Dawn of Science

4. Archimedes

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His work ranged from optics, mechanics and hydrodynamics to pure mathematics.

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Previous parts:
Resonance, Vol.15: p.498; p.590; 684.

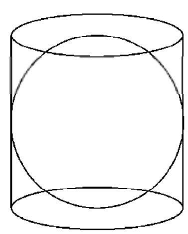


Figure 1. Tombstone of Archimedes.
Courtesy:
http://www.math.sunysb.edu/~tony/whatsnew/column/archimedes-0100/archimedes3.html

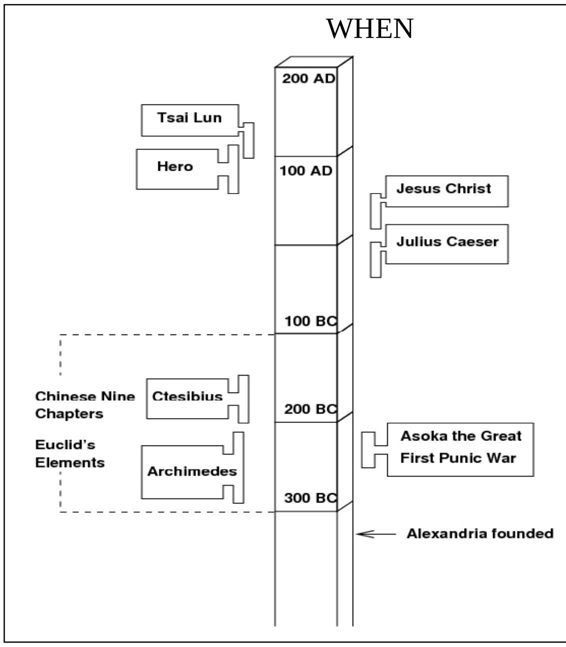
Keywords
Archimedes, volume of sphere, lever, hero of Alexandria, value of pi.

In 1965, while excavating for laying the foundation of a new hotel in Syracuse (Sicily), Italy, the steam-shovel unearthed a tombstone bearing the picture shown in *Figure* 1. This was the tombstone of Archimedes, one of the greatest scientists who ever lived and of whose equal there have been only two since then, Newton and Einstein.

It is rather ironic that such a scientist did not come from the centre of intellectual activity at the time – Alexandria (see 'Dawn of Science 3', *Resonance*, August 2010, p.684). He was born in Syracuse, about 287 BC. His father was an astronomer of considerable talent and repute. However, Archimedes did spend some time in Alexandria training under Euclid's students. He returned to his native town, possibly because of his close friendship with the King of Syracuse, Hieron II.

Archimedes thrived in Syracuse. No other scientist of ancient times, not even Thales, had so many tales and legends told about him; the most famous, of course, is the one about 'Eureka' and the principle of buoyancy. Archimedes was responsible for several inventions and discoveries in the branches of mechanics, hydrodynamics, optics and in pure mathematics, the details of which are spelt out in nine Greek treatises which are available to us.

He worked out the principle of lever in clear mathematical terms (even though several others before him had conjectured about it). "A small weight at larger distance from a fulcrum can balance a large weight nearer to the fulcrum." This led to the science of statics and to the notion of centre of gravity of bodies. Two of the volumes by Archimedes – *On the Equilibrium of Planes* and *On*



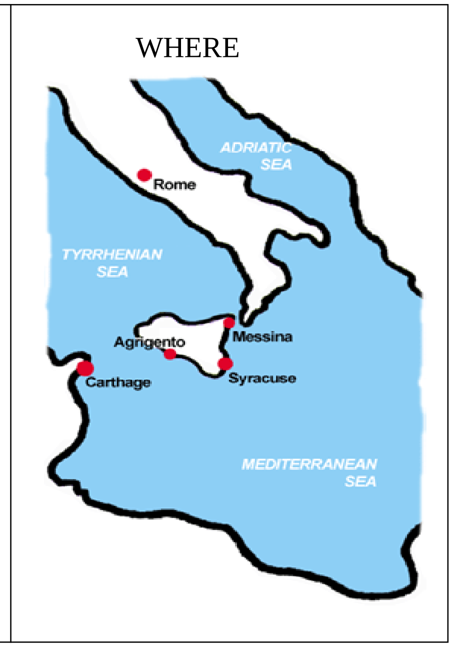


Figure 2. (left)

Figure 3. (right).

Courtesy: www.boglewood.com/sicily/vassal.html

Floating Bodies – elaborate on the implications of these concepts. In these volumes, Archimedes spends considerable time establishing the position of equilibrium of floating bodies of various shapes; the results are of considerable importance in naval architecture.

Archimedes used these principles in several practical devices. He is supposed to have perfected a hollow helical cylinder which, when rotated, served as a water pump (*Figure* 4). He also devised a heavenly globe and a model planetarium depicting the motion of the planets. His engineering tradition influenced several people (see *Box* 1).

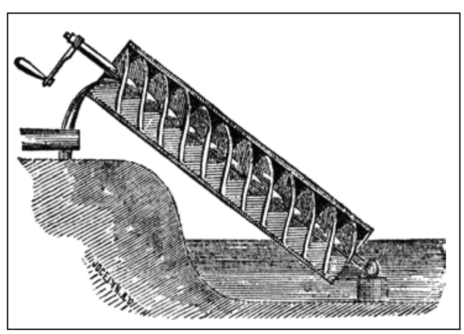


Figure 4. Archimedes' water pump.

Courtesy: http://picsdigger.com/domain/decodingtheheavens.com/

He was, however, a purist and did not really care too much for these applications. What he was most pleased with were the results he could obtain in the branch of pure mathematics – in the determination of areas and volumes of geometrical shapes. He came up with ingenious arguments – described in the treatises – *On Sphere and Cylinder* and *Method* – to show that the volume of a sphere is $4\pi R^3/3$ and its surface area

Box 1. Other Mechanical Inventions

The engineering tradition of Archimedes influenced several contemporaries and future generations. One among them who lived in the second century BC was Ctesibius who is responsible for devising a sensible water clock. He improved upon the more ancient Egyptian 'clepsydra', in which water dripping into a container at a steady rate made a pointer move, indicating time. Ctesibius made the whole device practical, compact and accurate. In fact, his clock was as accurate as the timepieces of the Middle Ages run by falling weights. It was only after the invention of the pendulum that the accuracy of measuring time was improved,

After about 120 BC Ptolemic Egypt fell into decadence and by 30 BC it was a Roman Province. Greek science was virtually over but for an occasional genius like Hero of Alexandria. His most famous invention was a hollow sphere with two tubes attached to it in which water could be boiled to make steam. The steam escaping through the tubes made the whole device spin. This was the first steam engine, though unfortunately it was only used in toys and by priests to deceive gullible believers. Hero also wrote extensively on mechanics elaborating on the principle of the lever and several simple machines, involving inclined planes, pulleys and levers.

Almost at the same time as Hero was devising the levers, an unknown Chinese named Tsai Lun, made a breakthrough in China. Chinese historians credit him for inventing the product we now call 'paper' from tree bark and rags. This event took place around AD 105 and in the coming centuries, paper-making spread westwards. Baghdad had this technique by AD 800 and Europe inherited it after the Crusades (after the 13th century). In the nineteen centuries since Tsai Lun, this invention is yet to be improved upon!

$4\pi R^2$. To obtain these results he had to use the notion of a solid being made up of a large number of extremely small pieces. If only he had used more compact and consistent notations, he would have discovered integral calculus! Another of his contributions was in the development of a technique for the computation of π which was used by several later workers as well (see *Box* 2).

Archimedes could not, unfortunately, end his life in peace. The king of Syracuse, Hieron II, had a treaty of alliance with Rome. After his death, his grandson Hieronymus ruled Syracuse. During his reign Rome suffered a disastrous defeat by Carthage and seemed to be quite lost. Hieronymus, misjudging the situation, switched loyalties to the winning side, Carthage. The Romans did not like this a bit, and once they recovered, they set a fleet commanded by Marcellus, thereby laying siege to Syracuse. This started the strange three-year war between the mighty Roman fleet and virtually a single man – Archimedes. The mechanical

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Box 2. A Piece of Pi

Every ancient civilisation, which built anything of significance, needed to know the length of the perimeter of a circle of a given diameter. They all knew that the ratio between the circumference and the diameter was a constant, roughly around 3. The question was to determine it exactly.

Some civilisations (like the ancient Hebrews) were happy with a value of 3; while others had gone for more accurate values; the Egyptians, for instance, used 22/7 and the Chinese had the value 355/133. It was Archimedes who devised a systematic method, which allowed one to compute the value of π to any desired accuracy.

His idea was to inscribe and superscribe polygons around a circle and measure the perimeters of these polygons. When the number of sides of the polygon increased, the circle got crowded between the polygons (see *Figure* A) and the perimeters of the polygons offered a good approximation to the perimeter of the circle. With tremendous patience, Archimedes used polygons of 96 sides and got for π the value of (3123/994) = 3.14185… which is off the correct value by only 1 part in 12,500!

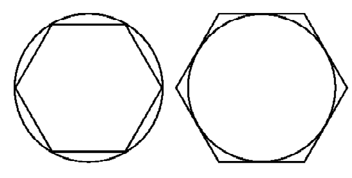

Figure A

inventions (*Figure* 5) Archimedes is supposed to have used in this war probably constituted the first massive application of superior technological knowledge in warfare. He is said to have constructed large mirrors and lenses to set Roman ships on fire and mechanical cranes to lift ships from the sea. (These details come from the description of Marcellus in the works of Plutarch, whose bias in favour of Greeks should not probably be overlooked!) The

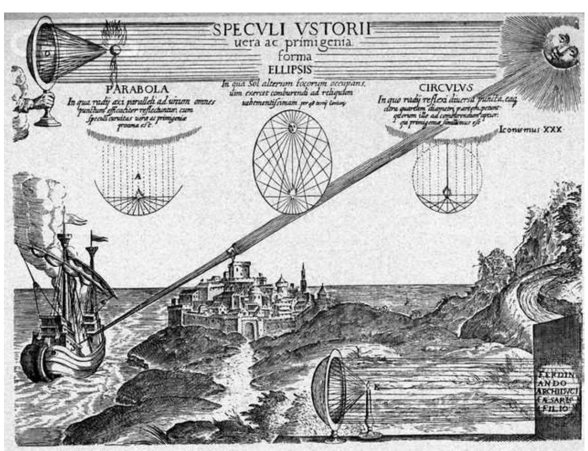

Figure 5. A 17th century engraving shows the arrangement Archimedes is supposed to have used to burn Roman ships.
Courtesy: www.corbisimages.com

Figure 6 (left). Archimedes used his discovery that different objects with the same density and weight displace equal amounts of fluid to test the purity of the gold crown of the King of Syracuse.

Courtesy: http://neatorama.cachefly.net/images/2008-05/archimedes-eureka.jpg

Figure 7 (right). Death of Archimedes at the hands of a Roman soldier (a sixteenth century mosaic)

Courtesy: http://www.livius.org/a/1/greeks/archimedes_circles.JPG

city fell after three years, though, and Archimedes was killed (212 BC) by a Roman soldier – apparently much to the disappointment of Marcellus. It is said that Marcellus arranged a proper funeral with a tombstone as desired by Archimedes.

Our knowledge about Archimedes and his times increased significantly in 2003 when historians of mathematics discovered long lost information in the form of an ancient parchment overwritten by monks nearly a thousand years ago (see [1]). This gives us information, among other things, about a curious puzzle called *stomachion* which involves fairly advanced concepts from combinatorics. The goal of the stomachion is to determine in how many ways a particular set of 14 pieces of varied planar figures can be put together to form a square. In 2003, mathematicians found that the answer is 17152!

Suggested Reading

[1] R Netz and W Noel, *The Archimedes Codex*, Weidenfeld & Nicolson, 2007.

[2] Petr Beckmann, *A History of Pi*, St. Martin's Griffin, 1976.

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