure 2.

Al-Khowsrizmi discusses in a systematic manner – among other things – the solution of algebraic equations up to the quadratic order.

The use of Indian numerals was picked up by many scholars and was taught in major cities. In particular, the use of zero became well established in these discussions. Al-Khowarizmi himself says explicitly: 'When nothing remains...put down a small circle so that the place be not empty... and the number of places is not diminished and one number is mistaken for the other.' However, the new system was not accepted by the average man easily; ultimately, what tilted the balance in its favour were not scholarly expositions but commercial considerations! For by the end of the first millennium, Italy had grown to be a major mercantile power around the Mediterranean. Italian ships were used for crusades, Italian bankers lent the money, and Venice, Genoa and Pisa rose as cities of prominence. The traders and merchants very quickly realised the advantages of the Indo–Arabic number system. Blessed by big business, the system stayed. For example, the *Margarita Philosophica* (the philosophic pearl), a beautifully illustrated encyclopaedia which was widely used as a university textbook in the early sixteenth century, authored by the monk Gregor Reisch (c.1467–1525) discusses arithmetic using Indo–Arabic numerals compared to the use of a counting board (*Figures* 3 and 4).

Al-Khowarizimi also wrote another influential work called *Al-jabr-wa'l Muquabala,* (which could be translated as 'The science

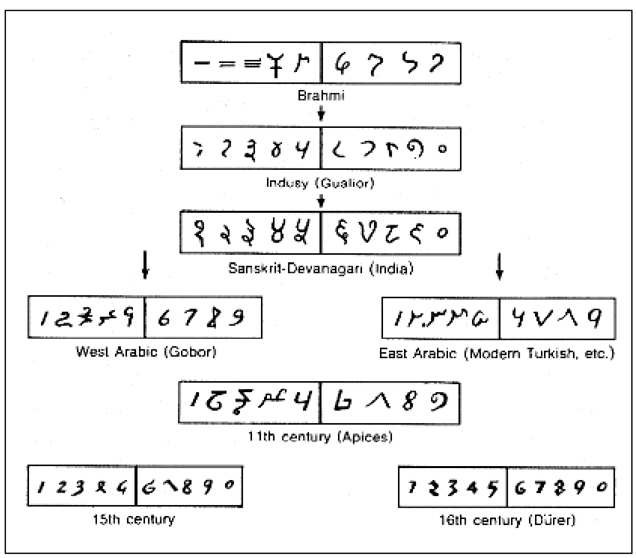


Figure 3. The genealogy of modern numerals.
Courtesy: V F Turchin, *The Phenomenon of Science*.

of transposition and cancellation'). Here, he gives a detailed exposition of the fundamentals of the subject, which has come to be called 'alegbra'. Al-Khowarizmi discusses in a systematic manner – among other things – the solution of algebraic equations up to the quadratic order. The clarity of discussion in this book has made later workers call Al-Khowarizmi the 'father of algebra'.

Figure 4. The title page of Gregor Reisch's Margarita Philosophica (1503). The seven 'liberal arts' are around the three-headed figure in the centre with arithmetica, with a counting board, seated in the middle.

Courtesy: Freiburg: Johann. Schott, 1503 [Rare Books Collection B765.R3 M2].

The synthesising power of Arabic civilisation also influenced trigonometry. This subject, well developed in both India and Greece due to the stimulus given by astronomical observations, attained a unified look in the hands of the Arabs. In Greece, it was developed by Aristarchus (310–230 BC), Hipparchus (around 140 BC) and most notably by Cladius Ptolemy (AD 85–165). In particular, Ptolemy constructed what he called a "table of chords" which is equivalent to the modern trigonometric table for the sine of an angle. He did this by using a very elegant geometrical procedure for all angles at half-degree intervals. This work, of course, was developed further by the Arabs. Abul-Wefa (AD 940–998), for example, produced the tables for sines and tangents at quarter-degree intervals; this table was used extensively by later scholars. Similar tables were constructed in the East by Aryabhata.

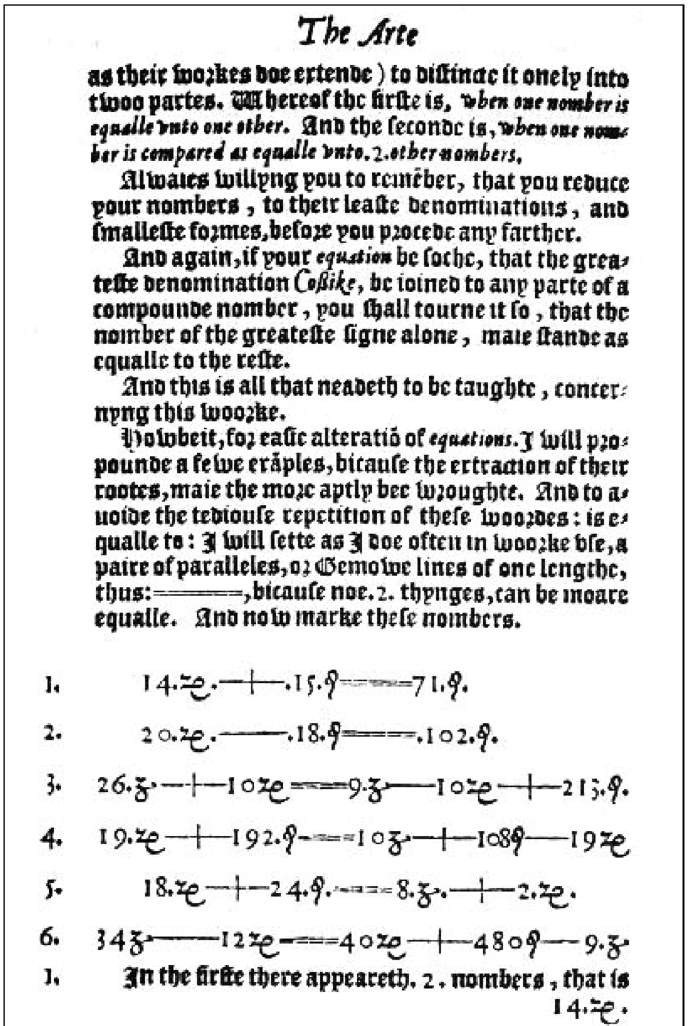
Incidentally, there is an interesting story behind the term 'sine'. In trigonometry, one associates with each angle certain ratios usually called sine, tangent and secant (three other ratios, cosine, cotangent and cosecant, arise as complements of these three ratios). Of these three, the terms 'tangent' and 'secant' have clear geometrical meaning and correspond to the standard definition in Euclidean geometry (see *Figure* 5). How did the word 'sine' originate? Surprisingly enough, it came from the Sanskrit term *jya-ardha* ('half of chord')! This is how it happened.

The Indian mathematician, Aryabhata (AD 475–550), used the term *jya-ardha* to denote what we now call sine. This term, abbreviated as *jya*, was converted phonetically as *jiba* by the Arabs. Following the standard Arabic practice of dropping the

Box 1. Algebraic Symbolism

The earliest discussions in algebra, both in the East and the West, were rhetorical. Questions and answers were given in the form of dialogues or discussions and no symbols were used. The first two mathematicians to realise the powers of symbolic manipulations were Diophantus in Greece (AD 250 ?) and Brahmagupta (AD 700 ?) in India.

Diophantus had symbols to denote unknown quantities, various powers of an unknown quantity, reciprocals and equality. He also used Greek letters to denote numerals. The system followed by Brahmagupta was more elaborate. Addition was indicated by just placing the terms next to each other, subtraction by placing a dot over the term to be deducted, multiplication by writing *bha* (the first letter of *bhavitha*, the product), and square root by the prefix *ka* (from the word *karana).* The first unknown in the problem is denoted by *ya* and additional unknowns were indicated by the initial syllables of various colours.



The Arte

as their workes doe extende) to distincte it onely into twoo partes. Whereof the firste is, *when one nomber is equalle unto one other.* And the seconde is, *when one nomber is compared as equalle unto .2. other nombers.*

Alwaies willyng you to remẽber, that you reduce your nombers, to their leaste denominations, and smalleste formes, before you procede any farther.

And again, if your *equation* be soche, that the greateste denomination *Cossike*, be ioined to any parte of a compounde nomber, you shall tourne it so, that the nomber of the greateste signe alone, maie stande as equalle to the reste.

And this is all that neadeth to be taughte, concernyng this woorke.

Howbeit, for easie alteratiõ of *equations*. I will propounde a fewe exãples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: =====, bicause noe .2. thynges, can be moare equalle. And now marke these nombers.

1. 14.ze.—+—.15.ϑ=====71.ϑ.
2. 20.ze.——.18.ϑ=====.102.ϑ.
3. 26.ʒ.—+—10ze=====9.ʒ——10ze—+—213.ϑ.
4. 19.ze—+—192.ϑ=====10ʒ—+—108ϑ——19ze
5. 18.ze—+—24.ϑ.=====8.ʒ.—+—2.ze.
6. 34ʒ——12ze=====40ze—+—480ϑ——9.ʒ.

1. In the firste there appeareth. 2. nombers, that is 14.ze.

Figure A. A page from Robert Recorde's The Whetstone of Witte (1557), showing the much longer equality sign.
Courtesty: http://www-groups. dcs.st-and.ac.uk/~history/Bookpages/Recorde4.jpeg

The various mathematical symbols we use today came into existence over the centuries. The 'equal' sign (=) was due to Robert Recorde (*Figure* A). The 'plus' and 'minus' signs first appeared in print in an arithmetic text by John Widman published in 1489. The signs for multiplication and proportion were due to William Oughtred (1574–1660). It was Déscartés who introduced the present compact notation with indices a, a^2, a^3, etc. Finally, π as a ratio between the circumference and the diameter of a circle was first used by the English writer, William Jones, in 1706.

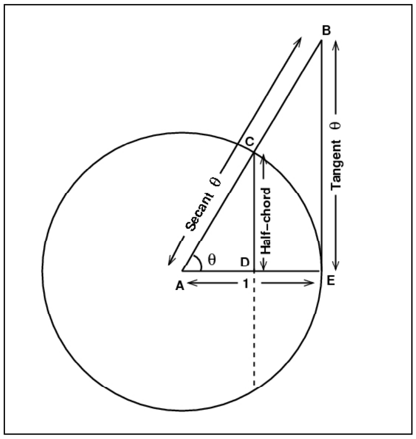


Figure 5. Geometrical meaning of trignometric ratios. In a circle of radius 1 unit, draw a tangent BE and secant BA from the external point B. The lengths of AB and BE give, respectively, the value of 'tangent θ' (or tan θ) and 'secant θ' (or sec θ). Similarly, the length of half-chord ('jya-ardha', in Sanskrit) CD is equal to sine θ .

vowels in the written version. this became just *jb*. Of course, the term *jiba* has no meaning in Arabic except in this technical context. Later writers, coming across *jb* as a shortened version for *jiba* (which appeared meaningless to them), decided to 'correct' it to *jaib* which is an Arabic word meaning 'cove' or 'bay'. Still later, Gherardo of Cremona, while translating technical terms from Arabic to Latin, literally translated *jaib* to the Latin equivalent *sinus*. This, in English, became 'sine'. That is how a cavity in our upper nose and a trigonometric ratio ended up having the same roots.

Suggested Reading

[1] V F Turchin, *The Phenomenon of Science*, Columbia University Press, 1977. Also available at: http://pespmc1.vub.ac.be/pos/default.html

[2] G Ifrah, *The Universal History of Numbers*, Penguin, 2005.

[3] Howard Eves, *Great Moments in Mathematics*, Vol. I, Mathematical Association of America, 1983.

Address for Correspondence
T Padmanabhan
IUCAA, Post Bag 4
Pune University Campus
Ganeshkhind
Pune 411 007 India.
Email:
paddy@iucaa.ernet.in
nabhan@iucaa.ernet.in