Dawn of Science

3. 'Ishango Bone' to Euclid

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Resonance, Vol.15: p.498, p.590.

Figure 1. Ishango bone.

Keywords
Euclid, Ishango bone, *Sulvasutras*, *Chiu Chang*.

From primitive counting and tally marks, mathematics progressed rather rapidly.

Even a prehistoric tribe would have needed the notion of counting – to make sure that all the cattle came home, or that the tribe outnumbered its enemies. The primitive form of counting involved setting up a correspondence between the objects to be counted and some other convenient set of objects like, for example, the fingers in one's hand. Even today, some primitive African hunters keep count of the number of wild boars they kill by collecting the tusks of each animal, and young girls in the Masai tribe – who live on the slopes of Mt. Kilimanjaro – wear brass rings around their necks in numbers equal to their ages.

This process of counting soon evolved into a more sophisticated process of keeping 'tally marks' on a bone or a stone. One such bone (Ishango bone, *Figure* 1) belonging to the period from 9000 to 6500 BC was found at the fishing village of Ishango (on the shores of Lake Edward in Congo) in 1962. If the markings on this bone are in fact tally marks, this is probably the earliest available record of a mathematical activity.

From such humble beginnings, mathematics progressed rather rapidly. Counting made one realise that 'one cow and one cow make two cows' just as 'one spear and one spear make two spears'. To extract from such a concrete experience the abstract idea that 'one and one make two' was the first major breakthrough in mathematical thought. This idea, which appears so obvious to us today, involves thinking of 'one' and 'two' as independent abstract entities existing on their own. (This abstraction has not been achieved by some tribal societies even today. For example, Fiji tribals distinguish ten boats (called bole) from ten coconuts (koro) and call thousand coconuts by a separate name, saloro!) In practical terms, this idea requires words in a language to describe these numbers as separate nouns. All ancient civilisations – Egyptian, Chinese, Babylonian and Indian developed such verbal descriptions for numbers at some stage in their development. In the early stages, words for numbers often originated from parts of the body, names of fingers, etc., and covered only small numbers. In fact, words originally existed only for 1, 2, rarely for 3 and for 'many'. For example, Egyptian and Chinese writings often identify 3 with many. The Egyptian word for water was just the word for wave repeated three times; in Chinese, 'forest' is '3 trees', 'fur' is '3 hair' and – quite chauvinistically – 'all' is '3 men' and 'gossip' '3 women'!

The written forms of these 'number words' formed the earliest of mathematical notations. Very soon, they were condensed into forms, which were more compact and useful. The most primitive and complicated among them which has, surprisingly enough, survived till today are the Roman numerals: I, V, X, L, C, etc. The most useful, of course, are the Arabic numerals, which we will discuss fully in a later installment. Other civilisations had their own symbols (*Box* 1).

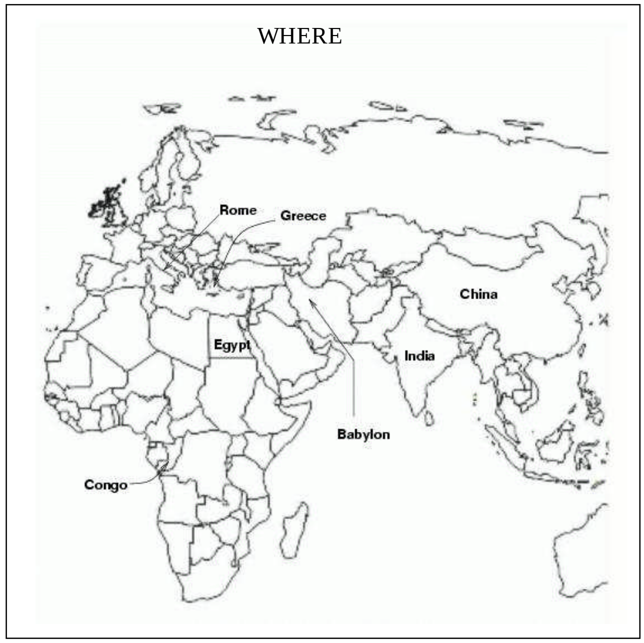

Figure 2.

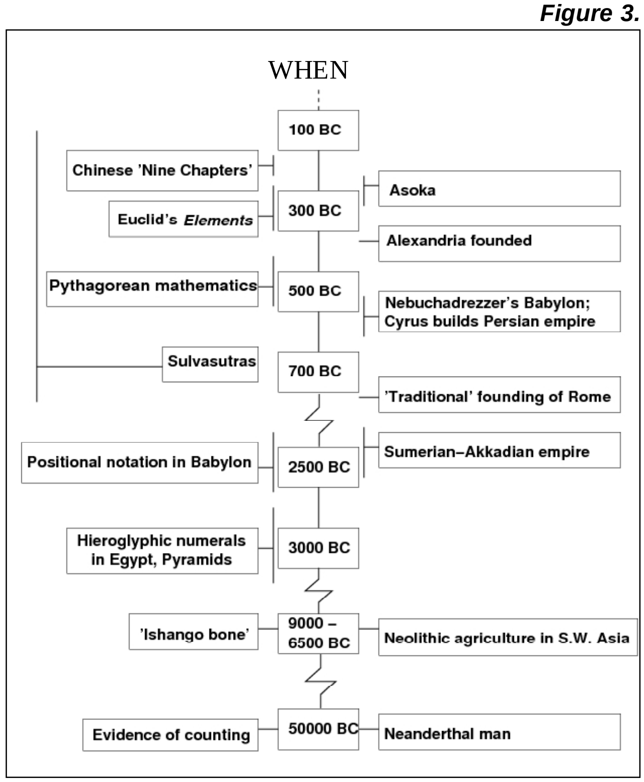

Figure 3.

Box 1. Positional Notation and *Sunya.*

Expressing arbitrarily large numbers using a small set of symbols required great ingenuity, involving three distinct ideas: (i) The positional notation in which the value of a symbol depends on its location, for example, we interpret 23 as '2 tens' and '3 ones' and 32 as '3 tens' and '2 ones', (ii) a convenient choice for the 'base' in the positional notation; we normally use 10 as the base so that 467 will stand for $4 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$, and (iii) a symbol for 'nothing' (0) which allows us to distinguish 203 from 23.

Different civilisations achieved varying degrees of success in this task. The Babylonians, the Egyptians, the Chinese, and the Indians all had the idea of positional notations. The Babylonians developed, as early as 3000–2000 BC, a positional system with a base of 60! However, to avoid having to use separate symbols for 1 and 59, they also used a grouping scheme based on 10. The Indian Kharosti numerals needed the use of three different groupings of 4, 10 and 20. The Chinese, whose language provides a separate pictographic character for each idea, used a notation which completely spells out the value of the number.

The last crucial step, a symbol for nothing, came much later. The earliest known written form of zero occurs in a text inscribed on the wall of a small temple near Gwalior in AD 870. It lists, among the gifts given by the king to the temple, a tract of land "…270 royal *hastas* long and 187 wide, for a flower garden." The *sunya* of India reached the West through the Arabs, becoming zero in the process.

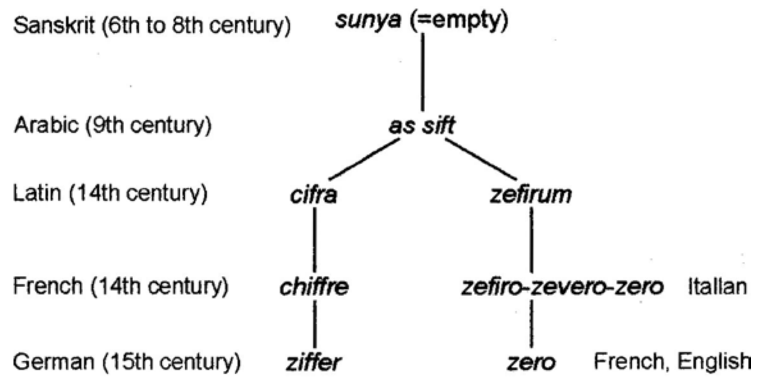

Figure A. The transmutation of sunya to zero through Arabic.

Next to counting, the ancients were preoccupied with sizes and shapes, which naturally led to the development of what we now call geometry. The Greeks, especially Thales and Pythagoras, contributed significantly to the development of geometry. But there is one name, which stands out above the rest and has exerted a lasting influence over the centuries. That was Euclid of Alexandria.

After the death of Alexander in 323 BC, the Macedonian Empire was divided into three, and Egypt came under the rule of Ptolemy

Soter (whose dynasty continued for nearly 250 years ending with Cleopatra). He chose Alexandria as his capital and opened the gates of the University of Alexandria to scholars from all over, making this city the centre of academic activity for centuries. One of the scholars was Euclid[1], the mathematician. Very little is known about his life; he lived around 300 BC, taught for a few years at the university and what is most important, compiled the monumental 13-part treatise, *Elements*. This work, which has dominated teaching of and thinking in geometry for the past two thousand years, definitely earns him a special mention in the annals of science.

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[1] *Resonance*, Vol.12, No.4, 2007.

We do not have a copy of *Elements* from Euclid's own time. All modern editions are based either on a version by Theon of Alexandria (a Greek commentator who lived 700 years after Euclid) or on an anonymous compilation found in the Vatican library. Greek commentaries on Euclid were translated by three Arabic scholars at different times in the Middle Ages (*Figure* 4). From the Arabic, it was translated to Latin; the first Latin translation was made in AD 1120 by Adelard of Bath (who had to travel disguised as a Muslim student in Spain to obtain an Arabic copy!). In all these versions, *Elements* comprises 13 books with a total of 465 theorems.

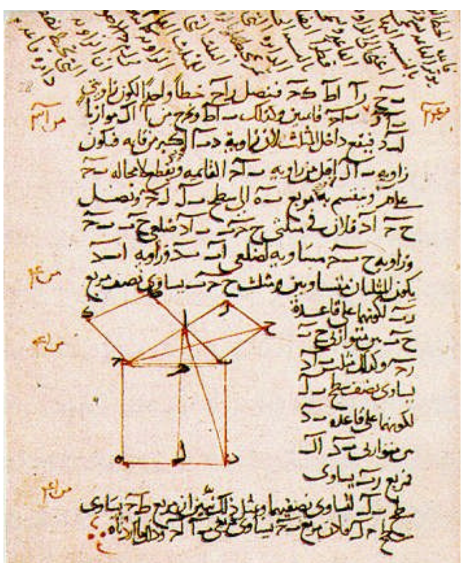

Figure 4. Al-Tusl's Arabic rendering (AD 1258) of Euclid's proof of the Pythagoras theorem.

Box 2. *Sulvasutras* and *Chiu Chang*

There are two other ancient mathematical texts of Oriental origin, which contain several interesting results. These are the Indian treatise called *Sulvasutras* dated by various historians, anywhere from 800 to 100 BC and the Chinese work *Chiu Chang Suan Shu* (*Nine Chapters on the Mathematical Art*) written probably around 250 BC.

The *Sulvasutras* contain, among other things (1) an explicit statement of Pythagoras theorem (in terms of length, breadth and diagonal of a rectangle), (2) several examples of Pythagorian triples – integers (a, b, c) satisfying the relation $a^2 + b^2 = c^2$, and (3) construction for producing a square equal in area to a rectangle (a problem, which incidentally, arises in making a falcon-shaped altar for sacrifices!) What is probably more important than these results is a discussion in *Apastamba Sulvasutra* of the area of a trapezium with bases 24 and 30 units and width 36 units. The text not only calculates the area correctly, but also gives a purely geometrical (Euclid-like) proof for the result!

The Chinese work *Nine Chapters* deals with a gamut of problems in elementary mathematics: operation of fractions, measurement of areas of rectangles, trapezia, triangles (which are done correctly), circle, circle segments and sectors (which are done approximately with pi taken as 3!), volumes of elementary solids (including the frustum of the pyramid), extraction of square and cube roots, and a system of linear equations.

Book 1 begins with the basic axioms and develops the theorems on the congruence of triangles, parallel lines and rectilinear figures. (Theorem 47, for example, is the Pythagoras theorem.) Book 2 deals with the algebraic results arising out of Pythagorean constructions while Book 3 deals with standard results on circles, tangents and secants. Books 4, 5 and 6 discuss geometrical constructions and similarity of figures and the last three books (11, 12 and 13) contain theorems on solid geometry. Probably the most remarkable and the least known volumes are Books 7 to 10. These discuss, not pure geometry, but elementary number theory! They contain some of the most fundamental results in this subject.

Euclid enlarged upon the work by Theudius, Exodus and Hippocrates of Cos – all of whom had contributed to various aspects of geometry and number theory.

This compilation, of course, was based on the earlier works of several people. Before Euclid, a compilation by Theudius was used at Alexandria. Euclid enlarged upon this work, drawing considerably on material developed by Theudius, Exodus and Hippocrates of Cos – all of whom had contributed to various aspects of geometry and number theory. In fact, historians have sometimes cynically commented that Euclid's exposition excelled only in those parts in which he had excellent sources at his disposal.

Even so, *Elements* is a notable achievement. The pattern of logic and order in this treatise was wholly due to Euclid, and this in itself was an outstanding contribution to the evolution of the subject. Euclid might not have been a first-rate mathematician but he was a first-rate teacher of mathematics, for his book remained in use, practically unchanged, for nearly 2,000 years!

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Suggested Reading

[1] Howard Eves, *Great Moments in Mathematics Before* 1650 (Dolciani Mathematical Expositions No 5), Mathematical Association of America, 1983.

[2] Georges Ifrah, *The Universal History of Numbers* - I, Penguin, 2005.